21) Solve over the unit ball: $\int\int\int_B \frac{dxdydz}{\sqrt{2+x^2+y^2+z^2}}$

Through a change of variables, this can be rewritten as the following:

$$\int_0^{\pi}\int_0^{2\pi}\int_{\rho=0}^{1} \frac{\rho^2}{\sqrt{2+\rho^2}} \sin\phi d\rho d\theta d\phi = \int_0^{\pi} \sin\phi \int_0^{2\pi}\int_{\rho=0}^{1} \frac{\rho^2}{\sqrt{2+\rho^2}} d\rho d\theta d\phi$$

There are two ways to solve for this:

i) Add zero:

$$\int_{\rho=0}^{1} \frac{\rho^2}{\sqrt{2+\rho^2}} d\rho = \int_{\rho=0}^{1} \frac{\rho^2+2-2}{\sqrt{2+\rho^2}} d\rho$$

$$= \int_{\rho=0}^{1} \frac{\rho^2+2}{\sqrt{2+\rho^2}} - \frac{2}{\sqrt{2+\rho^2}} d\rho$$

$$= \int_{\rho=0}^{1} \sqrt{2+\rho^2} d\rho - 2\int_{\rho=0}^{1} \frac{1}{\sqrt{2+\rho^2}} d\rho$$

These two integrals can be found in the table in the book, #43 and #36 respectively.

This gives you

$$\frac{\rho}{2}\sqrt{\rho^2+2} + \log(\rho+\sqrt{\rho^2+2}) - 2\log(\rho+\sqrt{\rho^2+2})|_0^1$$

$$= \frac{1}{2}\sqrt{3} - \log(1+\sqrt{3}) + \log(\sqrt{2})$$

ii) Integrate by parts: let $u = \rho$ and $du = 1d\rho$, $dv = (2+\rho^2)^{-\frac{1}{2}}\rho d\rho$, and $v = (2+\rho^2)^{\frac{1}{2}}$.

$$\int_{\rho=0}^{1} \frac{\rho^2}{\sqrt{2+\rho^2}} d\rho = \int_{\rho=0}^{1} \frac{\rho}{\sqrt{2+\rho^2}} \rho d\rho$$

$$= \int_{\rho=0}^{1} \frac{\rho^2}{\sqrt{2+\rho^2}} d\rho = uv|_0^1 - \int_0^1 vdu$$

$$= \rho(2+\rho^2)^{\frac{1}{2}}|_0^1 - \int_0^1 \sqrt{2+\rho^2} d\rho$$

This new integral is in the table in the book, #43. This gives you:

$$\sqrt{3} - (\frac{\rho}{2}\sqrt{\rho^2+2} + \log(\rho+\sqrt{\rho^2+2}))|_0^1$$

$$\sqrt{3} - (\frac{1}{2}\sqrt{3} + \log(1+\sqrt{3}) - \log(\sqrt{2}))$$

Using this, we then get

$$\int_0^{\pi}\int_0^{2\pi}\int_{\rho=0}^{1}\frac{\rho^2}{\sqrt{2+\rho^2}}\sin\phi d\rho d\theta d\phi = \int_0^{\pi}\sin\phi\int_0^{2\pi}\int_{\rho=0}^{1}\frac{\rho^2}{\sqrt{2+\rho^2}}d\rho d\theta d\phi$$

$$= \int_{\phi=0}^{\pi}\sin\phi\int_{\theta=0}^{2\pi}\frac{1}{2}\sqrt{3}-\log(1+\sqrt{3})+\log(\sqrt{2})d\theta d\phi$$

$$= \int_{\phi=0}^{\pi}(\sin\phi\frac{1}{2}\sqrt{3}-\log(1+\sqrt{3})+\log(\sqrt{2}))\theta|_{\theta=0}^{2\pi}d\phi$$

$$= \int_{\phi=0}^{\pi}2\pi(\sin\phi\frac{1}{2}\sqrt{3}-\log(1+\sqrt{3})+\log(\sqrt{2}))d\phi$$

$$= -\cos\phi 2\pi(\frac{1}{2}\sqrt{3}-\log(1+\sqrt{3})+\log(\sqrt{2}))|_{\phi=0}^{\pi}$$

$$= 2\pi(\sqrt{3}-2\log(1+\sqrt{3})+2\log(\sqrt{2}))$$

For problems 5-7, Find the limit of the sequence or explain why it does not converge.

5) $\frac{3n^2+2n-7}{n^2}$ By applying the limit for this sequence, we get $\lim_{n\to\infty}\frac{3n^2+2n-7}{n^2}=3$

6) $\frac{5n^3-n^2+7n+2}{3n^3+n^2-n+10}$ By applying the limit for this sequence, we get $\lim_{n\to\infty}\frac{5n^3-n^2+7n+2}{3n^3+n^2-n+10}=\frac{5}{3}$

7) $\frac{log(n)}{n}$ By applying the limit for this sequence, we get $\lim_{n\to\infty}\frac{log(n)}{n}=0$

8) Find the limit of the following series, or explain why it does not converge.

$$\sum_{k=0}^{n}(\frac{1}{3^k}) = (1+\frac{1}{3}+\frac{1}{9}+\frac{1}{21}+\cdots+\frac{1}{3^n})$$

Let $S_n=(1+\frac{1}{3}+\frac{1}{9}+\frac{1}{21}+\cdots+\frac{1}{3^n})$. Then:

$$\frac{1}{3}S_n=(\frac{1}{3}+\frac{1}{9}+\frac{1}{21}+\cdots+\frac{1}{3^n}+\frac{1}{3^{n+1}})$$

So,

$$\frac{2}{3}S_n=S_n-\frac{1}{3}S_n=1-\frac{1}{3^{n+1}}$$

$$\lim_{n\to\infty}\frac{2}{3}S_n=1$$

So

$$\lim n\to\infty S_n=\frac{3}{2}$$