Math 140: Calculus II: Spring '22 (Williams) Professor Steven J Miller: <u>sjm1@williams.edu</u>

Homepage: https://web.williams.edu/Mathematics/sjmiller/ public html/140Sp22/

Lecture 28: 4-25-22: https://youtu.be/-p-SGj3YOXo

https://web.williams.edu/Mathematics/sjmiller/public_html/140Sp22/talks2022/140Sp22_lecture28.pdf

Plan for the day: Lecture 28: April 25, 2022:

Topics: Review / Application Day



- Related rates: For a given length L, what cone has the greatest volume for a given surface area? (Note: how do we find surface area – it's hard!) How fast is the volume changing as the angle changes?
- Split an integer N as a sum of non-negative integers such that the product is maximized what is the best way to do this? Application to storing information!
- Can we prove a circle encloses the largest area for a given perimeter?

For a given length L, what cone has the greatest volume for a given surface area? (Note: how do we find surface area – it's hard!) How fast is the volume changing as the angle changes?

 $\int_{a}^{\infty} \mathcal{T}\left[\frac{1}{m}\left(y-6\right)\right]^{2} dy$ $y=0 \qquad \text{Tr} \qquad \int_{a}^{b} \left(y-6\right)^{2} dy$ $= m^{2} \qquad y=0$ Sume $y = m \times + b$ $X = - \frac{1}{m}(y - 6)$ $z T (y-6)^{3} = \frac{\pi 6}{3m^{2}}$ (f rod of length L, Den Mand 6 are functions of 2 Carchoose any mf (w, o] and then b is a known of mand L

Sufface area Recall for lensths of corres'. $\int_{t}^{t} \int_{t} \left(\frac{dy}{dx} \right)^{2} dx$ $\chi'(t) = d\chi$ $\chi = \chi_c$ $\int X'(t)^2 + Y'(t)^2 dt$ ((t)= (X(t), y(4)) $\begin{aligned} t = t_{i} \quad \lim_{k \neq i} \int_{a_{i}}^{b_{i}} \int_{a_{i}}^{d_{i}} \int_{a_{i}}^{z} (dt)^{2} + (\frac{dy}{dt})^{2} (dt)^{2} \\ t = t_{i} \quad z \quad (\sqrt{dx})^{2} + (\frac{dy}{dt})^{2} = \int \int_{a_{i}}^{z} \int_{a_{i}}^{d_{i}} \int_{a_{i}}^{z} dx \end{aligned}$

Surface Grea



add Surface area of dists what top 60thom at height y, radius is X=f'(y) f'= g then have X = g(y) length is $\int I + \left(\frac{dx}{dy}\right)^2 dy$ $\int 2\pi f'(y) \int [+[(f')]^2 dy$ y=9;

US: X= $\frac{1}{m}(7-6)$

Split an integer N as a sum of non-negative integers such that the product is maximized – what is the best way to do this? Application to storing information!

N = -N - N + 3N product is $3N^3$ N = -mN - mN + (2m+1)N

05 M-Joo product 200

product cs (zmti) m² N³

Clearly new use O as a summand (mates product Zerd)

Split an integer N as a sum of non-negative integers such that the product is maximized – what is the best way to do this? Application to storing information!

NEalt...tan with a...an maximized Know 1 E Qi ? 9 = 7 + 2 product 14 product (2 8=6+2 product 10 2= 5+2 Laie3 11 8 6 = 4 € 2 .. 6 a; e E 1, 2, 33 5= 3+2 product 4 Indictionst $4 = 2 \pm 2$

Split an integer N as a sum of non-negative integers such that the product is maximized – what is the best way to do this? Application to storing information!

Incolve NE 9,49,4,49,4,49,4,49,40,40,7 $N = a_1 + \dots + a_7 + a_q + \dots + (a_n + i)$ product is larger

 $\implies a_i \in \{2,3\}$

Split an integer N as a sum of non-negative integers such that the product is maximized – what is the best way to do this? Application to storing information!

Each Qi E { Z, 3} Which is better: more 3's on more Z's? 3+3 = 2+2+2 product q product

ans: all 3's and one o-two Z's

Cz

C~2,71828....

Split an integer N as a sum of non-negative Nonbers such that the product is maximized – what is the best way to do this? Application to storing information!

NE altaz--- tan maximize al---an Know each ac E [1,47 Solve for each n Case Zi nzzz Case I: N= 1 Soln: N=N Suln: actaz = N max alaz Færne Brun Problem! > a1= a7 = N/2

Split an integer N as a sum of non-negative Nonbers such that the product is maximized – what is the best way to do this? Application to storing information!

General Case By analysis There is at least one optimal choice Tragene have aitazt...+an=N is optimil for Deproduct If all not erval, whog assume a, 2az Farme Brun! Do bette with 91,402 and 91,402 6 Same Sun, larger product So optimal has an = N/n 11

Split an integer N as a sum of non-negative Nonbers such that the product is maximized – what is the best way to do this? Application to storing information!

For any n, best product as $f(n) = (N/n)^n$ La optimize our nE E1, 2, 3, ... 3 $g(x) = (M/x)^{X}$ ist lite integral tests replace rules with x $= \mathcal{O}^{\chi \log(n)} = \mathcal{O}^{\chi \log(n)} - \log(n)$ so not an endpoint... $g(\omega) = 0$ $g(i) \equiv N$

 $g(x) = e^{\chi \log(n/x)}$ $g'(x) = e^{-x \log(nx)} * \left(x \left[\log(n) - \log(x)\right]\right)'$ $= e^{-x \log(nx)} * \left[\log(n) - \log(x) - \frac{x}{x}\right]$ $= e^{\chi \log(N/\chi)} \times \left[\log(N) - \log(\chi) - \log(e) \right]$ $z e^{\chi \log(N/\chi)} = \log\left(\frac{N}{\chi e}\right)$ So Critical Point has g'(x) = 0 or $\log(\frac{1}{xe}) = 0$ so N(xe) = 1 = 2 X = N/e, each precedure e

 $g'(X) = g(X) \log(\frac{\pi}{xe})$ gosinie realmax realmax intege max is eithe Re smallest intege large Than We or largest itster smalle Than Nic > X-axis $\frac{+++++}{\nu(e)} \xrightarrow{g'(x)}$ Base 3 note efficient tha Base 2 for storage of info