HW #8

2.3 #12a, #13a 2.4 #1, #15

12) a) Use the linear approx. to approximate a suitable function $f(x,y)$ and thereby estimate the following:

$(0.99\, e^{0.02})^8$

$z = z_0 + [f_x(x_0,y_0)](x-x_0) + [f_y(x_0,y_0)](y-y_0)$

let $z = f(x,y) = (xe^y)^8$, $x_0 = 1$, $x = .99$, $y_0 = 0$, $y = .02$

We compute:

$\frac{\partial f}{\partial x} = 8(xe^y)^7 \cdot e^y$, so $f(1,0) = 8$

$\frac{\partial f}{\partial y} = 8(xe^y)^7 \cdot xe^y$, so $f(1,0) = 8$

so $z \approx 1 + [8](-.01) + [8](.02) = 1.08$

13) a) Compute the gradients of the following functions:

$f(x,y,z) = x\, e^{(-x^2-y^2-z^2)}$

$\nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$

let $u = -x^2-y^2-z^2$

$\frac{\partial u}{\partial x} = -2x$

$\frac{\partial u}{\partial y} = -2y$

$\frac{\partial u}{\partial z} = -2z$

$\frac{\partial f}{\partial x} = x \cdot e^{(-x^2-y^2-z^2)} \cdot (-2x) + e^{(-x^2-y^2-z^2)}$

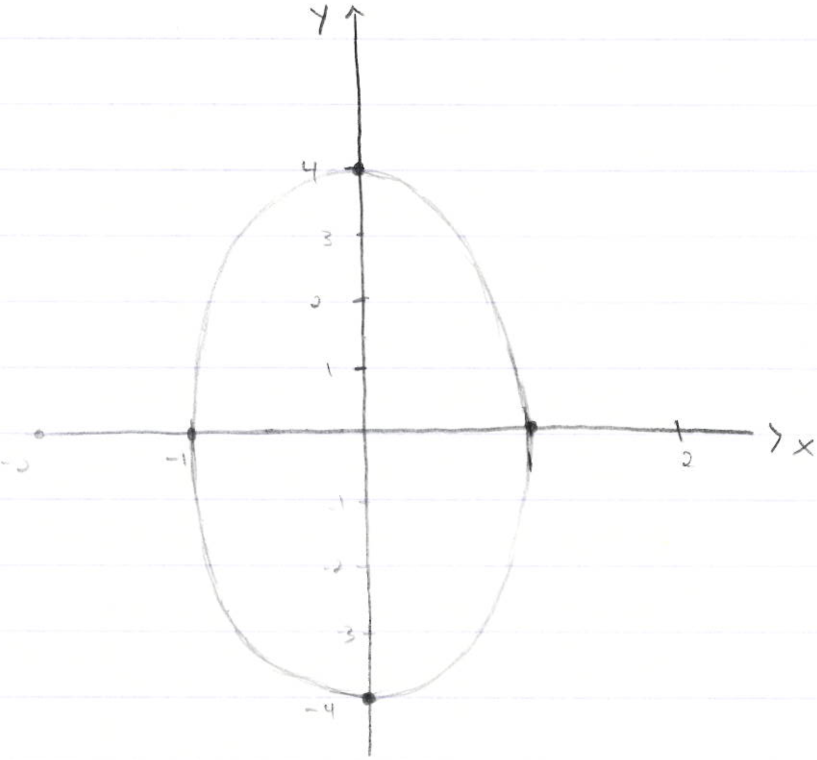
$= e^{(-x^2-y^2-z^2)}(-2x^2+1)$

$\frac{\partial f}{\partial y} = -2y \cdot x \cdot e^{(-x^2-y^2-z^2)}$

$\frac{\partial f}{\partial z} = -2z \cdot x \cdot e^{(-x^2-y^2-z^2)}$

$\nabla f = \left(e^{(-x^2-y^2-z^2)}(-2x^2+1),\ -2xye^{(-x^2-y^2-z^2)},\ -2xze^{(-x^2-y^2-z^2)}\right)$

1) Sketch the curves that are the images of the path $x = \sin(t)$ $y = 4\cos(t)$, where $0 \le t \le 2\pi$



15) Determine the equation of the tangent line to the path $(\sin(3t), \cos(3t), 2t^{5/2})$ when $t=1$

The equation for the tangent line is

$\ell(t) = c(t_0) + (t - t_0)c'(t_0)$. $\quad t_0 = 1$

$\frac{d}{dt} \sin(3t) = 3\cos(3t)$

$\frac{d}{dt} \cos(3t) = -3\sin(3t)$

$\frac{d}{dt} 2t^{5/2} = 5t^{3/2}$

$\ell(t) = c(1) + (t-1)c'(1) = (\sin(3), \cos(3), 2) + (t-1)(3\cos(3), -3\sin(3), 5)$

$= (\sin(3) - 3\cos(3), \cos(3) + 3\sin(3), -3) + t(3\cos(3), -3\sin(3), 5)$