MATH 105: PRACTICE PROBLEMS FOR CHAPTER 1 AND CALCULUS REVIEW: SPRING 2010

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Question 1 : These problems deal with equations of lines.

(1) Find the equation of the line going through the points (2,3) and (4,9).

(2) Find the equation of the line going through the points (2,3) and (-1,2).

(3) Find the equation of the line going through the point (2,3) with slope 3.

(4) Find the equation of the line going through the points (2,3,4) and (4,9,16).

(5) Find the equation of the line going through the point (2,3,4) in the direction (2,6,12).

(6) Is the point (4,19,26) on the line going through the point (2,3,4) in the direction (2,6,12)?

(7) Consider the lines in part (1) and part (2). Find all points on both lines.

Question 2 : These equations deal with vectors. For all problems below, let $\overrightarrow{P} = (1,2,3)$, $\overrightarrow{Q} = (4,9,6)$, $\overrightarrow{R} = (3,3,3)$, $\overrightarrow{v} = (3,7,3)$ and $\overrightarrow{w} = (2,1,0)$.

(1) Find $\overrightarrow{P} + \overrightarrow{R}$, $4\overrightarrow{P} - 3\overrightarrow{Q} + 2\overrightarrow{R}$, $(\overrightarrow{P} + 2\overrightarrow{Q}) \cdot \overrightarrow{R}$, and $(\overrightarrow{P} \times \overrightarrow{Q}) \times \overrightarrow{R}$.

(2) Find the plane containing $\overrightarrow{P}$ with two directions $\overrightarrow{v}$ and $\overrightarrow{w}$.

(3) Find the equation of the plane containing the vectors $\overrightarrow{P}$, $\overrightarrow{Q}$ and $\overrightarrow{R}$.

(4) Find the equation of the plane containing the point $\overrightarrow{P}$ whose normal is in the direction $(-3,6,-11)$.

(5) Find the equation of the plane containing $\overrightarrow{Q}$ with two directions $\overrightarrow{v}$ and $\overrightarrow{w}$.

(6) Find the area of the parallelogram, two of whose sides are $\overrightarrow{v}$ and $\overrightarrow{w}$.

(7) Find the cosine of the angle between $\overrightarrow{v}$ and $\overrightarrow{w}$.

Date: February 12, 2010.

(8) Find the length of $\overrightarrow{v}$, and find a vector of unit length in the same direction as $\overrightarrow{v}$.

(9) Find a vector perpendicular to both $\overrightarrow{v}$ and $\overrightarrow{w}$.

(10) Find a vector perpendicular to $\overrightarrow{v}$.

Question 3 : State the following results.

(1) The Pythagorean Formula.

(2) The Law of Cosines.

(3) The formula for the cosine of the angle between two vectors $\overrightarrow{P}$ and $\overrightarrow{Q}$.

(4) The formulas for the determinant of a 2×2 matrix A and a 3×3 matrix B, where

$$A = \begin{pmatrix} a & b \\ c & d \end{pmatrix}, \quad B = \begin{pmatrix} a & b & c \\ d & e & f \\ g & h & i \end{pmatrix}.$$

(5) Give a reason why we care about determinants.

(6) Give the formula for the cross product of two vectors; specifically, what is the cross product $\overrightarrow{v} \times \overrightarrow{w}$, where $\overrightarrow{v} = (v_1, v_2, v_3)$ and $\overrightarrow{w} = (w_1, w_2, w_3)$. Give three properties of the cross product.

(7) Give the formula for the inner (or dot) product of two vectors; specifically, what is $\overrightarrow{v} \cdot \overrightarrow{w}$ where $\overrightarrow{v} = (v_1, \dots, v_n)$ and $\overrightarrow{w} = (w_1, \dots, w_n)$. Give three properties of the inner product.

(8) Explain what the phrase *right hand screw rule* means, and why it is useful.

(9) Prove the triple product formula; specifically, if $\overrightarrow{A} = (a_1, a_2, a_3)$, $\overrightarrow{B} = (b_1, b_2, b_3)$ and $\overrightarrow{C} = (c_1, c_2, c_3)$ then

$$(\overrightarrow{A} \times \overrightarrow{B}) \cdot \overrightarrow{C} = \begin{vmatrix} a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \\ c_1 & c_2 & c_3 \end{vmatrix}.$$

Question 4 : (**Calculus Review**) Find the maximum and minimum values for $f(x) = \frac{1}{3}x^3 - 9x^2 + 80x + 1$ when $-20 \le x \le 40$. Use the first and second derivative tests to classify the local maximum and minimums, and sketch the curve.

Question 5 : (**Calculus Review**) Consider all rectangles with perimeter 100. Find the rectangle with largest area.

Question 6 : State the fundamental theorem of calculus (FTC). (1) Use the FTC to calculate the area under the curve $f(x) = x^2 + 2x + 1$ from $x = 1$ to $x = 4$; (2) use the FTC to calculate the area under the curve of $f(x) = \sin(x)$ from $x = 0$ to $x = \pi/2$. Note we may denote these areas by $\int_1^4 (x^2 + 2x + 1)dx$ and $\int_0^{\pi/2} \sin(x)dx$.

Question 7 : Find *all* the anti-derivatives of the following: (1) x^4; (2) $x^4 + 3x^5$; (3) $(x+6)^8$; (4) $(x^3 + 4x^2 + 1)^7 \cdot (3x^2 + 8x)$; (5) $\sin(x) - \cos(x) + e^x$.

Question 8 : State L'Hopital's rule. Determine

$$\lim_{x\to 0} \frac{\sin(x)}{x}, \quad \lim_{x\to 0} \frac{\sin(x)\cos(x) - x}{x^2}, \quad \lim_{x\to 2} \frac{x^2 - 4}{(x-2)\sin(x)}.$$