Math 105 - Multivariable Calculus (Miller) - Practice Midterms - 2010

First Practice Midterm

1. (20 points) Let $f(x, y) = 3xy + \cos x$ and $g(x, y) = 4x^2y + e^{xy}$. Find the derivatives of the following functions if possible; if it is not possible to find the derivative state why not: (1) $f(x)+g(x)$; (2) $f(x)g(x)$; (3) $f(g(x))$.

2. (20 points) Define or state the following.

1. State what it means for a function $f : \mathbb{R}^4 \to \mathbb{R}$ to be continuous at the point $(1, 7, 9, 3)$.
2. Describe the method of Lagrange Multipliers.

3. (20 points) Write down a formula for the second order Taylor Series expansion of a function $f : \mathbb{R}^2 \to \mathbb{R}$ at $(0, 0)$. Assume now $f(x, y) = 3x^2 + 4y^4 + 5x^3y^3 + \cos(xy)$. Calculate the gradient of f, evaluate it at the point $(0, 0)$, and determine the second order Taylor Series expansion for this function at $(0, 0)$.

4. (20 points) An airplane is flying at a constant height of 10,000 feet. The airplane travels in a circle of radius 1 and center (0,0, 10000) and notices the outside temperature at the point (x, y, z) is $x + 2y$. What is the hottest temperature the plane passes through? What is the coldest?

5. (20 points) Maximize the function $x^2 + y^2$ subject to $(x/2)^2 + y^2 = 1$.

Second Practice Midterm

1. (20 points) Let $f(x, y) = x^3y + \cos(xy)$, $g(u, v, w) = u + vw$, $h(r, s, t) = (r + s, r - t)$. Using the Chain Rule, compute $A(u, v, w) = f(g(u, v, w))$ if possible (and evaluate the derivative at $(0, 1, 0)$), or state why it is not possible; using the Chain Rule, compute $B(r, s, t) = f(h(r, s, t))$ if possible (and evaluate the derivative at $(0, 1, 0)$), or state why it is not possible.

2. (20 points) Let $f(x, y) = x^3y + \cos(xy)$. Compute all derivatives up to second order, and verify that the mixed partials are equal.

3. (20 points) Let f be a continuous function whose partial derivatives exist. Either prove that f is differentiable or give an example of such an f that is not differentiable.

4. (20 points) Let $f(x, y, z) = x^2+3xyz+e^z+4$. How fast is f increasing in the direction $(3/13, 4/13, 12/13)$? In which direction is f increasing fastest? Slowest?

5. (20 points) Find the maximum and minimum values of $f(x, y) = 5x^2 + 2y^2$ subject to $x^2 + y^2 \leq 100$.