MATH 105: QUIZ 4

INSTRUCTOR: STEVEN MILLER (SJM1@WILLIAMS.EDU)

NOTE: Write your name *and* section number. Each question is worth 10 points. You have 30 minutes to do the quiz, which is closed book. Do not use computers or any other devices.

Question 1: Write down the 3×2 matrix (three rows, two columns) where the entry in the i^{th} row and j^{th} column is the product of i and j. If this matrix acts on the column vector $(3,4)^T$ (for typographical reasons it is written as a row vector, with the T standing for transpose), what is the result?

Solution: The matrix is

$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{pmatrix}.$$

Acting on the column vector $(3,4)^T$ gives

$$\begin{pmatrix} 1 & 2 \\ 2 & 4 \\ 3 & 6 \end{pmatrix} \begin{pmatrix} 3 \\ 4 \end{pmatrix} = \begin{pmatrix} 1\cdot 3 + 2\cdot 4 \\ 2\cdot 3 + 4\cdot 4 \\ 3\cdot 3 + 6\cdot 4 \end{pmatrix} = \begin{pmatrix} 11 \\ 22 \\ 33 \end{pmatrix}.$$

Question 2: Find the tangent plane to $f(x,y,z) = x^2y^2z^2 + xyz$ at the point $(1,1,1)$.

Solution: We have

$$(Df)(x,y,z) \;=\; (\nabla f)(x,y,z) \;=\; \left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right).$$

We have

$$\frac{\partial f}{\partial x} \;=\; 2xy^2z^2 + yz, \quad \frac{\partial f}{\partial y} \;=\; 2yz^2x^2 + zx, \quad \frac{\partial f}{\partial z} \;=\; 2zx^2y^2 + xy.$$

Collecting gives

$$\left(\frac{\partial f}{\partial x}, \frac{\partial f}{\partial y}, \frac{\partial f}{\partial z}\right) \;=\; \left(2xy^2z^2 + yz, 2yz^2x^2 + zx, 2zx^2y^2 + xy\right).$$

Date: March 4, 2010.

The language is in error for this problem; I meant to ask for the tangent *hyperplane*. We have

$$u \ = \ f(x,y,z)$$

leads to

$$\begin{aligned} u &= f(1,1,1) + \frac{\partial f}{\partial x}(1,1,1)(x-1) + \frac{\partial f}{\partial y}(1,1,1)(y-1) + \frac{\partial f}{\partial y}(1,1,1)(z-1) \\ &= 2 + 3(x-1) + 3(y-1) + 3(z-1). \end{aligned}$$

Question 3: State a condition on a function f and its partial derivatives that implies f is differentiable. Give an example of a function satisfying these conditions. Do not hand in the following, but you should be able to discuss the main ingredients in the proof and how they are used.

Solution: If the partial derivatives exist and are continuous, then the function is differentiable.