

NAME:

Section:

Answer to Question 1:

Answer to Question 2:

Answer to Question 3:

Math 105: Quiz 1: Due Wednesday, February 8

Remember, just hand in the answers.

Instructions: The quiz is due by the start of class on Wednesday, and should be handed in on a separate sheet of paper than the HW. Make sure you include your name, section (section 1 or 2, or you can write 9am or 10am). Remember, the quizzes only count if at the end of the semester it helps your average. I will drop your lowest quiz grade. **You are on the honor system to work on the quiz by yourself, without consulting any material (ie, it is a closed book quiz).** There is no partial credit for quizzes; the answer is either right or wrong. As such, please only hand in your answers (if you hand in your work, please box your answer). I will provide solution keys to each quiz; if afterwards you are not sure if your logic was correct, I and the TAs are of course happy to look at your quiz in greater detail.

Question 1: Give the equation for the line going through the points $(1,2,3)$ and $(3,2,1)$.

Question 2: Give the equation for the line going through the points $(1,2,3)$ and $(2,4,6)$.

Question 3: Let v be a unit vector in the direction of the line in Question 1, and let w be a unit vector in the direction of the line in Question 2. Find the cosine of the angle between v and w .

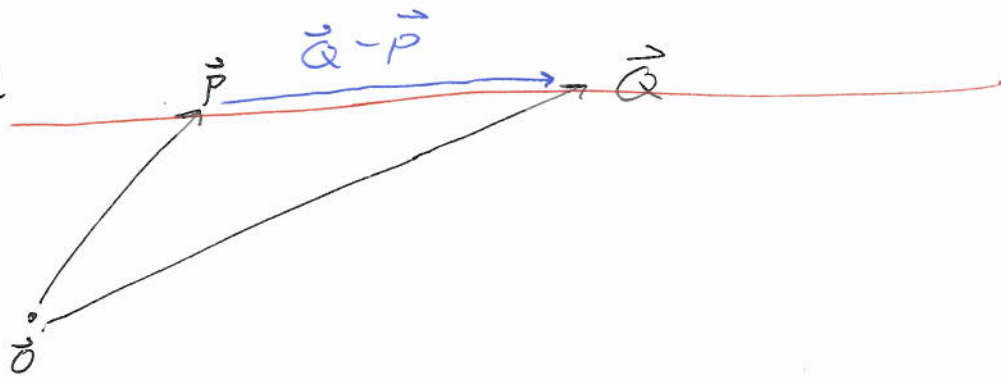
No time limit for
The first quiz

MATH 105: Quiz 1:

#1) Find eq of line ~~from~~ going through

$$\vec{P} = (1, 2, 3) \text{ and } \vec{Q} = (3, 2, 1)$$

SOLN:



$$\begin{aligned} \text{Direction of line is } \vec{Q} - \vec{P} &= (3, 2, 1) - (1, 2, 3) \\ &= (2, 0, -2) \end{aligned}$$

Eq of line ~~with~~ going through point $\vec{P}$ and in direction $\vec{v}$ is $(x, y, z) = \vec{P} + t\vec{v}$

$$\text{Thus have } (x, y, z) = (1, 2, 3) + t(2, 0, -2)$$

$$\text{or } (x, y, z) = (1+2t, 2, 3-2t)$$

$$\text{Ans! } (x, y, z) = (1+2t, 2, 3-2t)$$

MATH 105: Quiz #1

#2) Find eq of line going through $(1, 2, 3)$ and $(2, 4, 5)$

Soln: Direction is: $\vec{Q} = (2, 4, 6)$
 $\vec{P} = (1, 2, 3)$

$$\text{so dir is } \vec{Q} - \vec{P} = (2, 4, 6) - (1, 2, 3) \\ \vec{w} = (1, 2, 3)$$

Eq of line is

$$\begin{aligned} (x, y, z) &= \vec{P} + s \vec{w} \\ &= (1, 2, 3) + s(1, 2, 3) \\ &= 1+s, 2+2s, 3+3s \end{aligned}$$

$$\text{Ans: } (x, y, z) = (1+s, 2+2s, 3+3s)$$

MATH 105: QUIZ 1

#3) $\vec{v}$ unit vector in direction of $(2, 0, -2)$
and $\vec{w}$ unit vector in direction of $(1, 2, 3)$,
find cosine of angle between them.

SOLN 1: $\vec{A} = (2, 0, -2)$ so $\|\vec{A}\| = \sqrt{2^2 + 0^2 + (-2)^2} = \sqrt{8} = 2\sqrt{2}$

$$\vec{B} = (1, 2, 3) \text{ so } \|\vec{B}\| = \sqrt{1^2 + 2^2 + 3^2} = \sqrt{14}$$

$$\text{Thus } \vec{v} = \frac{\vec{A}}{2\sqrt{2}} = \left(\frac{2}{2\sqrt{2}}, 0, \frac{-2}{2\sqrt{2}} \right) = \left(\frac{1}{\sqrt{2}}, 0, -\frac{1}{\sqrt{2}} \right)$$

$$\vec{w} = \frac{\vec{B}}{\sqrt{14}} = \left(\frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right)$$

$$\text{Now } \vec{v} \cdot \vec{w} = \|\vec{v}\| \|\vec{w}\| \cos \theta \text{ so } \cos \theta = \frac{\vec{v} \cdot \vec{w}}{\|\vec{v}\| \|\vec{w}\|}$$

$$\text{As } \vec{v} \cdot \vec{w} = \frac{1}{\sqrt{2}} \frac{1}{\sqrt{14}} + 0 \cdot \frac{2}{\sqrt{14}} - \frac{1}{\sqrt{2}} \frac{3}{\sqrt{14}} = \frac{1}{2\sqrt{7}} - \frac{3}{2\sqrt{7}} = -\frac{1}{\sqrt{7}} = \frac{\sqrt{7}}{7}$$

$$\text{we have } \cos \theta = \frac{-\sqrt{7}/7}{1 \cdot 1} = -\frac{\sqrt{7}}{7}$$

SOLN 2: KEY POINT: ANGLE b/w $\vec{v}$ and $\vec{w}$ same as $\vec{A}$ and $\vec{B}$

$$\|\vec{A}\| \cdot \|\vec{B}\| \cos \theta = \vec{A} \cdot \vec{B} \Rightarrow \cos \theta = \frac{\vec{A} \cdot \vec{B}}{\|\vec{A}\| \|\vec{B}\|}$$

$$\text{As } \vec{A} \cdot \vec{B} = 2 \cdot 1 + 0 \cdot 2 - 2 \cdot 3 = -4$$

$$\text{Thus } \cos \theta = \frac{-4}{2\sqrt{2} \cdot \sqrt{14}} = \frac{-4}{4\sqrt{7}} = -\frac{1}{\sqrt{7}} = -\frac{\sqrt{7}}{7}$$