Math 105: Secs 1 and 2: Writing up Problems

#13b) Compute the gradient of

$$f(x,y,z) = \frac{xyz}{x^2+y^2+z^2}$$

Solution: We have $\text{grad}(f) = \nabla f = \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right)$

Now $\frac{\partial f}{\partial x}$ is the quotient rule: keep y and z fixed and vary x. Thus

$$\frac{\partial f}{\partial x} = \frac{\left[\frac{\partial}{\partial x}(xyz)\right](x^2+y^2+z^2) - xyz\,\frac{\partial}{\partial x}(x^2+y^2+z^2)}{(x^2+y^2+z^2)^2}$$

$$\left.\begin{aligned} &= \frac{yz(x^2+y^2+z^2) - xyz(2x)}{(x^2+y^2+z^2)^2} \\ &= \frac{yz(y^2+z^2-x^2)}{(x^2+y^2+z^2)^2} \end{aligned}\right\} \text{both answers are fine}$$

Similarly compute $\partial f/\partial y$ and $\partial f/\partial z$

$$\nabla f = \left(\frac{yz(y^2+z^2-x^2)}{(x^2+y^2+z^2)^2}, \frac{xz(x^2+z^2-y^2)}{(x^2+y^2+z^2)^2}, \frac{xy(x^2+y^2-z^2)}{(x^2+y^2+z^2)^2}\right)$$

Section 2.4) #2) Sketch $x = 2\sin t,\ y = 4\cos t$ when $0 \le t \le 2\pi$

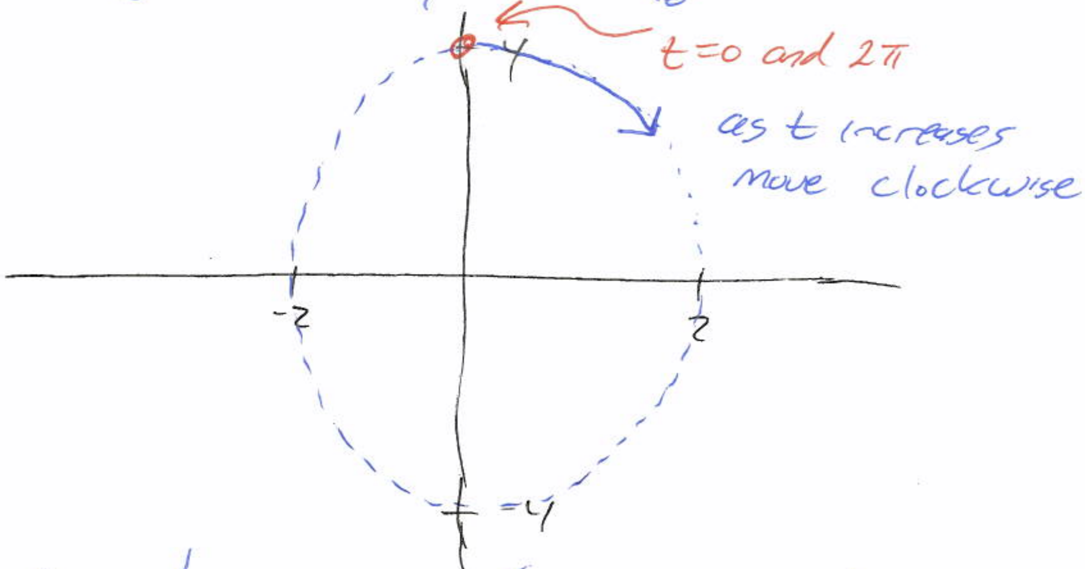
Soln: So $c(t) = (x(t), y(t))$

$$= (2\sin t, 4\cos t)$$

We note this is the equation of an ellipse:

$$\left(\frac{x(t)}{2}\right)^2 + \left(\frac{y(t)}{4}\right)^2 = 1$$

as this is just $\frac{4\sin^2 t}{4} + \frac{16\cos^2 t}{16} = 1$ ✓



When $t=0$ have $c(0) = (2\sin 0, 4\cos 0) = (0,4)$

$t = 2\pi$ have $c(2\pi) = (2\sin 2\pi, 4\cos 2\pi) = 0,4$

Note if t is small and positive, $c(t)$ is in the first quadrant - moving clockwise