HW Due Fri February 26

Section 2.3

2. Evaluate $\partial z/\partial x$, $\partial z/\partial y$ for given function

b)

$$z = \log\sqrt{1+xy} \; ; \quad (1,2)\ (0,0)$$
$$= \log\left((1+xy)^{1/2}\right)$$
$$= \frac{1}{2}\log(1+xy)$$

$$\frac{\partial z}{\partial x} = \frac{1}{2} \cdot \frac{1}{(1+xy)} \cdot (y) = \frac{y}{2(1+xy)}$$

at $(1,2)$, $\partial z/\partial x = \frac{2}{2(1+2)} = \frac{2}{6} = \frac{1}{3}$

at $(0,0)$, $\partial z/\partial x = 0$

Similarly,

$$\frac{\partial z}{\partial y} = \frac{x}{2(1+xy)}$$

at $(1,2)$ $\partial z/\partial y = 1/6$

at $(0,0)$ $\partial z/\partial y = 0$.

4.

a) $f(x,y) = \frac{2xy}{(x^2+y^2)^2} = 2xy \cdot (x^2+y^2)^{-2}$

$$\frac{\partial f}{\partial x} = 2y \cdot (x^2+y^2)^{-2} + 2xy \cdot -2(x^2+y^2)^{-3} \cdot 2x$$

$$= \frac{2y}{(x^2+y^2)^2} + \frac{-8x^2y}{(x^2+y^2)^3}$$

b) $f(x,y) = \frac{x}{y} + \frac{y}{x} = xy^{-1} + yx^{-1}$

$\frac{\partial f}{\partial x} = \frac{1}{y} + \frac{-y}{x^2} = \frac{x-y^2}{x^2 y}$

continuous for $x \neq 0$ and $y \neq 0$

$\frac{\partial f}{\partial y} = \frac{-x}{y^2} + \frac{1}{x} = \frac{y^2 - x^2}{xy^2}$

continuous for $x \neq 0$ and $y \neq 0$

5. Find equation of plane tangent to the surface

$z = x^2 + y^3$ at $(3,1,10)$

$\partial z/\partial x = 2x = 3(3) = 9 \qquad x_0 = 3,\ y_0 = 1,\ z_0 = f(x_0, y_0) = 10$

$\partial z/\partial y = 3y^2 = 3(1^2) = 3$

$z = 9(x-3) + 3(y-1) + 10.$

7. Compute the matrix of partial derivs.

c. $f: \mathbb{R}^3 \to \mathbb{R}^2 \quad f(x,y,z) = (x + e^z + y,\ yx^2)$

$f_1(x,y,z) = x + e^z + y \qquad f_2(x,y,z) = yx^2$

$$Df(x,y,z) = \begin{array}{cc} & \begin{array}{ccc} \partial f_1/\partial x_1 & \partial f_1/\partial y_1 & \partial f/\partial z \end{array} \\ \begin{array}{c} \\ \partial f_2 \end{array} & \begin{bmatrix} e^z + y & x + e^z & e^z \\ 2xy & x^2 & 0 \end{bmatrix} \end{array}$$

or: $\frac{\partial f_1}{\partial x} = e^z + y \qquad \frac{\partial f_1}{\partial y} = x + e^z \qquad \frac{\partial f_1}{\partial z} = e^z$

$\frac{\partial f_2}{\partial x} = 2xy \qquad \frac{\partial f_2}{\partial y} = x^2 \qquad \frac{\partial f_2}{\partial z} = 0$

Additional notes on the homework problems:

For Problem #4, the important part is figuring out what the domain of the function is. For #4a, the domain is all (x,y) other than (0,0), while for #4b it is all (x,y) such that xy is not zero (in other words, everything but the coordinate axes). You might notice that this is exactly the set you looked at on an earlier homework problem, when you were asked to show this set is open.

The domain of a function is where it is defined. Here are two examples to help understand domains. Consider $f(x) = x(1-x)^2$, where x represents the tax rate and f(x) the revenue the government collects. For most cases, it shouldn't make sense to talk about f(-1) or f(2); tax rates typically are neither negative nor larger than 100%.

For another example, consider the geometric series formula: $g(r) = 1 + r + r^2 + r^3 + \ldots = 1/(1-r)$. If we take r=2, if we use 1/(1-r) we get -1, though everyone should be suspicious of 1 + 2 + 4 + 8 + 16 + ... equalling a negative number!

For functions, it is very important to know where we evaluate them. In physics we frequently talk about point masses and the gravitational or electrical fields they induce. If we have a point mass, it attracts other masses to itself; the closer the other mass, the stronger the attraction. What would be the attraction if our second mass were on top of the first? This should not be allowed to happen; two pieces of matter cannot occupy the same space. Thus, we do not talk about the force when the distance is zero.

These two functions both have natural domains where they make sense. If we take the partial derivatives in the domain where they are defined, everything is fine. These domains are open sets, and the partial derivatives are continuous there.