HW 2.5: 2g, 4, 7, 12

2g $f: \mathbb{R}^2 \to \mathbb{R}$, $(x,y) \to x^4 - y^4$

$\frac{\partial f}{\partial x} = 4x^3$

$\frac{\partial f}{\partial y} = -4y^3$

We know that x^4 and y^4 are differentiable by the ~~powerfo~~ power rule, and because we know ~~that~~ each component of f is differentiable, then by the sum rule, f is differentiable.

(this ~~was covered~~ concept was covered in one of your previous homeworks).

$$\left[\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}\right] = \left[4x^3, -4y^3\right]$$

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Section 2.5, #4

```
f[u_, v_] := (u^2 + v^2) / (u^2 - v^2);
U[x_, y_] := Exp[-x - y]
V[x_, y_] := Exp[x y]
```

In[245]:= `f[U[x, y], V[x, y]]`

Out[245]= $$\frac{e^{-2x-2y} + e^{2xy}}{e^{-2x-2y} - e^{2xy}}$$

The above is a formula for f[u[x, y], v[x, y]]. We can forget about the chain rule and just take the partial derivatives by brute force.

We first find the partial of f with respect to x.

In[246]:= `D[f[U[x, y], V[x, y]], x]`

Out[246]= $$-\frac{\left(e^{-2x-2y} + e^{2xy}\right)\left(-2\, e^{-2x-2y} - 2\, e^{2xy}\, y\right)}{\left(e^{-2x-2y} - e^{2xy}\right)^2} + \frac{-2\, e^{-2x-2y} + 2\, e^{2xy}\, y}{e^{-2x-2y} - e^{2xy}}$$

If we wanted it, here is partial f partial y.

In[247]:= `D[f[U[x, y], V[x, y]], y]`

Out[247]= $$-\frac{\left(e^{-2x-2y} + e^{2xy}\right)\left(-2\, e^{-2x-2y} - 2\, e^{2xy}\, x\right)}{\left(e^{-2x-2y} - e^{2xy}\right)^2} + \frac{-2\, e^{-2x-2y} + 2\, e^{2xy}\, x}{e^{-2x-2y} - e^{2xy}}$$

Note your answer may look different,
depending on whether or not, or how, you simplify your work.

7. Let $f: \mathbb{R}^3 \to \mathbb{R}$ and $g: \mathbb{R}^3 \to \mathbb{R}$ be differentiable.

Prove that

$$\nabla(fg) = f\nabla g + g\nabla f$$

So, going from Left to right we take the partials of f and g while respecting the product rule:

$$\nabla(fg) = f\frac{\partial g}{\partial x} + g\frac{\partial f}{\partial x} + f\frac{\partial g}{\partial y} + g\frac{\partial f}{\partial y} + f\frac{\partial g}{\partial z} + g\frac{\partial f}{\partial z}$$

Rearranging

$$= f\underbrace{\left(\frac{\partial g}{\partial x} + \frac{\partial g}{\partial y} + \frac{\partial g}{\partial z}\right)}_{\vec{\nabla} g} + g\underbrace{\left(\frac{\partial f}{\partial x} + \frac{\partial f}{\partial y} + \frac{\partial f}{\partial z}\right)}_{\vec{\nabla} f}$$

$$= f\vec{\nabla} g + g\vec{\nabla} f$$

12. Suppose temp at the point (x,y,z) in Space is $T(x,y,z) = x^2+y^2+z^2$,
let a particle follow the right circular helix
$\sigma(t) = (\cos t, \sin t, t)$ and let
$T(t)$ be its temperature at time t.

a) What is $T'(t)$.

Notice: By $T(t)$, the book means $T(\sigma(t))$ this is an abuse of notation because it should have a different name for each function

Say $Temp(x,y,z) = x^2+y^2+z^2$
$\sigma(t) = (\cos t, \sin t, t)$

So,
$$T(\sigma(t)) = \underbrace{\cos^2 t + \sin^2 t}_{} + t^2$$
$$= 1 + t^2$$
So, $T'(t) = 2t$

b) Find an approx. value for the temp at $t = (\pi/2) + .01$.
We notice that .01 is much smaller that $(\pi/2)$, so an approx value would be.

$T(\pi/2) = \frac{\pi^2}{4} + 1$.