# Math 140: Calculus II: Spring '22 (Williams) Professor Steven J Miller: sjm1@williams.edu 

## Homepage:

https://web.williams.edu/Mathematics/sjmiller/ public html/140Sp22/

Lecture 28: 4-25-22: httos///routubeep-. SGisizoxo
https://web.williams.edu/Mathematics/sjmiller/public html/140Sp22/talks2022/140Sp22 lecture28.pdf

## Plan for the day: Lecture 28: April 25, 2022:

Topics: Review / Application Day


- Related rates: For a given length $L$, what cone has the greatest volume for a given surface area? (Note: how do we find surface area - it's hard!) How fast is the volume changing as the angle changes?
- Split an integer N as a sum of non-negative integers such that the product is maximized - what is the best way to do this? Application to storing information!
- Can we prove a circle encloses the largest area for a given perimeter?

For a given length $L$, what cone has the greatest volume for a given surface area? (Note: how do we find surface area - it's hard!) How fast is the volume changing as the angle changes?

$$
\begin{aligned}
& \text { Udume } \\
& \\
& \hdashline \\
& \hdashline \\
& \hdashline
\end{aligned}
$$

If rod of length $L$, Then

$$
\begin{aligned}
\int_{y=0}^{b} & \pi\left[\frac{1}{m}(y-b)\right]^{2} d y \\
& =\frac{\pi}{m^{2}} \int_{y=0}^{b}(y-b)^{2} d y \\
& =\left.\frac{\pi}{m^{2}} \frac{(y-b)^{3}}{3}\right|_{0} ^{b}=\frac{\pi b^{3}}{3 m^{2}}
\end{aligned}
$$

$m$ and $b$ are functions of 2
can choose cns $n \in(-\infty, 0]$ and then $b$ is a Auction of nard $L$

Su-face area
Recall for lershs of curvas:

$$
\begin{aligned}
& \int_{x=x_{i}}^{x f} \sqrt{1+\left(\frac{d y}{d x}\right)^{2}} d x \\
& \int_{t=t_{i}}^{x_{f}} \sqrt{x^{\prime}(t)^{2}+y^{\prime}(t)^{2}} d t=\frac{d x}{d t} \\
& \int_{t=t i}^{t s} \sqrt{\left(\frac{d x}{d t}\right)^{2}(d t)^{2}+\left(\frac{d y}{d t}\right)^{2}(d t)^{2}} \\
& =\int(d x)=(x(t), y(t)) \\
& (d x)^{2}+(d y)^{2}
\end{aligned}=\int \sqrt{1+\left(\frac{d y}{d x}\right)^{2}} d x .
$$

Surface Area

ald surface area of disks wither tool bottom
at height $y$, radius is $x=f^{-1}(y)$
$f^{-1}=g$ then have $x=g(y)$
leash is $\sqrt{1+\left(\frac{d x}{d y}\right)^{2}} d y$

$$
\int_{y=y_{i}}^{y_{f}} 2 \pi f^{-1}(y) \sqrt{1+\left[\left(f^{-1}\right)^{1}\right]^{2}} d y
$$

us: $x=\frac{1}{m}(y-b)$

$$
\text { so } \frac{d x}{d y}=\frac{1}{m}
$$

Split an integer N as a sum of non-negative integers such that the product is maximized - what is the best way to do this? Application to storing information!

$$
\begin{array}{r}
N=-N-N+S N \quad \text { product is } 3 N^{3} \\
N=-m N-N N+(2 m+1) N \\
\\
\text { product cs }(2 n+1) m^{2} N^{3} \\
\text { as } n \rightarrow \infty \\
\text { product } \rightarrow \infty \\
\text { Clearly new use } O \text { as a sunnand (nates product } 2 e r \text { ) }
\end{array}
$$

Split an integer N as a sum of non-negative integers such that the product is maximized - what is the best way to do this? Application to storing information!

$$
N=a_{c}+\cdots+a_{n} \text { with } a_{1} \cdots a_{n} \text { naxinezed }
$$

know

$$
\begin{array}{ll}
1 & \leq a_{i} \\
\vdots & \\
9 & =7+2
\end{array} \quad \text { product } 14.1 \leq a_{i} \leq 3
$$

Split an integer $N$ as a sum of non-negative integers such that the product is maximized - what is the best way to do this? Application to storing information!
Imagine


$$
N=a_{1}+\cdots+a_{2}+a_{9}+\cdots+\left(a_{1}+1\right)
$$

product us large

$$
\Longrightarrow
$$

$$
a_{i} \in\{2,3\}
$$

Split an integer N as a sum of non-negative integers such that the product is maximized - what is the best way to do this? Application to storing information!
Each $a_{i} \in\{2,3\}$
Which is betuter: more 3's or noe 2's?

$$
\underbrace{3+3+\underbrace{2+2+2}_{\text {product }}}_{\substack{\text { product } \\ q}}
$$

Ans: all 3's and ore o-two 2's


Split an integer N as a sum of non-negative numbers such that the product is maximized - what is the best way to do this? Application to storing information!
$N=a_{1}+a_{2} \ldots+a_{n}$ naxinire $a_{1} \ldots a_{n}$
Know each $a_{i} \in[1,4]$
Solve for each n

Case 1: $n=1$
soln: $N=N$

Case 2: $n=2=$
sols: $a_{1}+a_{2}=0$ max $a_{1} a_{2}$ Farmer Brown Pollen!

$$
\Rightarrow a_{1}=a_{2}=N / Z
$$

Split an integer N as a sum of non-negative numbers such that the product is maximized - what is the best way to do this? Application to storing information!
Geneal Case
By analysis There is at lest one optimal choice Imagine have $a_{1}+a_{2}+\cdots+a_{n}=N$ optimal for Re product

If all not eivel, whey assure $a_{1}<a_{2}$
Farmer Brown! Do bette with $\frac{a_{1}+a_{2}}{2}$ and $\frac{a_{1}+a_{2}}{2}$
$\leftrightarrow$ Sane sun, large product
So optimal has $a_{n}=N / n$

Split an integer $N$ as a sum of non-negative numbers such that the product is maximized - what is the best way to do this? Application to storing information!
For ans $n$, best product is $f(0)=(N / n)^{n}$
$\rightarrow$ optimize ore $n \in\{1,2,3, \ldots\}$

$$
\begin{aligned}
g(x) & =(N / x)^{x} j^{u s t l} \operatorname{lite} \text { inter. test. rplaceinbx } \\
= & e^{x \log (N / x)}=e^{x[\log (N)-\log (x)]}
\end{aligned}
$$

$g(1)=N \quad g(\infty)=0$ so not an endpoint...

$$
\begin{aligned}
g(x) & =e^{x \log (\omega / x)} \\
g^{\prime}(x) & =e^{x \log (\omega x)} *\left(x[\log (v)-\log (x))^{\prime}\right. \\
& =e^{x \log (N / x)} *\left[\log (N)-\log (x)-x \frac{1}{x}\right] \\
& =e^{x \log (\omega / x)} *[\log (v)-\log (x)-\log ((e)] \\
& =e^{x \log (N / x)} * \log \left(\frac{\omega}{x e}\right)
\end{aligned}
$$

so Critical Point has $g^{\prime}(x)=0$ or $\log \left(\frac{\tau}{x e}\right)=0$ so $N / x e=1 \Rightarrow x=N / e$, each puce of sine $e$

$$
g^{\prime}(x)=\underbrace{g(x)}_{\text {posinuce }} \log \left(\frac{v}{x e}\right)
$$



Base 3 mote efficient ha Base 2 for store of info

