5.4| 1b.)

First, we recall that $\cos^2\theta = (1+\cos 2\theta)/2$ and compute the integral as written:

$$\int_0^{\pi/2}\int_0^{\cos\theta}\cos\theta\,dr\,d\theta = \int_0^{\pi/2}\left(r\cos\theta\Big]_{r=0}^{\cos\theta}\right)d\theta$$

$$= \int_0^{\pi/2}\cos^2\theta\,d\theta = \int_0^{\pi/2}\left(1+\frac{\cos 2\theta}{2}\right)d\theta$$

$$= \frac{\theta}{2} + \frac{\sin 2\theta}{4}\Big]_0^{\pi/2} = \pi/4$$

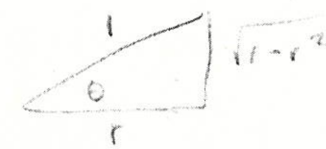
* θ extends from 0 to $\cos^{-1}(r_0)$, so the region can be described as

$$0 \le r \le 1 \quad \text{and} \quad 0 \le \theta \le \cos^{-1}(r)$$

and changing the order of int. gives

$$\int_0^1\int_0^{\cos^{-1}(r)}\cos\theta\,d\theta\,dr = \int_0^1\left(\sin\theta\Big]_{\theta=0}^{\cos^{-1}(r)}\right)dr$$

$$= \int_0^1 \sin(\underbrace{\cos^{-1}(r)}_{\text{same as arccos}})\,dr$$



$\sin(\cos^{-1}(r)) = \sqrt{1-r^2}$

$$\int_0^1 \underbrace{\sqrt{1-r^2}}\,dr = \frac{\pi}{4}$$

can use eq. 38 in book (identities page) to evaluate this integral

5.4) #4

Show that $\frac{1}{2}(1-\cos 1) \leq \iint_{[0,1]\times[0,1]} \frac{\sin x}{1+(xy)^4}\,dx\,dy \leq 1$

.54

* Note that sinx is nonnegative in this region.

when $0 \leq x \leq 1$, $0 \leq y \leq 1$, ~~$1 \geq \frac{1}{1+(xy)^4} \geq \frac{1}{2}$~~ $\quad 1 \geq \frac{1}{1+(xy)^4} \geq \frac{1}{2}$

Estimate with $\frac{1}{1+(xy)^4} = \frac{1}{2}$

So, the lower bound is

$$\frac{1}{2}\int_0^1\int_0^1 \sin x \; dy\,dx$$

inside: $\quad y\sin x \Big]_0^1 = 1 \sin x$

outside: $\quad \int_0^1 \sin x \, dx = -\cos x \Big]_0^1$

$$= 1 - \cos 1.$$

So the lower bound is

$$\frac{1-\cos 1}{2}.$$

Page 364; #14.

Evaluate the integral,

$$\int_0^1 \int_0^x \int_0^y (y + xz)\, dz\, dy\, dx$$

<u>inside</u>: $\displaystyle\int_0^y y + xz \, dz$

$\frac{1}{n+1} z^{n+1}$

$$= yz + \frac{x z^2}{2} \Big]_0^y = y^2 + \frac{x y^2}{2}$$

<u>middle</u> $\displaystyle\int_0^x y^2 + \frac{1}{2} x y^2 \, dy$

$$= \frac{y^3}{3} + \frac{1}{2} x \frac{y^3}{3} \Big]_0^x = \frac{1}{3} x^3 + \frac{1}{6} x^4$$

<u>outside</u>: $\displaystyle\int_0^1 \frac{1}{3} x^3 + \frac{1}{6} x^4 \, dx$

$$= \frac{1}{3} \frac{x^4}{4} + \frac{1}{6} \frac{x^5}{5} \Big]_0^1 = \frac{1}{12}(1)^4 + \frac{1}{30}(1)^5 = \frac{1}{12} + \frac{1}{30}$$