MATH 105 SOLUTION KEYS

MURAT KOLOGLU

HOMEWORK 5

Section 2.1.

Exercise (1). Sketch the level curves and graphs of the following functions:

(a) $f : \mathbb{R}^2 \to \mathbb{R}, (x, y) \mapsto x - y + 2$. The level set of value c is the set of all x and y such that $f(x, y) = c$. For this function, this means that $x - y + 2 = c$, which we may rewrite as $y = x + 2 - c$. This is the equation of a line with slope -1 and y-intercept $2 - c$. We plot some level sets in Figure 1, and graph of the function in Figure 2.

Date: March 7, 2010.

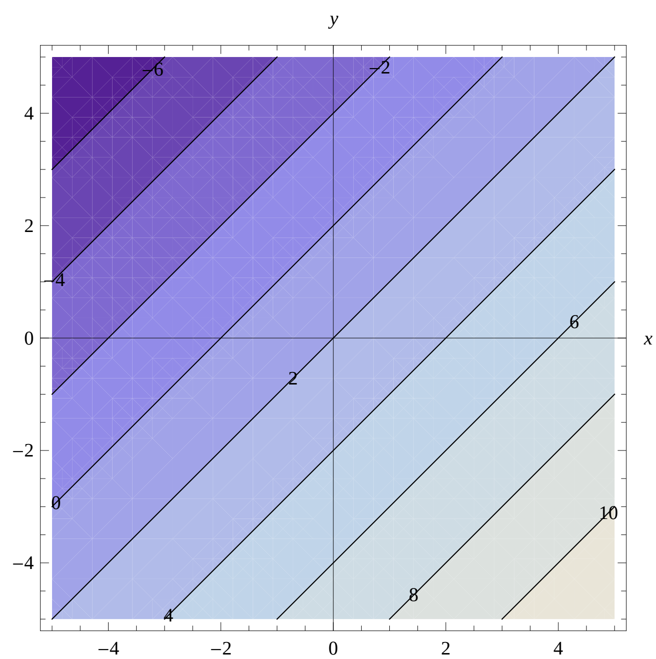

FIGURE 1. Level curves of f(x,y)=x-y+2.

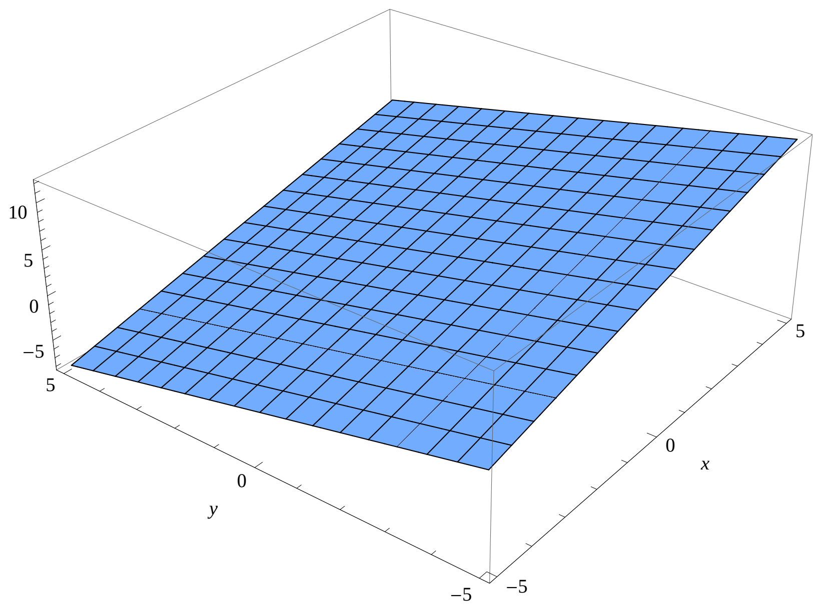

FIGURE 2. Plot of f(x,y)=x-y+2.

(c) $f : \mathbb{R}^2 \to \mathbb{R}, (x, y) \mapsto -xy$. Once again, the level set of value c is the set of all x and y such that $f(x, y) = c$. For this function this means that $-xy = c$, which we may write as $y = -\frac{c}{x}$. For $c = 0$ we have $x = 0$ or $y = 0$, which means the level curve of $c = 0$ is the x- and y-axes. For $c \neq 0$ this is a collection of curves that go to 0 as the function approaches $\pm\infty$ and go to ∞ and $-\infty$ as the function approaches 0 from the right and the left. We plot some level sets in Figure 3.

The function itself resembles a saddle. For $c > 0$, the function has negative values in the first and third quadrants of the xy plane. Conversely, for $c < 0$, the function has positive values in the second and fourth quadrants of the xy plane. We plot the graph of the function in Figure 4.

Exercise (24). Sketch or describe the surfaces in $\mathbb{R}^3$ *of the equations presented:*

$$\frac{y^2}{9} + \frac{z^2}{4} = 1 + \frac{x^2}{16}$$

The best way to start graphing a surface is to make certain critical observations. For example, when $z = 0$, the function is living in the xy plane.

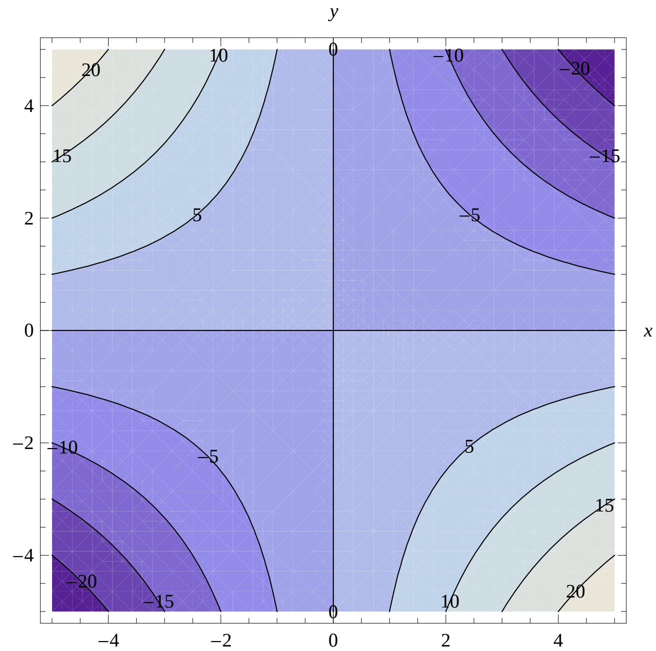

FIGURE 3. Level curves of f(x,y)=-xy.

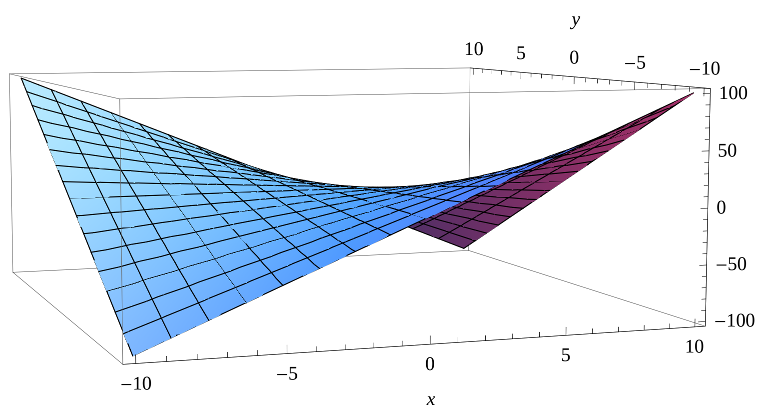

FIGURE 4. Plot of f(x,y)=-xy.

For $z = 0$ the equation takes the form

$$\begin{aligned} \frac{y^2}{9} &= 1 + \tfrac{x^2}{16} \\ y &= \pm 3\sqrt{1 + \tfrac{x^2}{16}} \end{aligned}$$

which describes two hyperbolas. These hyperbolas are distinctly visible in Figure 6. If we were to let $z = c$ and think of this as a question of level sets

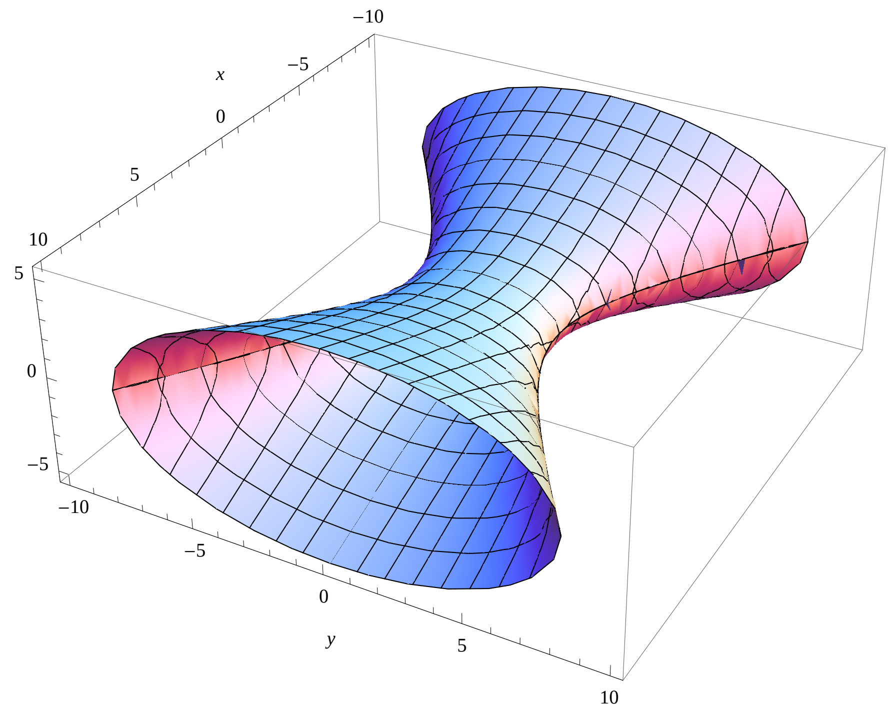

FIGURE 5. The surface $\frac{y^2}{9} + \frac{z^2}{4} = 1 + \frac{x^2}{16}$.

we would have

$$y = \pm 3\sqrt{1 + \frac{x^2}{16} - \frac{c^2}{4}}$$

which still describes hyperbolas, that get less 'curvier' and come closer to the xz plane as $|c|$ gets larger than 0.

Now that we have the xy plane handled, let's repeat the process for the xz and yz planes. When $y = 0$ we are in the xz plane with the equation

$$z = \pm 2\sqrt{1 + \frac{x^2}{16}}$$

Once again this describes two hyperbolas. By now we have the skeleton of our surface, it is between four hyperbolas, all extending along the x direction. To fill in the rest, let $x = c$. Then we have

$$\frac{y^2}{9} + \frac{z^2}{4} = 1 + \frac{c^2}{16}$$

This describes a collection of ellipses. So the hyperbolas are connected smoothly with ellipses. The graph of the surface can be seen in Figure 5 and in Figure 6.

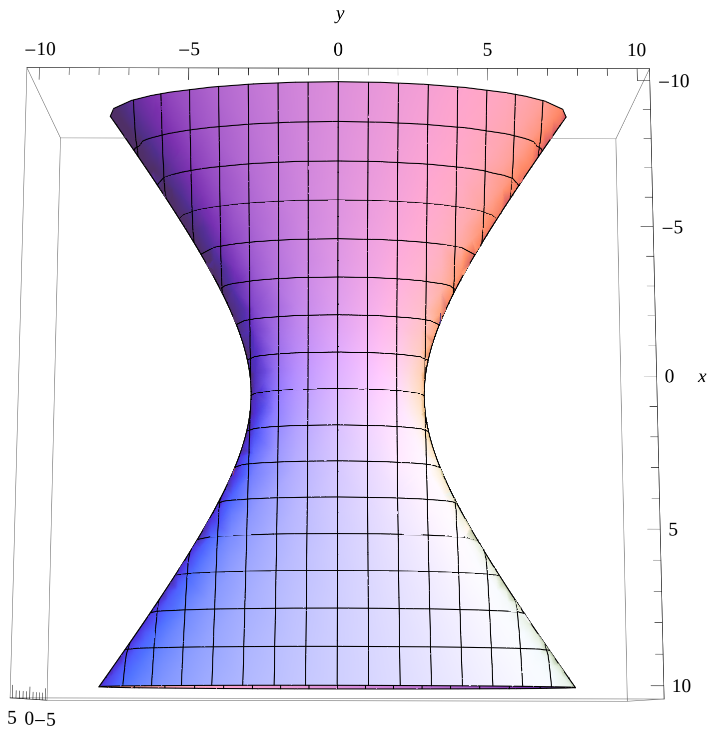

FIGURE 6. Top view of the surface $\frac{y^2}{9} + \frac{z^2}{4} = 1 + \frac{x^2}{16}$. Notice how the $z = 0$ hyperbolas are visible on the edges.