Solutions to

HW # 19

Sect. 5.1 # 1ac, 5.2 # 1b, Page 174 # 7e
Page 90 : 18a or 18b

Sect 5.1

1)
a) $\int_{-1}^{1}\int_{0}^{1}(x^4y+y^2)\,dy\,dx = \int_{-1}^{1} \frac{x^4y^2}{2}+\frac{y^3}{3}\Big|_0^1\,dx$

$= \int_{-1}^{1} \frac{x^4}{2}+\frac{1}{3}\,dx = \frac{x^5}{10}+\frac{x}{3}\Big|_{-1}^{1} = \frac{26}{30} = \frac{13}{15}$

c) $\int_0^1\int_0^1 (xye^{x+y})\,dy\,dx$

Note: for $a,b \in \mathbb{R}$ (a and b are real);

$\int axe^{x+b}\,dx = a(x-1)e^{x+b}$, which gives

$\int_0^1 x(y-1)e^{y+x}\Big|_0^1\,dx = \int_0^1 xe^x\,dx = (x-1)e^x\Big|_0^1$

$= 1$

Sect 5.2 # 1b

$$\iint_R y e^{xy}\, dA = \int_0^1 \int_0^1 y e^{xy}\, dx\, dy$$

Set: $\underline{u = e^{xy}}$

$du = y e^{xy}\, dx$

$$= \int_0^1 \int_1^{e^y} du\, dy = \int_0^1 e^y - 1\, dy = e^y - y \Big|_0^1$$

$= e - 2$

Page 174 # 7e

Find an equation of the tangent plane of $f(x,y) = \sqrt{x^2+y^2}$ at $(x_0, y_0) = (1,1)$

Our definition from Page 133 gives:

$$z = f(x_0,y_0) + \left[\frac{\partial f}{\partial x}(x_0,y_0)\right](x - x_0) + \left[\frac{\partial f}{\partial y}(x_0,y_0)\right](y - y_0)$$

$$\frac{\partial f}{\partial x} = x(x^2+y^2)^{-1/2}, \quad \frac{\partial f}{\partial y} = y(x^2+y^2)^{-1/2}$$

$$z = \sqrt{2} + \frac{\sqrt{2}}{2}(x-1) + \frac{\sqrt{2}}{2}(y-1) = \sqrt{2}\left(1 + \frac{x}{2} - \frac{1}{2} + \frac{y}{2} - \frac{1}{2}\right)$$

$$z = \sqrt{2}\left(\frac{x}{2} + \frac{y}{2}\right) = \frac{\sqrt{2}}{2}(x+y)$$

Page 90 #18a and 18b

a) For this problem, it is sufficient to show that the components of a equal the components of a'. We start with the equality; $\bar{a}\cdot\bar{b} = \bar{a}'\cdot\bar{b}$, but these are both dot products.

$$\bar{a}\cdot\bar{b} = a_1 b_1 + a_2 b_2 + a_3 b_3 + \dots + a_n b_n$$

and similarly:

$$\bar{a}'\cdot\bar{b} = a_1' b_1 + a_2' b_2 + \dots + a_n' b_n$$

Because the b components in every part of the sum are the same, it suffices to say

$$a_1 b_1 = a_1' b_1,\ a_2 b_2 = a_2' b_2, \dots\dots a_n b_n = a_n' b_n$$

and it follows that $a_1 = a_1', a_2 = a_2' \dots a_n = a_n'$ for all components of $\bar{a}$ and $\bar{a}'$. Therefore, $\bar{a}$ and $\bar{a}'$ are the same vector.

b) We take the identity:

$(a+b)\times c = (a\times c) + (b\times c)$

and assume

$a\times c = a'\times c$

Then,

$a\times c - a'\times c = 0$

and

$(a-a')\times c = 0$

if c is parallel to $(a-a')$ then for $c \neq 0$, it is also possible that $(a-a') \neq 0$, so a does not necessarily equal a'.