"If you look up 'Intelligence' in the new volumes of the Encyclopaedia Britannica," he had said, "you'll find it classified under the following three heads: Intelligence, Human; Intelligence, Animal; Intelligence, Military. My stepfather's a perfect specimen of Intelligence, Military."

—ALDOUS HUXLEY (*Point Counter Point*)

. . . a science is said to be useful if its development tends to accentuate the existing inequalities in the distribution of wealth, or more directly promotes the destruction of human life.

—G. H. HARDY

1 Mathematics in Warfare

By FREDERICK WILLIAM LANCHESTER

THE PRINCIPLE OF CONCENTRATION.
THE "N-SQUARE" LAW.

THE Principle of Concentration. It is necessary at the present juncture to make a digression and to treat of certain fundamental considerations which underlie the whole science and practice of warfare in all its branches. One of the great questions at the root of all strategy is that of *concentration*; the concentration of the whole resources of a belligerent on a single purpose or object, and concurrently the concentration of the main strength of his forces, whether naval or military, at one point in the field of operations. But the principle of concentration is not in itself a strategic principle; it applies with equal effect to purely tactical operations; it is on its material side based upon facts of a purely scientific character. The subject is somewhat befogged by many authors of repute, inasmuch as the two distinct sides—the moral concentration (the narrowing and fixity of purpose) and the material concentration—are both included under one general heading, and one is invited to believe that there is some peculiar virtue in the word *concentration*, like the "blessed word Mesopotamia," whereas the truth is that the word in its two applications refers to two entirely independent conceptions, whose underlying principles have nothing really in common.

The importance of concentration in the material sense is based on certain elementary principles connected with the means of attack and defence, and if we are properly to appreciate the value and importance of concentration in this sense, we must not fix our attention too closely upon the bare fact of concentration, but rather upon the underlying principles, and seek a more solid foundation in the study of the controlling factors.

The Conditions of Ancient and Modern Warfare Contrasted. There is an important difference between the methods of defence of primitive

times and those of the present day which may be used to illustrate the point at issue. In olden times, when weapon directly answered weapon, the act of defence was positive and direct, the blow of sword or battleaxe was parried by sword and shield; under modern conditions gun answers gun, the defence from rifle-fire is rifle-fire, and the defence from artillery, artillery. But the defence of modern arms is indirect: tersely, the enemy is prevented from killing you by your killing him first, and the fighting is essentially collective. As a consequence of this difference, the importance of concentration in history has been by no means a constant quantity. Under the old conditions it was not possible by any strategic plan or tactical manœuvre to bring other than approximately equal numbers of men into the actual fighting line; one man would ordinarily find himself opposed to one man. Even were a general to concentrate twice the number of men on any given portion of the field to that of the enemy, the number of men actually wielding their weapons at any given instant (so long as the fighting line was unbroken), was, roughly speaking, the same on both sides. Under present-day conditions all this is changed. With modern long-range weapons—fire-arms, in brief—the concentration of superior numbers gives an immediate superiority in the active combatant ranks, and the numerically inferior force finds itself under a far heavier fire, man for man, than it is able to return. The importance of this difference is greater than might casually be supposed, and, since it contains the kernel of the whole question, it will be examined in detail.

In thus contrasting the ancient conditions with the modern, it is not intended to suggest that the advantages of concentration did not, to some extent, exist under the old order of things. For example, when an army broke and fled, undoubtedly any numerical superiority of the victor could be used with telling effect, and, before this, pressure, as distinct from blows, would exercise great influence. Also the bow and arrow and the cross-bow were weapons that possessed in a lesser degree the properties

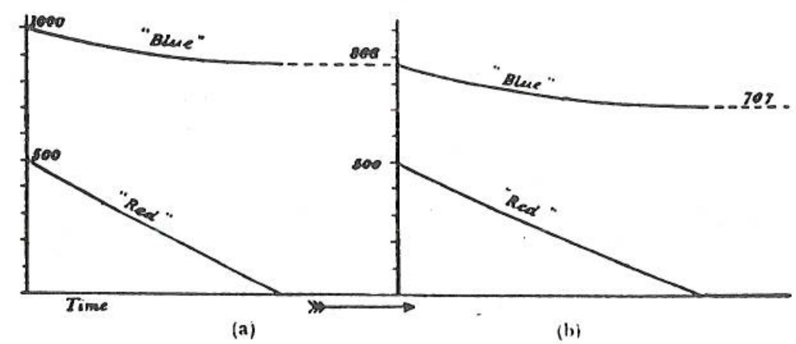

FIGURE 1

of fire-arms, inasmuch as they enabled numbers (within limits) to concentrate their attack on the few. As here discussed, the conditions are contrasted in their most accentuated form as extremes for the purpose of illustration.

Taking, first, the ancient conditions where man is opposed to man, then, assuming the combatants to be of equal fighting value, and other conditions equal, clearly, on an average, as many of the "duels" that go to make up the whole fight will go one way as the other, and there will be about equal numbers killed of the forces engaged; so that if 1,000 men meet 1,000 men, it is of little or no importance whether a "Blue" force of 1,000 men meet a "Red" force of 1,000 men in a single pitched battle, or whether the whole "Blue" force concentrates on 500 of the "Red" force, and, having annihilated them, turns its attention to the other half; there will, presuming the "Reds" stand their ground to the last, be half the "Blue" force wiped out in the annihilation of the "Red" force [1] in the first battle, and the second battle will start on terms of equality—*i.e.*, 500 "Blue" against 500 "Red."

Modern Conditions Investigated. Now let us take the modern conditions. If, again, we assume equal individual fighting value, and the combatants otherwise (as to "cover," etc.) on terms of equality, each man will in a given time score, on an average, a certain number of hits that are effective; consequently, the number of men knocked out per unit time will be directly proportional to the numerical strength of the opposing force. Putting this in mathematical language, and employing symbol b to represent the numerical strength of the "Blue" force, and r for the "Red," we have:—

$$\frac{db}{dt} = -r \times c \quad . \; . \; . \; . \quad (1)$$

and

$$\frac{dr}{dt} = -b \times k \quad . \; . \; . \; . \quad (2)$$

in which t is time and c and k are constants ($c = k$ if the fighting values of the individual units of the force are equal).

The reduction of strength of the two forces may be represented by two conjugate curves following the above equations. In Figure 1 (a) graphs are given representing the case of the "Blue" force 1,000 strong encountering a section of the "Red" force 500 strong, and it will be seen that the "Red" force is wiped out of existence with a loss of only about 134 men of the "Blue" force, leaving 866 to meet the remaining 500 of the

[1] This is not strictly true, since towards the close of the fight the last few men will be attacked by more than their own number. The main principle is, however, untouched.

"Red" force with an easy and decisive victory; this is shown in Figure 1 (b), the victorious "Blues" having annihilated the whole "Red" force of equal total strength with a loss of only 293 men.

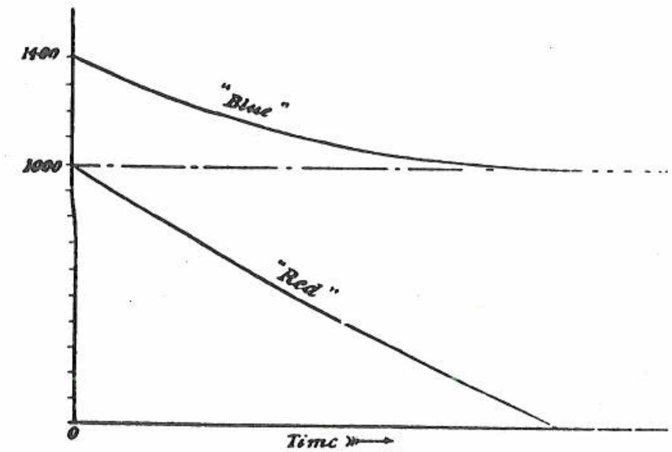

FIGURE 2*a*

In Figure 2*a* a case is given in which the "Red" force is inferior to the "Blue" in the relation 1: $\sqrt{2}$ say, a "Red" force 1,000 strong meeting a "Blue" force 1,400 strong. Assuming they meet in a single pitched battle fought to a conclusion, the upper line will represent the "Blue" force, and it is seen that the "Reds" will be annihilated, the "Blues" losing only 400 men. If, on the other hand, the "Reds" by superior strategy compel the "Blues" to give battle divided—say into two equal armies—then, Figure

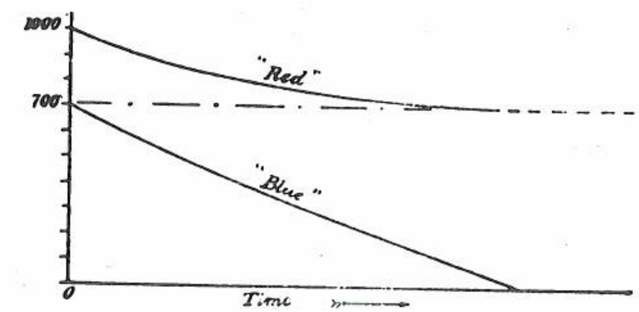

FIGURE 2*b*

2*b*, in the first battle the 700 "Blues" will be annihilated with a loss of only 300 to the "Reds" and in the second battle the two armies will meet on an equal numerical footing, and so we may presume the final battle of the campaign as drawn. In this second case the result of the second battle is presumed from the initial equality of the forces; the curves are not given.

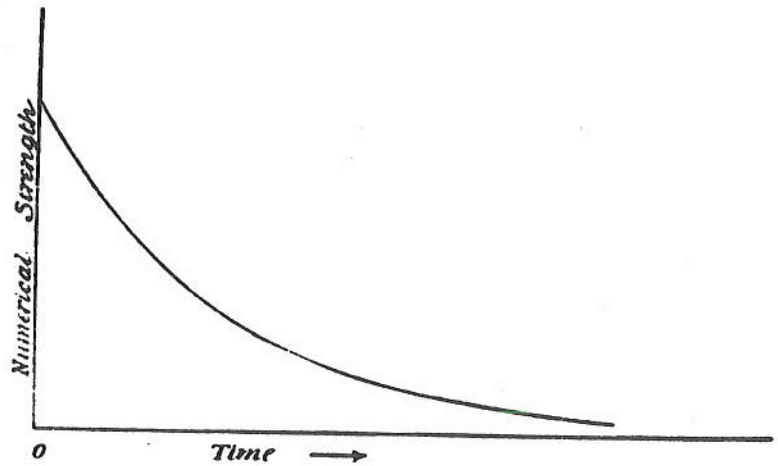

FIGURE 3

In the case of equal forces the two conjugate curves become coincident; there is a single curve of logarithmic form, Figure 3; the battle is prolonged indefinitely. Since the forces actually consist of a finite number of finite units (instead of an infinite number of infinitesimal units), the end of the curve must show discontinuity, and break off abruptly when the last man is reached; the law based on averages evidently does not hold rigidly when the numbers become small. Beyond this, the condition of two equal curves is unstable, and any advantage secured by either side will tend to augment.

Graph representing Weakness of a Divided Force. In Figure 4*a*, a pair of conjugate curves have been plotted backwards from the vertical datum representing the finish, and an upper graph has been added representing the total of the "Red" force, which is equal in strength to the "Blue" force for any ordinate, on the basis that the "Red" force is divided into two portions as given by the intersection of the lower graph. In Figure 4*b*, this diagram has been reduced to give the same information in terms *per cent.* for a "Blue" force of constant value. Thus in its application Figure 4*b* gives the correct percentage increase necessary in the fighting value of, for example, an army or fleet to give equality, on the assumption that political or strategic necessities impose the condition of dividing the said army or fleet into two in the proportions given by the lower graph, the enemy being able to attack either proportion with his full strength. Alternatively, if the constant (= 100) be taken to represent a numerical

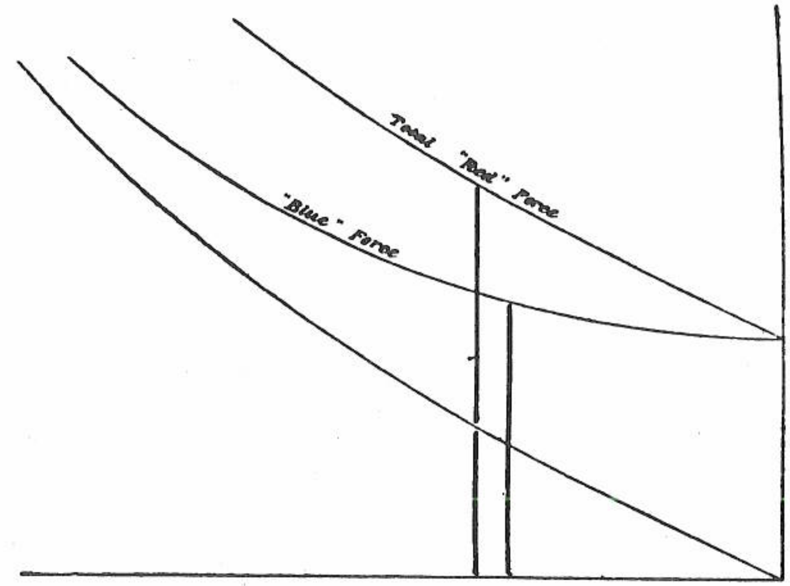

FIGURE 4*a*

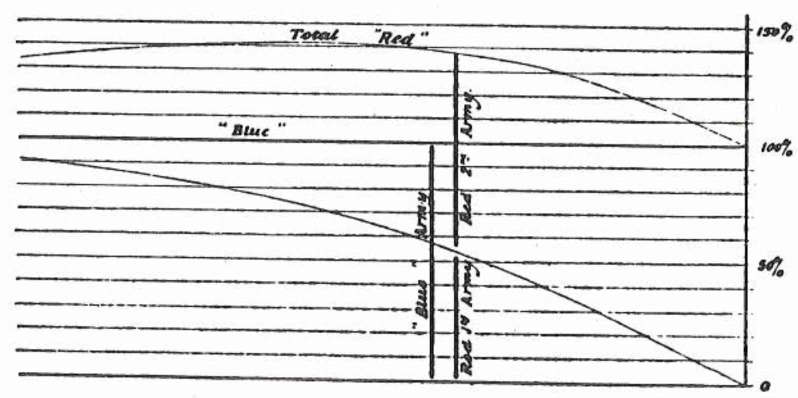

FIGURE 4*b*

strength that would be deemed sufficient to ensure victory against the enemy, given that both fleets engage in their full strength, then the upper graph gives the numerical superiority needed to be equally sure of victory, in case, from political or other strategic necessity, the fleet has to be divided in the proportions given. In Figure 4*b* abscissæ have no quantitative meaning.

Validity of Mathematical Treatment. There are many who will be inclined to cavil at any mathematical or semi-mathematical treatment of the present subject, on the ground that with so many unknown factors, such as the morale or leadership of the men, the unaccounted merits or demerits of the weapons, and the still more unknown "chances of war," it is ridiculous to pretend to calculate anything. The answer to this is simple: the direct numerical comparison of the forces engaging in conflict or available in the event of war is almost universal. It is a factor always carefully reckoned with by the various military authorities; it is discussed *ad nauseam* in the Press. Yet such direct counting of forces is in itself a tacit acceptance of the applicability of mathematical principles, but confined to a special case. To accept without reserve the mere "counting of the pieces" as of value, and to deny the more extended application of mathematical theory, is as illogical and unintelligent as to accept broadly and indiscriminately the balance and the weighing-machine as instruments of precision, but to decline to permit in the latter case any allowance for the known inequality of leverage.

Fighting Units not of Equal Strength. In the equations (1) and (2), two constants were given, c and k, which in the plotting of the Figures 1 to $4b$ were taken as equal; the meaning of this is that the fighting strength of the individual units has been assumed equal. This condition is not necessarily fulfilled if the combatants be unequally trained, or of different morale. Neither is it fulfilled if their weapons are of unequal efficiency. The first two of these, together with a host of other factors too numerous to mention, cannot be accounted for in an equation any more than can the quality of wine or steel be estimated from the weight. The question of weapons is, however, eminently suited to theoretical discussion. It is also a matter that (as will be subsequently shown) requires consideration in relation to the main subject of the present articles.

Influence of Efficiency of Weapons. Any difference in the efficiency of the weapons—for example, the accuracy or rapidity of rifle-fire—may be represented by a disparity in the constants c and k in equations (1) and (2). The case of the rifle or machine-gun is a simple example to take, inasmuch as comparative figures are easily obtained which may be said fairly to represent the fighting efficiency of the weapon. Now numerically equal forces will no longer be forces of equal strength; they will only be of equal strength if, when in combat, their losses result in no change in their numerical proportion. Thus, if a "Blue" force initially 500 strong, using a magazine rifle, attack a "Red" force of 1,000, armed with a single breech-loader, and after a certain time the "Blue" are found to have lost 100 against 200 loss by the "Red," the proportions of the forces will have suffered no change, and they may be regarded (due to the superiority of the "Blue" arms) as being of equal strength.

If the condition of equality is given by writing M as representing the

efficiency or value of an individual unit of the "Blue" force, and N the same for the "Red," we have:—

Rate of reduction of "Blue" force:—

$$\frac{db}{dt} = -\mathrm{N}\,r \times \text{constant} \quad . \quad . \quad (3)$$

and "Red,"

$$\frac{dr}{dt} = -\mathrm{M}\,b \times \text{constant} \quad . \quad . \quad (4)$$

And for the condition of equality,

$$\frac{db}{b\,dt} = \frac{dr}{r\,dt},$$

or

$$\frac{-\mathrm{N}\,r}{b} = \frac{-\mathrm{M}\,b}{r},$$

or

$$\mathrm{N}\,r^2 = \mathrm{M}\,b^2 \quad . \quad . \quad . \quad (5)$$

In other words, the fighting strengths of the two forces are equal when the *square of the numerical strength multiplied by the fighting value of the individual units are equal.*

The Outcome of the Investigation. The n-square Law. It is easy to show that this expression (5) may be interpreted more generally; the *fighting strength* of a force may be broadly defined as proportional to *the square of its numerical strength multiplied by the fighting value of its individual units.*

Thus, referring to Figure $4b$, the sum of the squares of the two portions of the "Red" force are for all values equal to the square of the "Blue" force (the latter plotted as constant); the curve might equally well have been plotted directly to this law as by the process given. A simple proof of the truth of the above law as arising from the differential equations (1) and (2), p. 2140, is as follows:—

In Figure 5, let the numerical values of the "blue" and "red" forces be represented by lines b and r as shown; then in an infinitesimally small interval of time the change in b and r will be represented respectively by db and dr of such relative magnitude that $db/dr = r/b$ or,

$$b\,db = r\,dr \quad (1)$$

If (Figure 5) we draw the squares on b and r and represent the increments db and dr as small finite increments, we see at once that the *change of area* of b^2 is $2b\,db$ and the *change of area* of r^2 is $2r\,dr$ which according to the foregoing (1), are equal. Therefore the difference between the two squares is constant

$$b^2 - r^2 = \text{constant}.$$

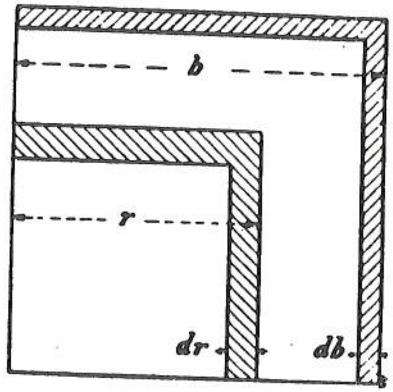

FIGURE 5

If this constant be represented by a quantity q^2 then $b^2 = r^2 + q^2$ and q represents the numerical value of the remainder of the blue "force" after annihilation of the red. Alternatively q represents numerically a second "red" army of the strength necessary in a *separate action* to place the red forces on terms of equality, as in Figure 4*b*.

A Numerical Example. As an example of the above, let us assume an army of 50,000 giving battle in turn to two armies of 40,000 and 30,000 respectively, equally well armed; then the strengths are equal, since $(50{,}000)^2 = (40{,}000)^2 + (30{,}000)^2$. If, on the other hand, the two smaller armies are given time to effect a junction, then the army of 50,000 will be overwhelmed, for the fighting strength of the opposing force, 70,000 is no longer equal, but is in fact nearly twice as great—namely, in the relation of 49 to 25. Superior morale or better tactics or a hundred and one other extraneous causes may intervene in practice to modify the issue, but this does not invalidate the mathematical statement.

Example Involving Weapons of Different Effective Value. Let us now take an example in which a difference in the fighting value of the unit is a factor. We will assume that, as a matter of experiment, one man employing a machine-gun can punish a target to the same extent in a given time as sixteen riflemen. What is the number of men armed with the machine gun necessary to replace a battalion a thousand strong in the field? Taking the fighting value of a rifleman as unity, let $n =$ the number required. The fighting strength of the battalion is, $(1{,}000)^2$ or,

$$n = \sqrt{\frac{1{,}000{,}000}{16}} = \frac{1{,}000}{4} = 250$$

or one quarter the number of the opposing force.

This example is instructive; it exhibits at once the utility and weakness of the method. The basic assumption is that the fire of each force is definitely *concentrated* on the opposing force. Thus the enemy will concentrate on the one machine-gun operator the fire that would otherwise be distributed over four riflemen, and so on an average he will only last for one quarter the time, and at sixteen times the efficiency during his short life he will only be able to do the work of four riflemen in lieu of sixteen, as one might easily have supposed. This is in agreement with the equation. The conditions may be regarded as corresponding to those prevalent in the Boer War, when individual-aimed firing or sniping was the order of the day.

When, on the other hand, the circumstances are such as to preclude the possibility of such concentration, as when searching an area or ridge at long range, or volley firing at a position, or "into the brown," the basic conditions are violated, and the value of the individual machine-gun operator becomes more nearly that of the sixteen riflemen that the power of his weapon represents. The same applies when he is opposed by shrapnel fire or any other weapon which is directed at a position rather than the individual. It is well thus to call attention to the variations in the conditions and the nature of the resulting departure from the conclusions of theory; such variations are far less common in naval than in military warfare; the individual unit—the ship—is always the gunner's mark. When we come to deal with aircraft, we shall find the conditions in this respect more closely resemble those that obtain in the Navy than in the Army; the enemy's aircraft individually rather than collectively is the air-gunner's mark, and the law herein laid down will be applicable.

The Hypothesis Varied. Apart from its connection with the main subject, the present line of treatment has a certain fascination, and leads to results which, though probably correct, are in some degree unexpected. If we modify the initial hypothesis to harmonise with the conditions of long-range fire, and assume the fire concentrated on a certain area known to be held by the enemy, and take this area to be independent of the numerical value of the forces, then, with notation as before, we have—

$$\left.\begin{aligned} -\frac{db}{dt} &= b \times \mathrm{N}\, r \\ -\frac{dr}{dt} &= r \times \mathrm{M}\, b \end{aligned}\right\} \times \text{constant.}$$

or

$$\frac{\mathrm{M}\, db}{dt} = \frac{\mathrm{N}\, dr}{dt}$$

or the rate of loss is independent of the numbers engaged, and is directly as the efficiency of the weapons. Under these conditions the fighting strength of the forces is directly proportional to their numerical strength; there is no direct value in concentration, *qua* concentration, and the advantage of rapid fire is relatively great. Thus in effect the conditions approximate more closely to those of ancient warfare.

An Unexpected Deduction. Evidently it is the business of a numerically superior force to come to close quarters, or, at least, to get within decisive range as rapidly as possible, in order that the concentration may tell to advantage. As an extreme case, let us imagine a "Blue" force of 100 men armed with the machine gun opposed by a "Red" 1,200 men armed with the ordinary service rifle. Our first assumption will be that both forces are spread over a front of given length and at long range. Then the "Red" force will lose 16 men to the "Blue" force loss of one, and, if the combat is continued under these conditions, the "Reds" must lose. If, however, the "Reds" advance, and get within short range, where each man and gunner is an individual mark, the tables are turned, the previous equation and conditions apply, and, even if "Reds" lose half their effective in gaining the new position, with 600 men remaining they are masters of the situation; their strength is $600^2 \times 1$ against the "Blue" $100^2 \times 16$. It is certainly a not altogether expected result that, in the case of fire so deadly as the modern machine-gun, circumstances may arise that render it imperative, and at all costs, to come to close range.

Examples from History. It is at least agreed by all authorities that on the field of battle concentration is a matter of the most vital importance; in fact, it is admitted to be one of the controlling factors both in the strategy and tactics of modern warfare. It is aptly illustrated by the important results that have been obtained in some of the great battles of history by the attacking of opposing forces before concentration has been effected. A classic example is that of the defeat by Napoleon, in his Italian campaign, of the Austrians near Verona, where he dealt with the two Austrian armies in detail before they had been able to effect a junction, or even to act in concert. Again, the same principle is exemplified in the oft-quoted case of the defeat of Jourdan and Moreau on the Danube by the Archduke Charles in 1796. It is evident that the conditions in the broad field of military operations correspond in kind, if not in degree, to the earlier hypothesis, and that the law deduced therefrom, that the fighting strength of a force can be represented by the square of its numerical strength, does, in its essence, represent an important truth.

THE "N-SQUARE" LAW IN ITS APPLICATION

The n-square Law in its Application to a Heterogeneous Force. In the preceding article it was demonstrated that under the conditions of mod-

ern warfare the fighting strength of a force, so far as it depends upon its numerical strength, is best represented or measured by the square of the number of units. In land operations these units may be the actual men engaged, or in an artillery duel the gun battery may be the unit; in a naval battle the number of units will be the number of capital ships, or in an action between aeroplanes the number of machines. In all cases where the individual fighting strength of the component units may be different it has been shown that if a numerical fighting value can be assigned to these units, the fighting strength of the whole force is as the square of the number multiplied by their individual strength. Where the component units differ among themselves, as in the case of a fleet that is not homogeneous, the measure of the total of fighting strength of a force will be *the square of the sum of the square roots of the strengths of its individual units.*

Graphic Representation. Before attempting to apply the foregoing, either as touching the conduct of aerial warfare or the equipment of the fighting aeroplane, it is of interest to examine a few special cases and applications in other directions and to discuss certain possible limitations. A convenient graphic form in which the operation of the *n-square* law can be presented is given in Figure 6; here the strengths of a number of separate armies or forces successively mobilised and brought into action are represented numerically by the lines *a, b, c, d, e,* and the aggregate fighting strengths of these armies are given by the lengths of the lines A, B, C, D, E, each being the hypotenuse of a right-angle triangle, as indicated. Thus two forces or armies *a* and *b*, if acting separately (in point of time), have only the fighting strength of a single force or army

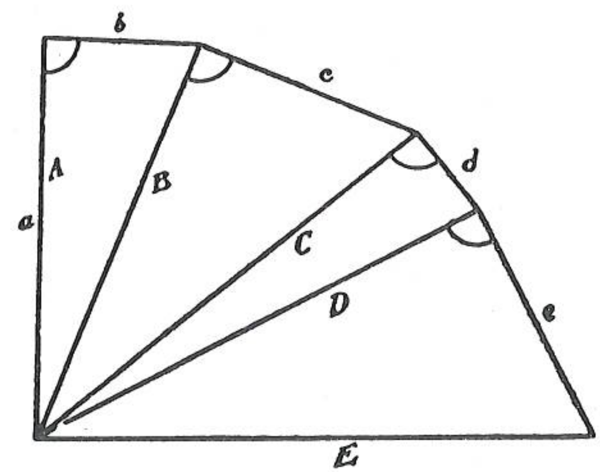

FIGURE 6

represented numerically by the line B. Again, the three separate forces, *a, b,* and *c,* could be met on equal terms in three successive battles by a single army of the numerical strength C, and so on.

Special or Extreme Case. From the diagram given in Figure 6 arises a special case that at first sight may look like a *reductio ad absurdum,* but which, correctly interpreted, is actually a confirmation of the *n-square* law. Referring to Figure 6, let us take it that the initial force (army or fleet), is of some definite finite magnitude, but that the later arrivals *b, c, d,* etc., be very small and numerous detachments—so small, in fact, as to be reasonably represented to the scale of the diagram as infinitesimal quantities. Then the lines *b, c, d, e, f,* etc., describe a polygonal figure approximating to a circle, which in the limit becomes a circle, whose radius is represented by the original force *a,* Figure 7. Here we have graphically represented the result that the fighting value of the added forces, no matter what their numerical aggregate (represented in Figure 7 by the circumferential line), is zero. The correct interpretation of this

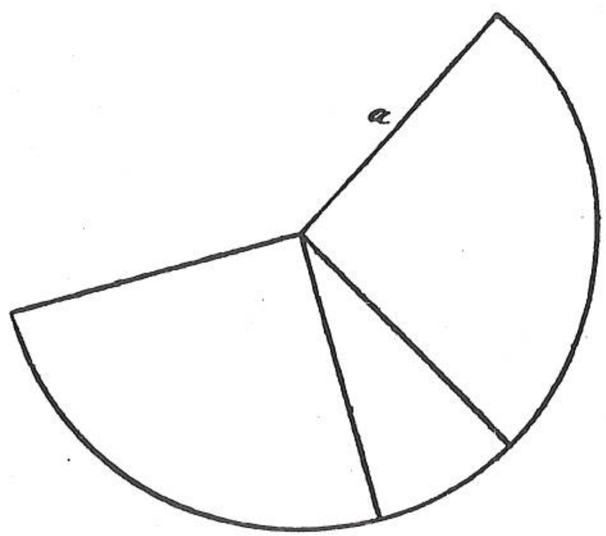

FIGURE 7

is that in the open a small force attacking, or attacked by one of overwhelming magnitude is wiped out of existence without being able to exact a toll even comparable to its own numerical value; it is necessary to say *in the open,* since, under other circumstances, the larger force is unable to bring its weapons to bear, and this is an essential portion of the basic hypothesis. In the limiting case when the disparity of force is extreme, the capacity of the lesser force to effect anything at all becomes negligible.

There is nothing improbable in this conclusion, but it manifestly does not apply to the case of a small force concealed or "dug in," since the hypothesis is infringed. Put bluntly, the condition represented in Figure 7 illustrates the complete impotence of small forces in the presence of one of overwhelming power. Once more we are led to contrast the ancient conditions, under which the weapons of a large army could not be brought to bear, with modern conditions, where it is physically possible for the weapons of ten thousand to be concentrated on one. Macaulay's lines

> "In yon strait path a thousand
> May well be stopped by three,"

belong intrinsically to the methods and conditions of the past.

The N-square Law in Naval Warfare. We have already seen that the *n-square* law applies broadly, if imperfectly, to military operations; on land however, there sometimes exist special conditions and a multitude of factors extraneous to the hypothesis whereby its operation may be suspended or masked. In the case of naval warfare, however, the conditions more strictly conform to our basic assumptions, and there are comparatively few disturbing factors. Thus, when battle fleet meets battle fleet, there is no advantage to the defender analogous to that secured by the entrenchment of infantry. Again, from the time of opening fire, the individual ship is the mark of the gunner, and there is no phase of the battle or range at which areas are searched in a general way. In a naval battle every shot fired is aimed or directed at some definite one of the enemy's ships; there is no firing on the mass or "into the brown." Under the old conditions of the sailing-ship and cannon of some 1,000 or 1,200 yards maximum effective range, advantage could be taken of concentration within limits; and an examination of the latter 18th century tactics makes it apparent that with any ordinary disparity of numbers (probably in no case exceeding 2 to 1) the effect of concentration must have been not far from that indicated by theory. But to whatever extent this was the case, it is certain that with a battle-fleet action at the present day the conditions are still more favourable to the weight of numbers, since with the modern battle range—some 4 to 5 miles—there is virtually no limit to the degree of concentration of fire. Further than this, there is in modern naval warfare practically no chance of coming to close quarters in ship-to-ship combats, as in the old days.

Thus the conditions are to-day almost ideal from the point of view of theoretical treatment. A numerical superiority of ships of individually equal strength will mean definitely that the inferior fleet at the outset has to face the full fire of the superior, and as the battle proceeds and the smaller fleet is knocked to pieces, the initial disparity will become worse and worse, and the fire to which it is subjected more and more concentrated. These are precisely the conditions taken as the basis of the investigation from which the *n-square* law has been derived. The same observations will probably be found to apply to aerial warfare when air fleets engage in conflict, more especially so in view of the fact that aeroplane can attack aeroplane in three dimensions of space instead of being limited to two, as is the case with the battleship. This will mean that even with weapons of moderate range the degree of fire concentration possible will be very great. By attacking from above and below, as well as from all points of the compass, there is, within reason, no limit to the number of machines which can be brought to bear on a given small force of the enemy, and so a numerically superior fleet will be able to reap every ounce of advantage from its numbers.

Individual Value of Ships or Units. The factor the most difficult to assess in the evaluation of a fleet as a fighting machine is (apart from the *personnel*) the individual value of its units, when these vary amongst themselves. There is no possibility of entirely obviating this difficulty, since the fighting value of any given ship depends not only upon its gun armament, but also upon its protective armour. One ship may be stronger than another at some one range, and weaker at some longer or shorter range, so that the question of fleet strength can never be reduced quite to a matter of simple arithmetic, nor the design of the battleship to an exact science. In practice the drawing up of a naval programme resolves itself, in great part at least, into the answering of the prospective enemy's programme type by type and ship by ship. It is, however, generally accepted that so long as we are confining our attention to the main battle fleets, and so are dealing with ships of closely comparable gun calibre and range, and armour of approximately equivalent weight, the fighting value of the individual ship may be gauged by the weight of its "broadside," or more accurately, taking into account the speed with which the different guns can be served, by the weight of shot that can be thrown per minute. Another basis, and one that perhaps affords a fairer comparison, is to give the figure for the *energy per minute* for broadside fire, which represents, if we like so to express it, the horsepower of the ship as a fighting machine. Similar means of comparison will probably be found applicable to the fighting aeroplane, though it may be that the *downward fire* capacity will be regarded as of vital importance rather than the broadside fire as pertaining to the battleship.

Applications of the n-square Law. The *n-square* law tells us at once the price or penalty that must be paid if elementary principles are outraged by the division of our battle fleet [2] into two or more isolated detachments. In this respect our present disposition—a single battle fleet or "Grand" fleet—is far more economical and strategically preferable as a

[2] Capital ships:—Dreadnoughts and Super-Dreadnoughts.

defensive power to the old-time distribution of the Channel Fleet, Mediterranean Fleet, etc. If it had been really necessary, for any political or geographical reason, to maintain two separate battle fleets at such distance asunder as to preclude their immediate concentration in case of attack, the cost to the country would have been enormously increased. In the

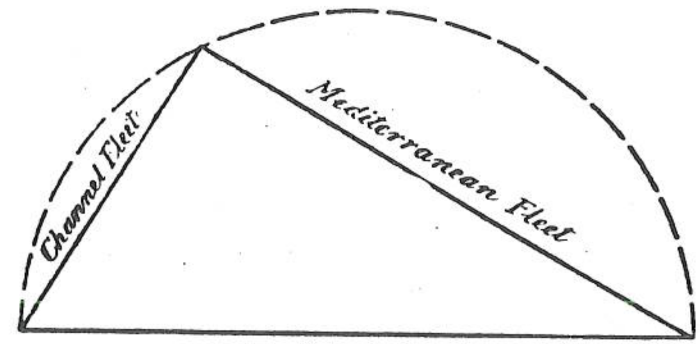

FIGURE 8—Single or "Grand" Fleet of Equal Strength (Lines give numerical values).

case, for example, of our total battle fleet being separated into two equal parts, forming separate fleets or squadrons, the increase would require to be fixed at approximately 40 per cent.—that is to say, in the relation of 1 to $\sqrt{2}$; more generally the solution is given by a right-angled triangle, as in Figure 8. In must not be forgotten that, even with this enormous increase, the security will not be so great as appears on paper, for the enemy's fleet, having met and defeated one section of our fleet, may succeed in falling back on his base for repair and refit, and emerge later with the advantage of strength in his favour. Also one must not overlook the demoralizing effect on the *personnel* of the fleet first to go into action, of the knowledge that they are hopelessly outnumbered and already beaten on paper—that they are, in fact, regarded by their King and country as "cannon fodder." Further than this, presuming two successive fleet actions and the enemy finally beaten, the cost of victory in men and *matériel* will be greater in the case of the divided fleet than in the case of a single fleet of equal total fighting strength, in the proportion of the total numbers engaged—that is to say, in Figure 8, in the proportion that the two sides of the right-angled triangle are greater than the hypotenuse.

In brief, however potent political or geographical influences or reasons may be, it is questionable whether *under any circumstances* it can be considered sound strategy to divide the main battle fleet on which the defence of a country depends. This is to-day the accepted view of every naval strategist of repute, and is the basis of the present distribution of Great Britain's naval forces.

Fire Concentration the Basis of Naval Tactics. The question of fire concentration is again found to be paramount when we turn to the consideration and study of naval tactics. It is worthy of note that the recognition of the value of any definite tactical scheme does not seem to have been universal until quite the latter end of the 18th century. It is even said that the French Admiral Suffren, about the year 1780, went so far as to attribute the reverses suffered by the French at sea to "the introduction of tactics" which he stigmatised as "the veil of timidity";[3] the probability is that the then existing standard of seamanship in the French Navy was so low that anything beyond the simplest of manœuvres led to confusion, not unattended by danger. The subject, however, was, about that date, receiving considerable attention. A writer, Clerk, about 1780, pointed out that in meeting the attack of the English the French had adopted a system of defence consisting of a kind of running fight, in which, initially taking the "lee gage," they would await the English attack in line ahead, and having delivered their broadsides on the leading English ships (advancing usually in line abreast), they would bear away to leeward and take up position, once more waiting for the renewal of the attack, when the same process was repeated. By these tactics the French obtained a concentration of fire on a small portion of the English fleet, and so were able to inflict severe punishment with little injury to themselves.[4] Here we see the beginnings of sound tactical method adapted to the needs of defence.

Up to the date in question there appears to have been no studied attempt to found a scheme of attack on the basis of concentration; the old order was to give battle in parallel columns or lines, ship to ship, the excess of ships, if either force were numerically superior, being doubled on the rear ships of the enemy. It was not till the "Battle of the Saints," in 1782, that a change took place; Rodney (by accident or intention) broke away from tradition, and cutting through the lines of the enemy, was able to concentrate on his centre and rear, achieving thereby a decisive victory.

British Naval Tactics in 1805. The Nelson "Touch." The accident or experiment of 1782 had evidently become the established tactics of the British in the course of the twenty years which followed, for not only do we find the method in question carefully laid down in the plan of attack given in the Memorandum issued by Nelson just prior to the Battle of Trafalgar in 1805, but the French Admiral Villeneuve[5] confidently asserted in a note issued to his staff in anticipation of the battle that:—

[3] Mahan, "Sea Power," page 425.
[4] Incidentally, also, the scheme in question had the advantage of subjecting the English to a raking fire from the French broadsides before they were themselves able to bring their own broadside fire to bear.
[5] "The Enemy at Trafalgar," Ed. Fraser; Hodder and Stoughton, page 54.

"The British Fleet will not be formed in a line-of-battle parallel to the combined fleet according to the usage of former days. Nelson, assuming him to be, as represented, really in command, will seek to break our line, envelop our rear, and overpower with groups of his ships as many as he can isolate and cut off." Here we have a concise statement of a definite tactical scheme based on a clear understanding of the advantages of fire concentration.

It will be understood by those acquainted with the sailing-ship of the period that the van could only turn to come to the assistance of those in the rear at the cost of a considerable interval of time, especially if the van should happen to be to leeward of the centre and rear. The time taken to "wear ship," or in light winds to "go about" (often only to be effected by manning the boats and rowing to assist the manœuvre), was by no means an inconsiderable item. Thus it would not uncommonly be a matter of some hours before the leading ships could be brought within decisive range, and take an active part in the fray.

Nelson's Memorandum and Tactical Scheme. In order further to embarrass the the enemy's van, and more effectively to prevent it from coming into action, it became part of the scheme of attack that a few ships, a comparatively insignificant force, should be told off to intercept and engage as many of the leading ships as possible; in brief, to fight an independent action on a small scale; we may say admittedly a losing action. In this connection Nelson's memorandum of October 9 is illuminating. Nelson assumed for the purpose of framing his plan of attack that his own force would consist of forty sail of the line, against forty-six of the com-

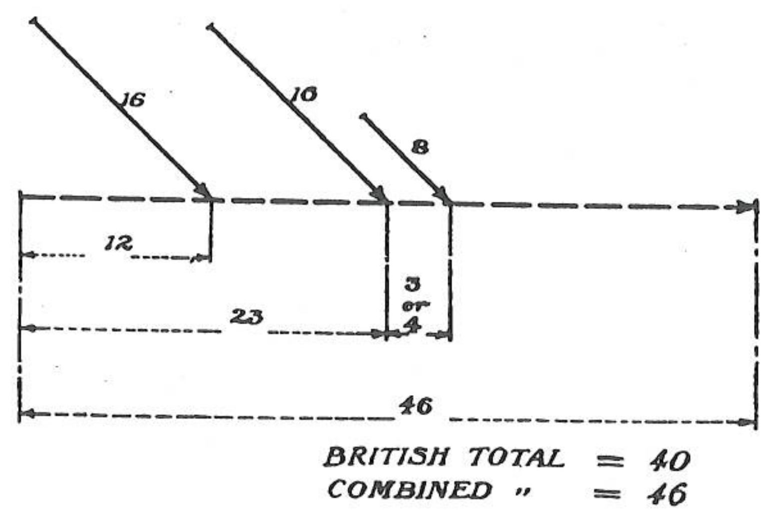

FIGURE 9

bined (French and Spanish) fleet. These numbers are considerably greater, as things turned out, than those ultimately engaged; but we are here dealing with the memorandum, and not with the actual battle. The British Fleet was to form in two main columns, comprising sixteen sail of the line each, and a smaller column of eight ships only. The plan of attack prescribed in the event of the enemy being found in line ahead was briefly as follows:—One of the main columns was to cut the enemy's line about the centre, the other to break through about twelve ships from the rear, the smaller column being ordered to engage the rear of the enemy's van three or four ships ahead of the centre, and to frustrate, as far as possible, every effort the van might make to come to the succour of the threatened centre or rear. Its object, in short, was to prevent the van of the combined fleet from taking part in the main action. The plan is shown diagrammatically in Figure 9 (p. 2155).

Nelson's Tactical Scheme Analysed. An examination of the numerical values resulting from the foregoing disposition is instructive. The force with which Nelson planned to envelop the half—*i.e.*, 23 ships—of the combined fleet amounted to 32 ships in all; this according to the n^2 law would give him a superiority of fighting strength of almost exactly two to one,[6] and would mean that if subsequently he had to meet the other half of the combined fleet, without allowing for any injury done by the special eight-ship column, he would have been able to do so on terms of equality. The fact that the van of the combined fleet would most certainly be in some degree crippled by its previous encounter is an indication and measure of the positive advantage of strength provided by the tactical scheme. Dealing with the position arithmetically, we have:—

Strength of British (in arbitrary n^2 units),

$$32^2 + 8^2 = 1088$$

And combined fleet,

$$23^2 + 23^2 = 1058$$

British advantage 30

Or, the numerical equivalent of the remains of the British Fleet (assuming the action fought to the last gasp), $= \sqrt{30}$ or 5½ ships.

If for the purpose of comparison we suppose the total forces had engaged under the conditions described by Villeneuve as "the usage of former days," we have:—

Strength of combined fleet,	46^2	$= 2116$
" British "	40^2	$= 1600$
Balance in favour of enemy	....	516

[6] $23 \times \sqrt{2} = 32.5$.

Or, the equivalent numerical value of the remainder of the combined fleet, assuming complete annihilation of the British, $= \sqrt{516} = 23$ ships approximately.

Thus we are led to appreciate the commanding importance of a correct tactical scheme. If in the actual battle the old-time method of attack had been adopted, it is extremely doubtful whether the superior seamanship and gunnery of the British could have averted defeat. The actual forces on the day were 27 British sail of the line against the combined fleet numbering 33, a rather less favourable ratio than assumed in the Memorandum. In the battle, as it took place, the British attacked in two columns instead of three, as laid down in the Memorandum; but the scheme of concentration followed the original idea. The fact that the wind was of the lightest was alone sufficient to determine the exclusion of the enemy's van from the action. However, as a study the Memorandum is far more important than the actual event, and in the foregoing analysis it is truly remarkable to find, firstly, the definite statement of the cutting the enemy into two *equal* parts—according to the *n-square* law the exact proportion corresponding to the reduction of his total effective strength to a minimum; and, secondly, the selection of a proportion, the nearest whole-number equivalent to the $\sqrt{2}$ ratio of theory, required to give a fighting strength equal to tackling the two halves of the enemy on level terms, and the detachment of the remainder, the column of eight sail, to weaken and impede the leading half of the enemy's fleet to guarantee the success of the main idea. If, as might fairly be assumed, the foregoing is more than a coincidence,[7] it suggests itself that Nelson, if not actually acquainted with the *n-square* law, must have had some equivalent basis on which to figure his tactical values.

[7] Although we may take it to be a case in which the dictates of experience resulted in a disposition now confirmed by theory, the agreement is remarkable.