Math 140: Calculus II: Spring ‘22 (Williams)

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Homepage:
https://web.williams.edu/Mathematics/sjmiller/public_html/140Sp22/

Lecture 11: 3-2-22: https://youtu.be/r91RvC9kPHc

https://web.williams.edu/Mathematics/sjmiller/public_html/140Sp22/talks2022/140Sp22_lecture11.pdf

Lecture: u-substitution

Plan for the day: Lecture 11: March 2, 2022:

Topics

Bring it Over

u-substitution

$$I = \int_0^{\pi} e^{cx} \cos x dx. \quad = uv\Big|_0^{\pi} - \int_0^{\pi} v\, du$$

$u = \cos x$ $\qquad$ $dv = e^{cx} dx$

$du = -\sin x dx$ $\qquad$ $v = \frac{1}{c} e^{cx}$

$$I = \frac{1}{c} \cos x \cdot e^{cx} \Big|_0^{\pi} + \frac{1}{c} \int_0^{\pi} \sin x \cdot e^{cx} dx$$

$$= -\frac{e^{c\pi}}{c} - \frac{1}{c} + \frac{1}{c} \int_0^{\pi} \sin x \cdot e^{cx} dx$$

$u = \sin x$ $\qquad$ $dv = e^{cx} dx$

$du = \cos x dx$ $\qquad$ $v = \frac{1}{c} e^{cx}$

$$I = -\frac{e^{c\pi}+1}{c} + \frac{1}{c}\left[\frac{1}{c} \sin x \cdot e^{cx} \Big|_0^{\pi} - \frac{1}{c} \int_0^{\pi} \cos x \cdot e^{cx} dx\right]$$

$$I = -\frac{e^{c\pi}+1}{c} - \frac{1}{c^2} I \Rightarrow \left(1 + \frac{1}{c^2}\right) I = -\frac{e^{c\pi}+1}{c}$$

$$I = \frac{-c^2}{c^2+1} \cdot \frac{e^{c\pi}+1}{c} = -\frac{c\,(e^{c\pi}+1)}{c^2+1}$$

In[•]:=
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Manipulate[Plot[Cos[x] Exp[c x], {x, 0, Pi}], {c, 0, 1}]
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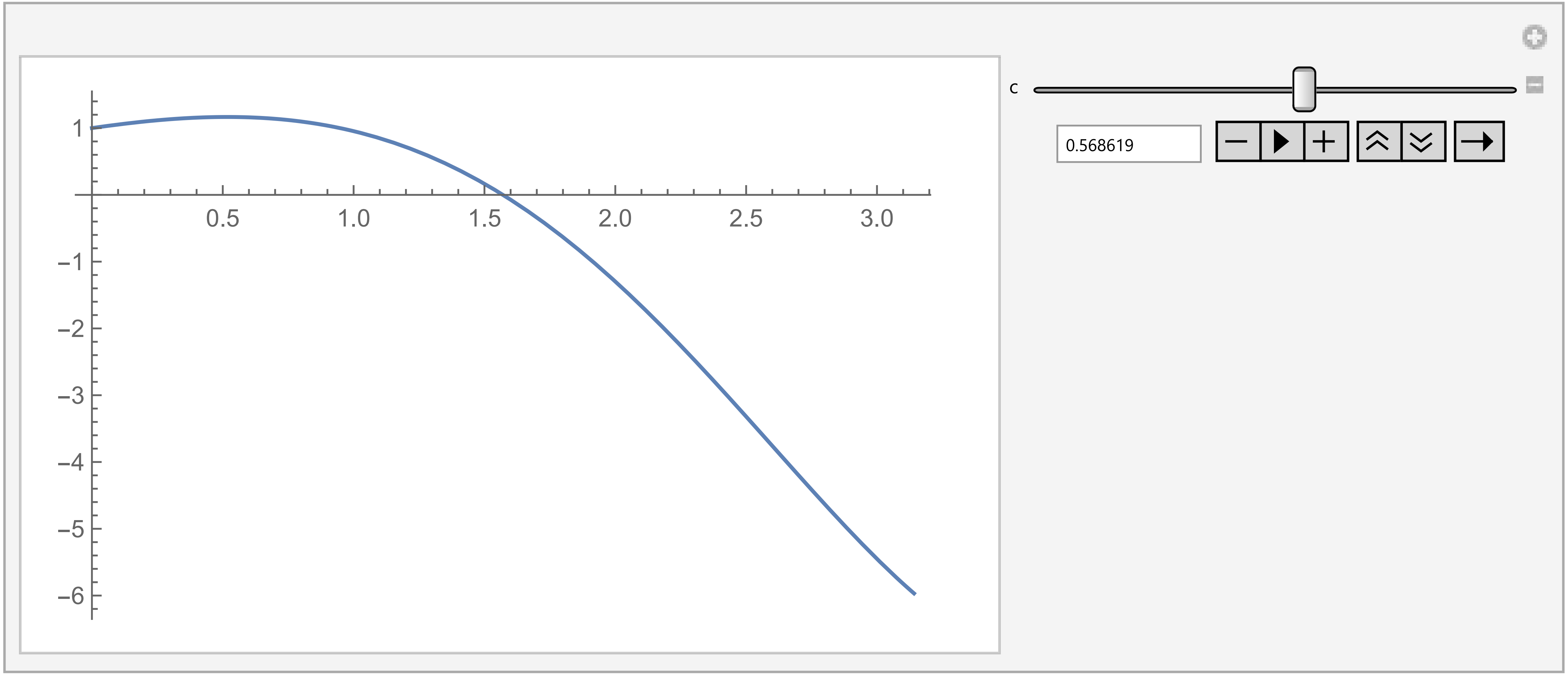
Out[•]=



THE METHOD OF u-SUBSTITUTION

$$\int [f(g(x))]' \, dx = \int f'(g(x)) \, g'(x) \, dx$$

$$f(g(x)) = \int f(u) \, du$$

$u = g(x)$

$\frac{du}{dx} = g'(x) \rightarrow du = g'(x)dx$

When $u = g(x)$, need This to be a increasing or decreasing function

$$\int_{-2}^{2} \frac{1}{1+x^2}\,dx = 2\int_0^2 \frac{1}{1+x^2}\,dx$$

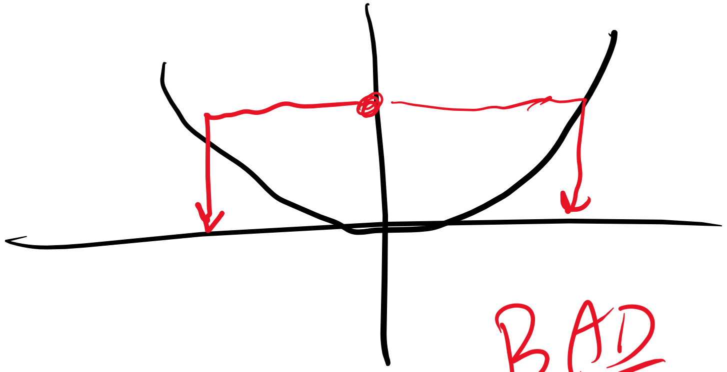
$u = x^2$ $du = 2x\,dx$ so $dx = \frac{du}{2x} = \frac{du}{2\sqrt{u}}$

$x: -2$ to 2 Then $u: 4 \to 4$

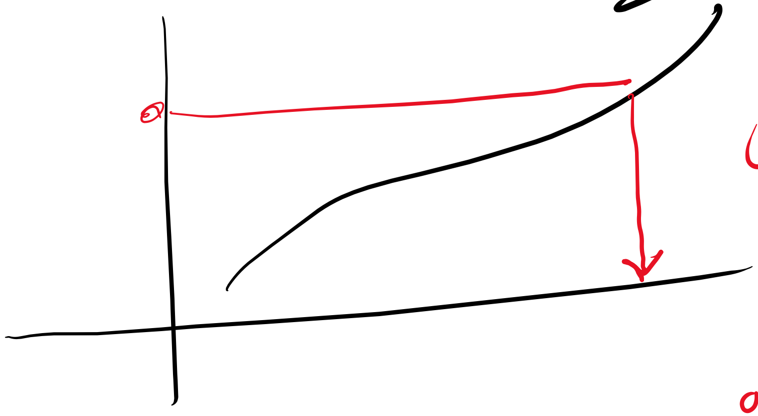
"equals" $\int_{u=4}^{4} \frac{1}{1+u}\frac{1}{2\sqrt{u}}\,du = 0$

Absurd!

$y = g(x)$



BAD



$g'(x) > 0$ or $g'(x) < 0$ on interval

http://math.oxford.emory.edu/site/math111/uSubstitution/#:~:text=Let%20us%20consider%20a%20few%20examples%20of%20this,that%20d%20x%20%3D%201%203%20d%20u.

Find $\int \sqrt{3x+4}\,dx$

Find $\int t(5+3t^2)^8\,dt$

Find $\int x^2\sqrt{1+x}\,dx$

https://www.math.ucdavis.edu/~kouba/CalcTwoDIRECTORY/usubdirectory/USubstitution.html

$$\int \frac{x^2+1}{x^3+3x}\,dx \ .$$

$$\int \frac{\sin(\ln x)}{x}\,dx \ .$$

$$\int \frac{3}{x\ln x}\,dx \ .$$

$$\int (x+3)(x-1)^5\,dx$$

$$\int \frac{x+5}{2x+3}\,dx \ .$$

$$\int \frac{(3+\ln x)^2(2-\ln x)}{4x}\,dx$$

$$\int_0^9 \sqrt{4-\sqrt{x}}\,dx \ .$$

Find $\int_0^8 \sqrt{3x+4}\,dx$

$u = 3x+4$

$x: 0 \to 8$ so $u: 4 \to 28$

$u = 3x+4$

$\frac{du}{dx} = 3$

$du = 3dx$

or $dx = \frac{1}{3}du$

$= \int_4^{28} u^{1/2} \frac{1}{3} du$

$= \frac{1}{3} \int_{u=4}^{28} u^{1/2} du$

$= \frac{1}{3} \frac{2}{3} u^{3/2} \Big|_4^{28} = \frac{2}{9}\left(28^{3/2} - 4^{3/2}\right)$

$g(x) = 3x+4$, $g'(x) = 3 > 0$ increasing!

$$\int_0^9 \sqrt{4-\sqrt{x}}\,dx = \int_0^9 (4-x^{1/2})^{1/2}\,dx$$

$u = 4 - x^{1/2}$

$u = 4 - x^{1/2}$ $\quad x: 0 \to 9 \quad u: 4 \to 1$

$x^{\frac{1}{2}} = 4 - u$

$\frac{du}{dx} = -\frac{1}{2} x^{-1/2} < 0$ so decreasing

$du = -\frac{1}{2} x^{-\frac{1}{2}} dx$ so $dx = -2x^{1/2} du$

$dx = -2(4-u)\,du$

Integral is $\int_{u=4}^{1} u^{1/2}(-2)(4-u)\,du = -2\int_{u=4}^{1} u^{1/2}(4-u)\,du$

$= 2\int_{u=1}^{4} u^{1/2}(4-u)\,du$

THOREAU!

$= 2\left[\int_{u=1}^{4} 4u^{1/2}\,du - \int_{u=1}^{4} u^{3/2}\,du\right]$

$= 2\left[4\,\frac{2}{3}u^{3/2}\Big|_1^4 - \frac{2}{5}u^{5/2}\Big|_1^4\right] = 2\left(\frac{8}{3}(8-1) - \frac{2}{5}(32-1)\right)$

$$\int \frac{3}{x \ln x}\,dx = 3 \int \frac{1}{x \ln x}\,dx = 3 \int \frac{1}{\ln x}\,\frac{1}{x}\,dx$$

$u = \ln(x)$

$\frac{du}{dx} = \frac{1}{x}$ $\quad du = \frac{1}{x}\,dx$

Integral is $3 \int \frac{1}{u}\,du = 3\ln(u) + C$

$= 3\ln(\ln x) + C$

$$\int \frac{x^2+1}{x^3+3x}\,dx .$$

$u = x^3 + 3x$

$\frac{du}{dx} = 3x^2 + 3 = 3(x^2+1)$

so $du = 3(x^2+1)\,dx$ or $\frac{1}{3}du = (x^2+1)\,dx$

Integral is $\int \frac{1}{u}\,\frac{1}{3}\,du = \frac{1}{3}\ln(u) + C$

$= \frac{1}{3}\ln(x^3+3x) + C$

$= \frac{1}{3}\left[\ln(x) + \ln(x^2+3)\right] + C$

$$\int (x+3)(x-1)^5 \, dx$$

$u = x - 1$ $\qquad \frac{du}{dx} = 1 \implies du = dx$

$x + 3 = ?$

$u = x - 1$ Then $x = u + 1$ so $x + 3 = u + 4$

THOREAU!

Integral $= \int (u+4) u^5 \, du$

$$= \int u^6 \, du + 4 \int u^5 \, du$$

$$= \frac{1}{7} u^7 + \frac{4}{6} u^6 + C$$

$$= \frac{1}{7}(x-1)^7 + \frac{2}{3}(x-1)^6 + C$$

$$\int (x+3)(x-1)^5\,dx$$

$$x+3 = x-1+4$$

$$= \int (x-1+4)(x-1)^5\,dx$$

$$= \int (x-1)^6\,dx + 4\int (x-1)^5\,dx$$

$$= \frac{(x-1)^7}{7} + \frac{4}{6}(x-1)^6 + C$$