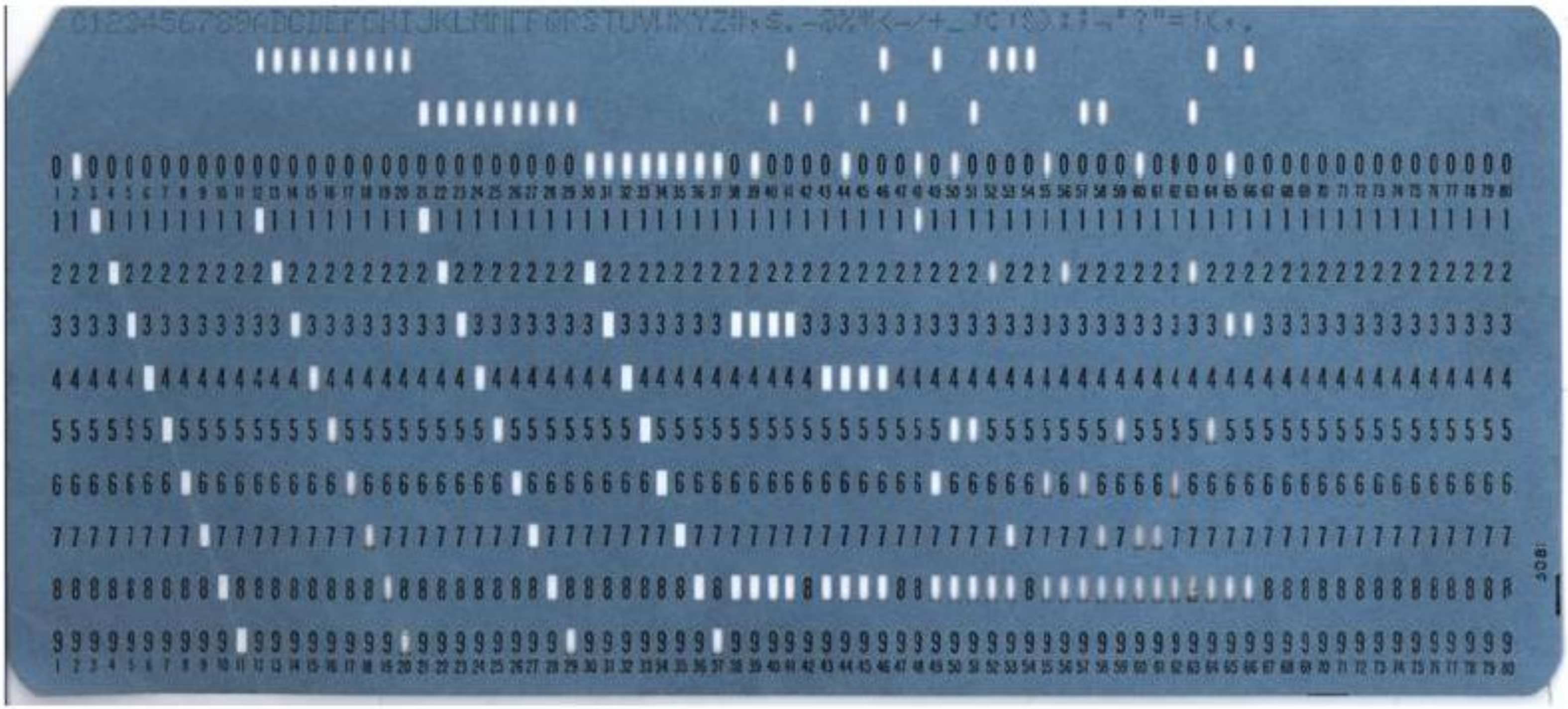
Images for Streaming Video Lecture

Professor Steven J Miller

Williams College

From UPC Symbols to Target

6 39382 00039 3



IBM punchcard. From Gwern via Wikimedia Commons.

1111111,	0010110,	1010101,	0111100,
0110011,	1011010,	0011001,	1110000,
0001111,	1100110,	0100101,	1001100,
1000011,	0101010,	1101001,	0000000;

Hamming (7,4) Code

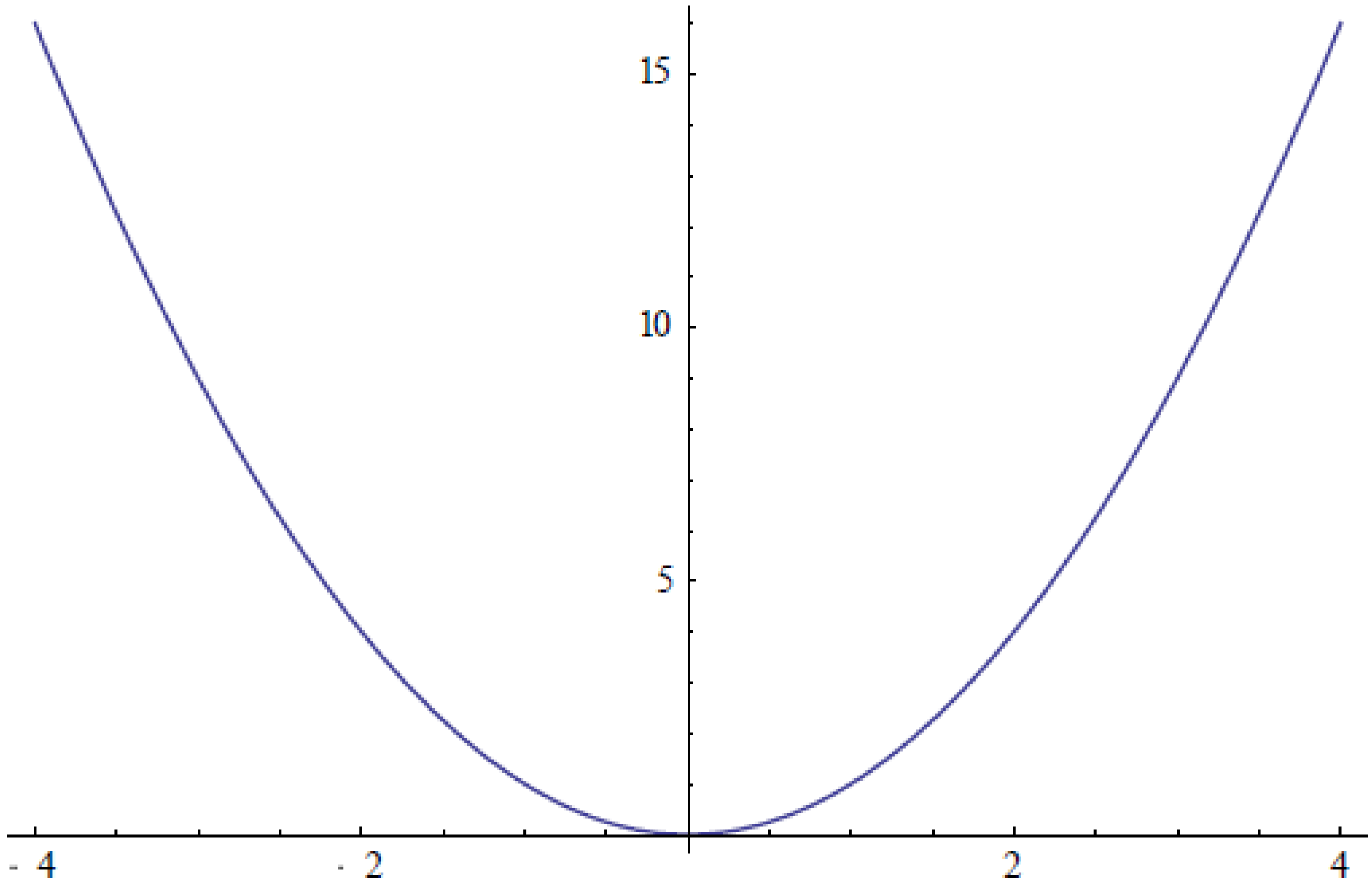


Figure 1: The plot of $y = x^2$ from -4 to 4.

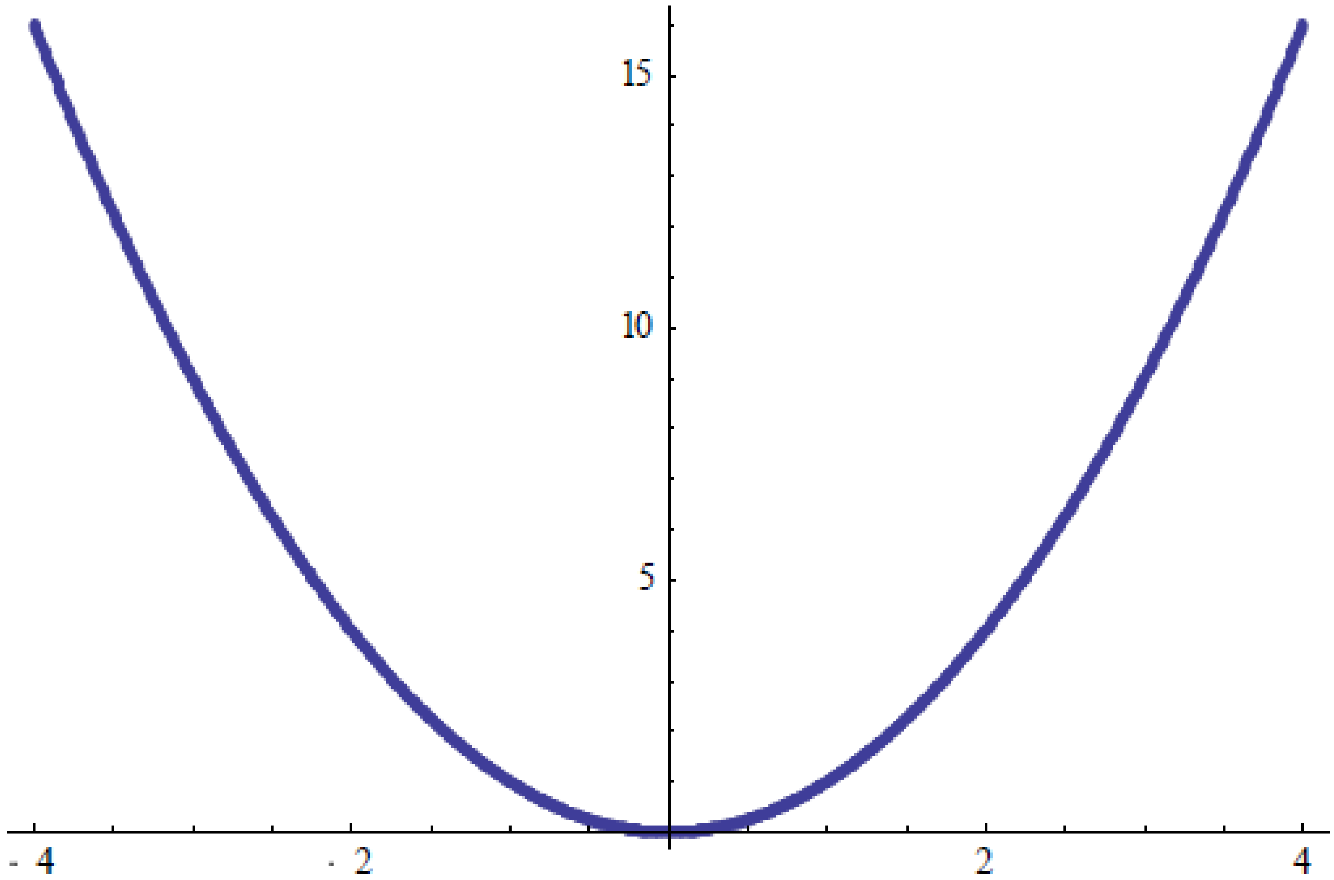


Figure 2: Approximating the plot of $y = x^2$ by sampling the function 100 times in each interval of length 1 from -4 to 4, with the 800 points equally spaced.

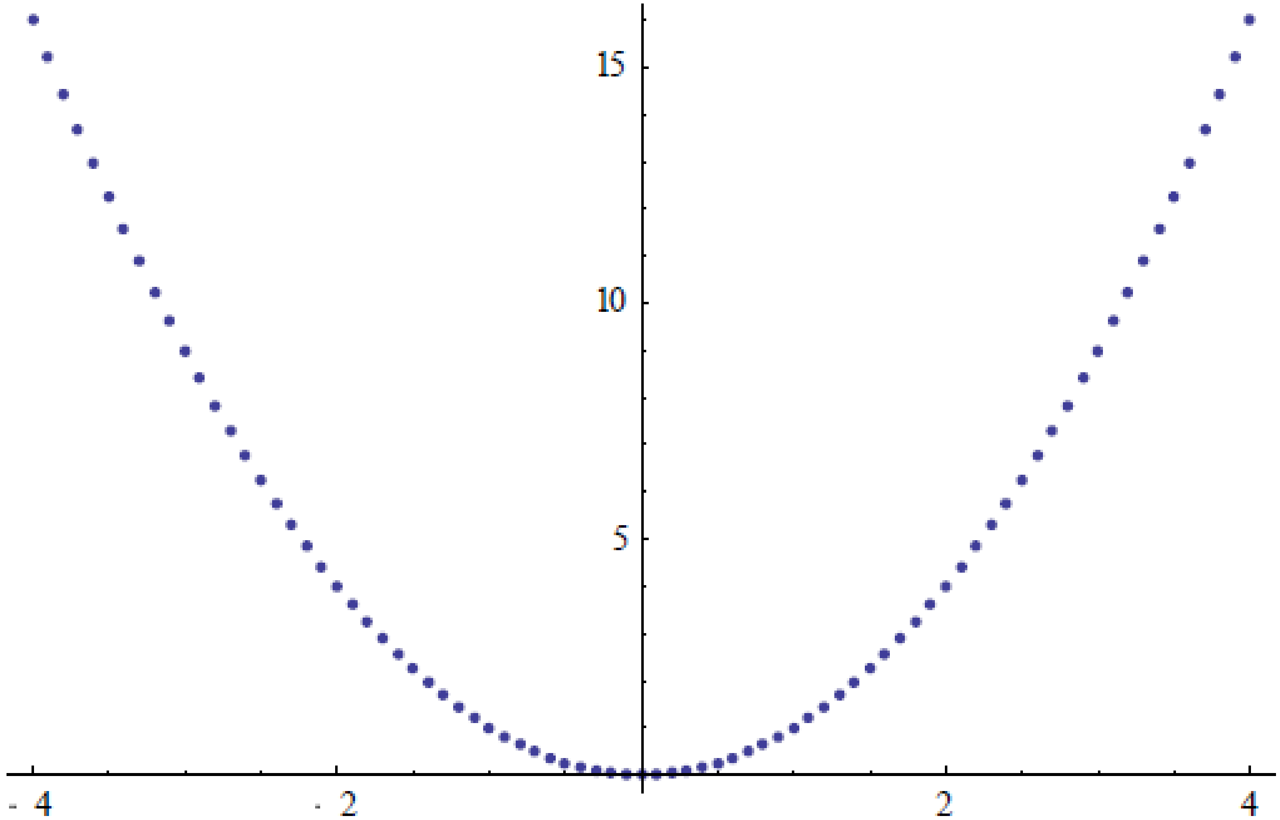


Figure 3: Approximating the plot of $y = x^2$ by sampling the function 10 times in each interval of length 1 from -4 to 4, with the 800 points equally spaced.

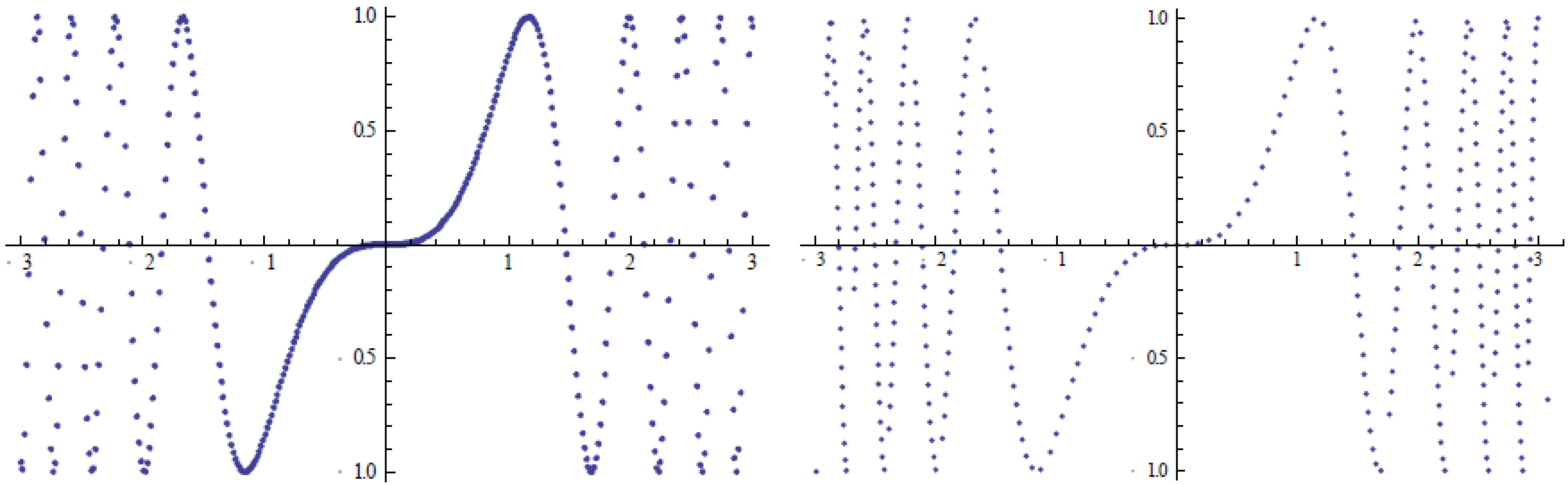


Figure 4: Two plots of the same function, sampled 365 times from -3 to 3.

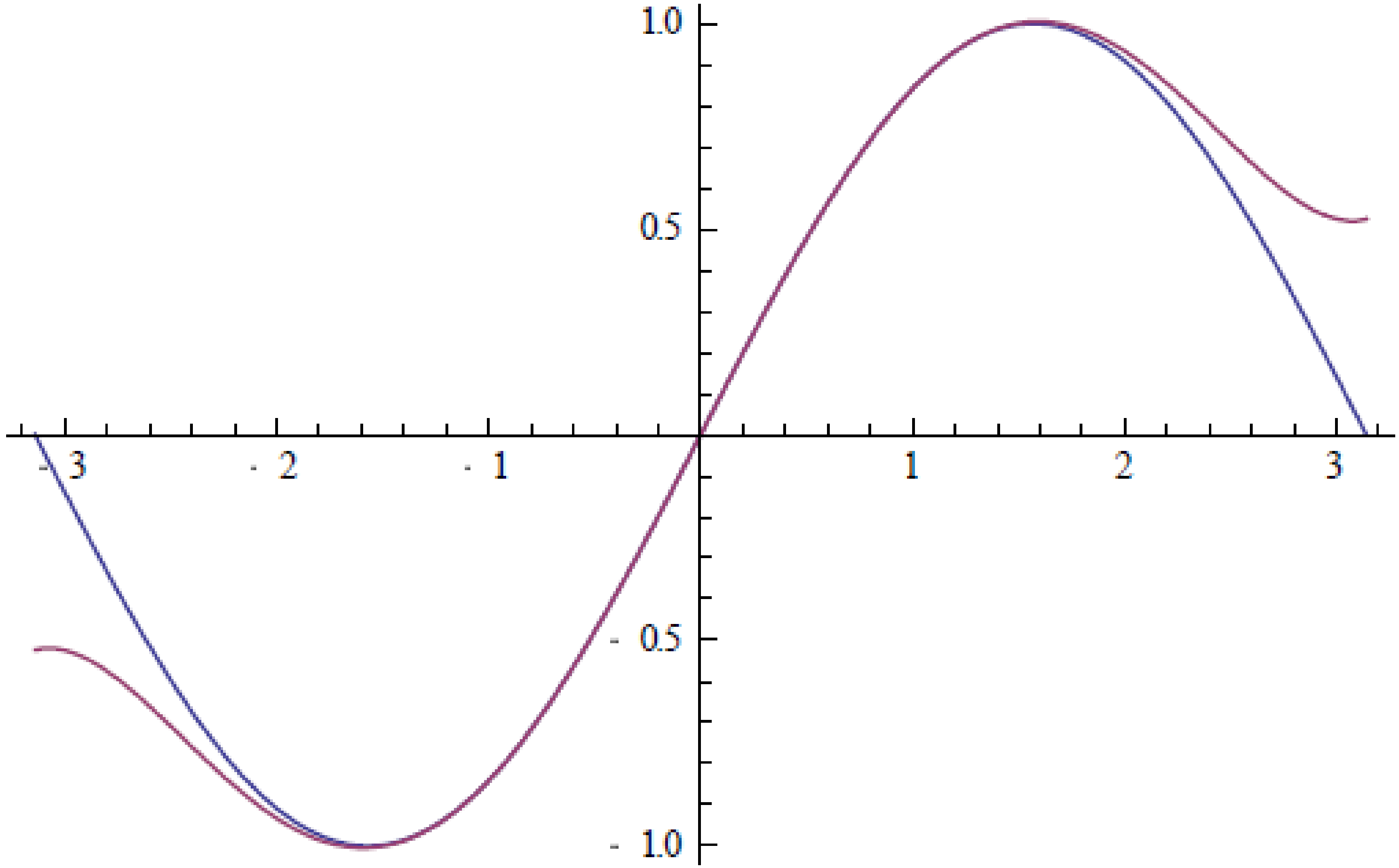


Figure 5: Plot of $y = \sin(x)$ and $y = x - x^3/6 + x^5/120$.

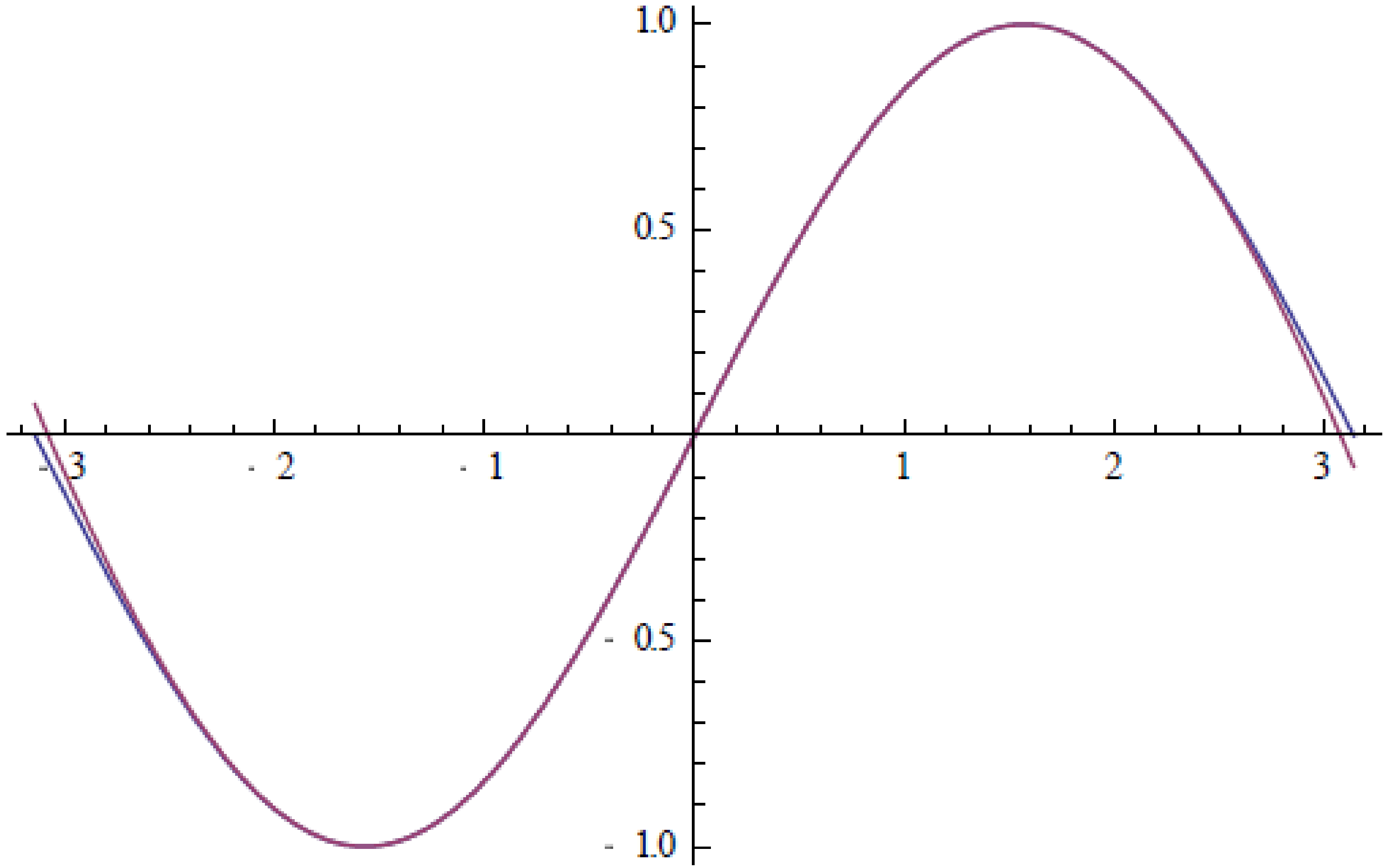


Figure 6: Plot of $y = \sin(x)$ and $y = x - x^3/6 + x^5/120 - x^7/5040$.

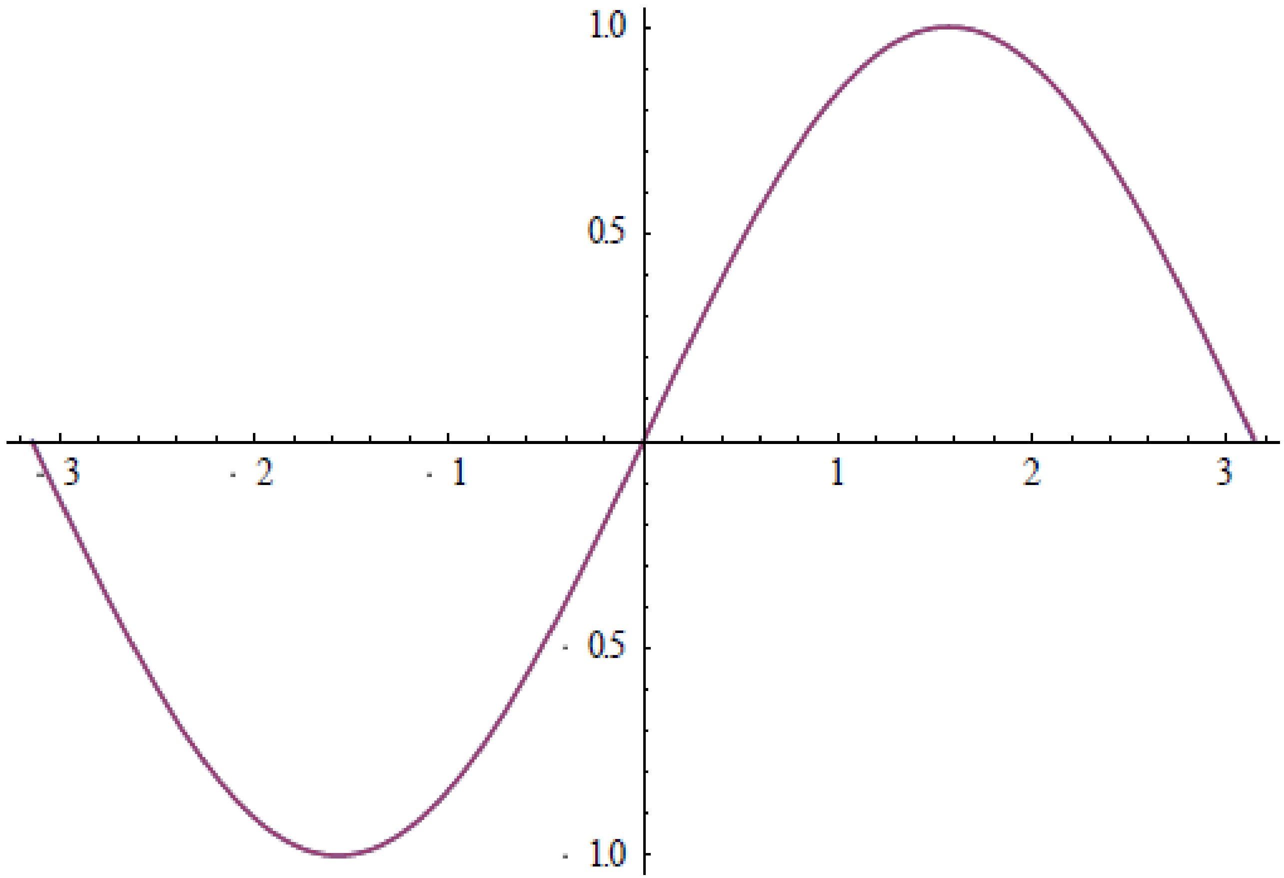


Figure 7: Plot of $y = \sin(x)$ and $y = x - x^3/6 + x^5/120 - x^7/5040 + x^9/362880$.

Taylor Series: write a function as a linear combination of $1, x, x^2, x^3, x^4, \ldots$

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$$

Fourier Series: write a function as a linear combination of 1, sin(x), cos(x), sin(2x), cos(2x), sin(3x), cos(3x), ….

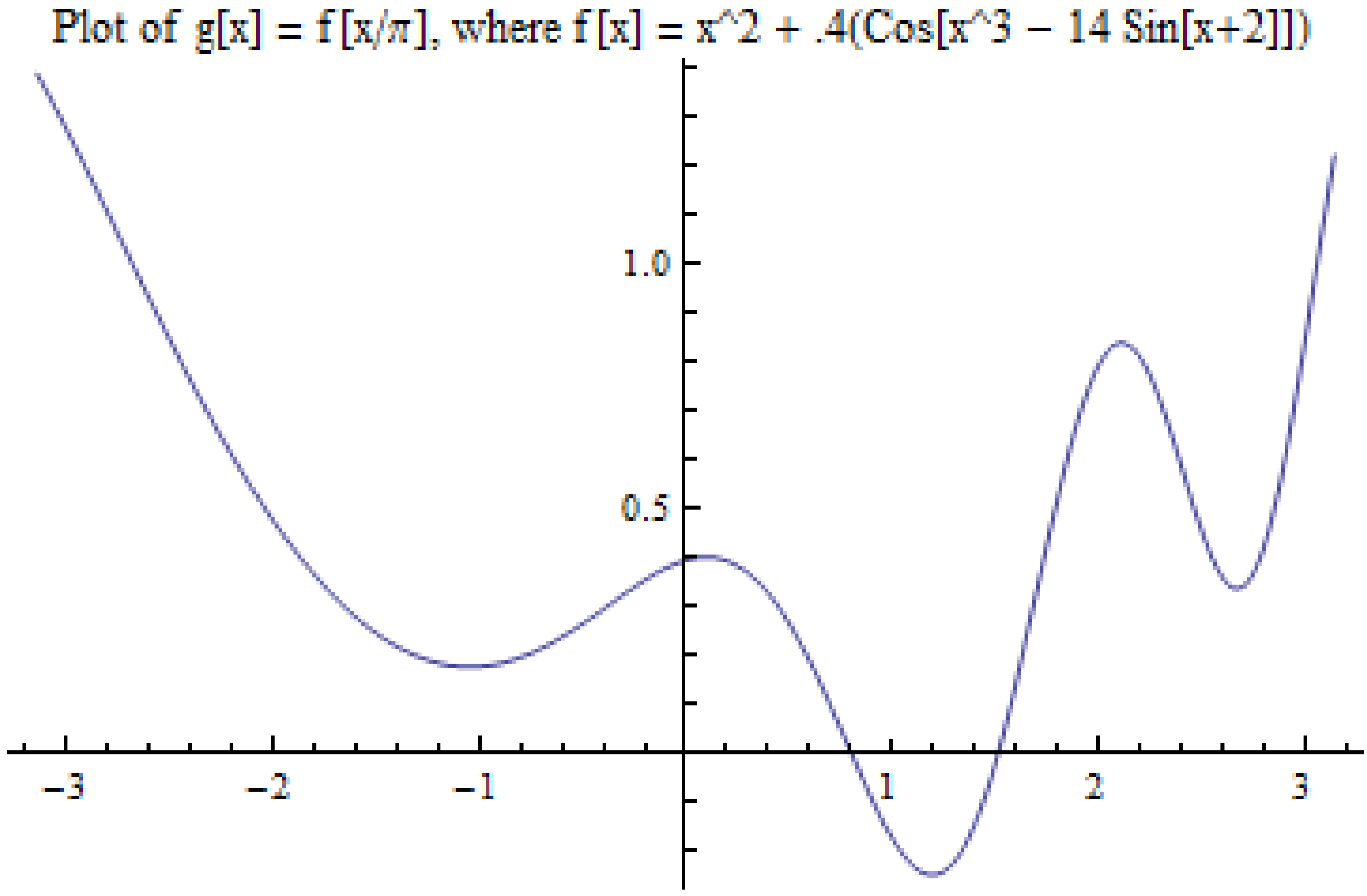
$$f(x) = a_0 + a_1 \sin(x) + b_1 \cos(x) + a_2 \sin(2x) + b_2 \cos(2x) + a_3 \sin(3x) + b_3 \cos(3x) + \ldots$$

Advantage of Fourier series over Taylor series:

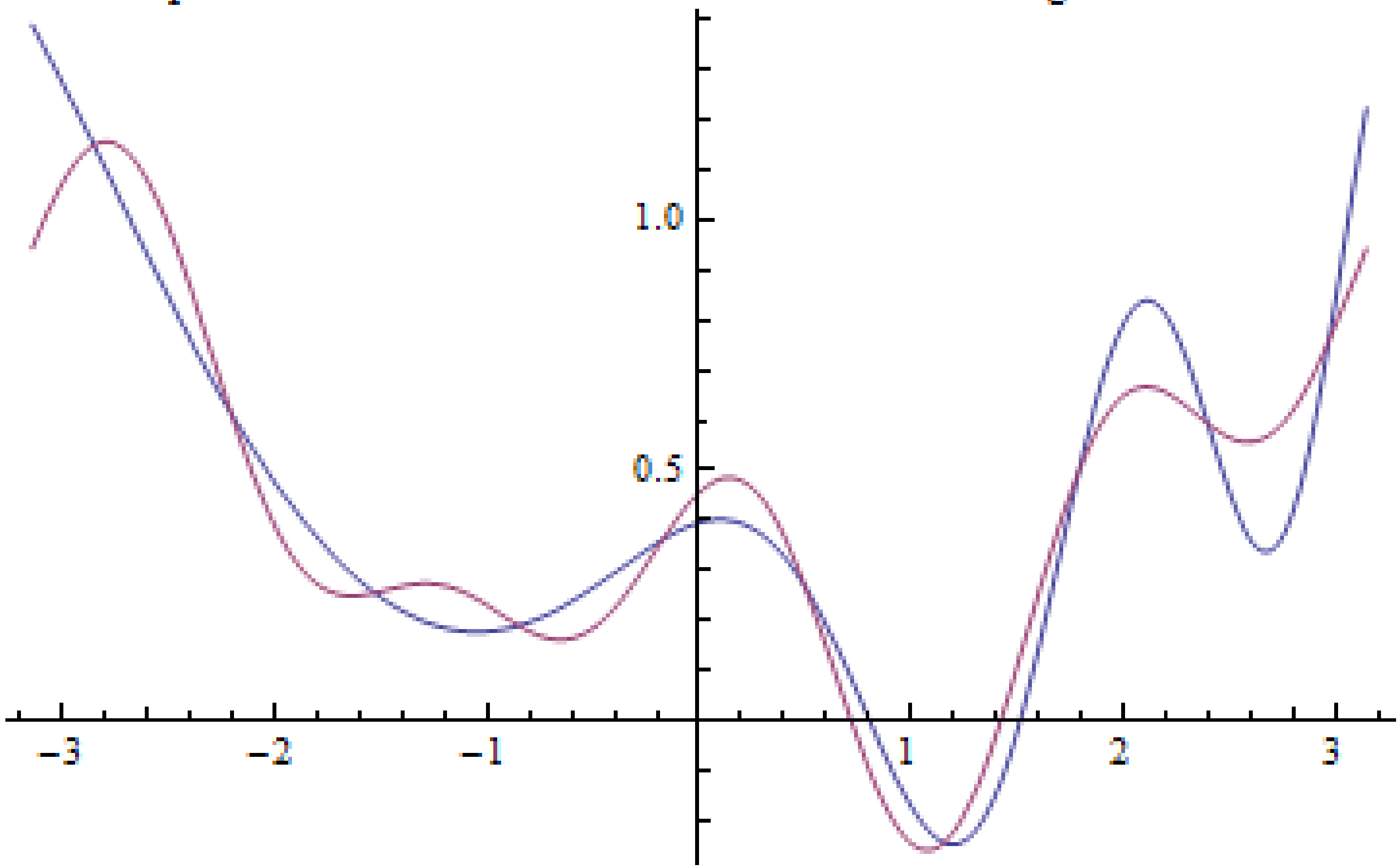
Taylor series require differentiation, Fourier only requires integrability.

We have for example:

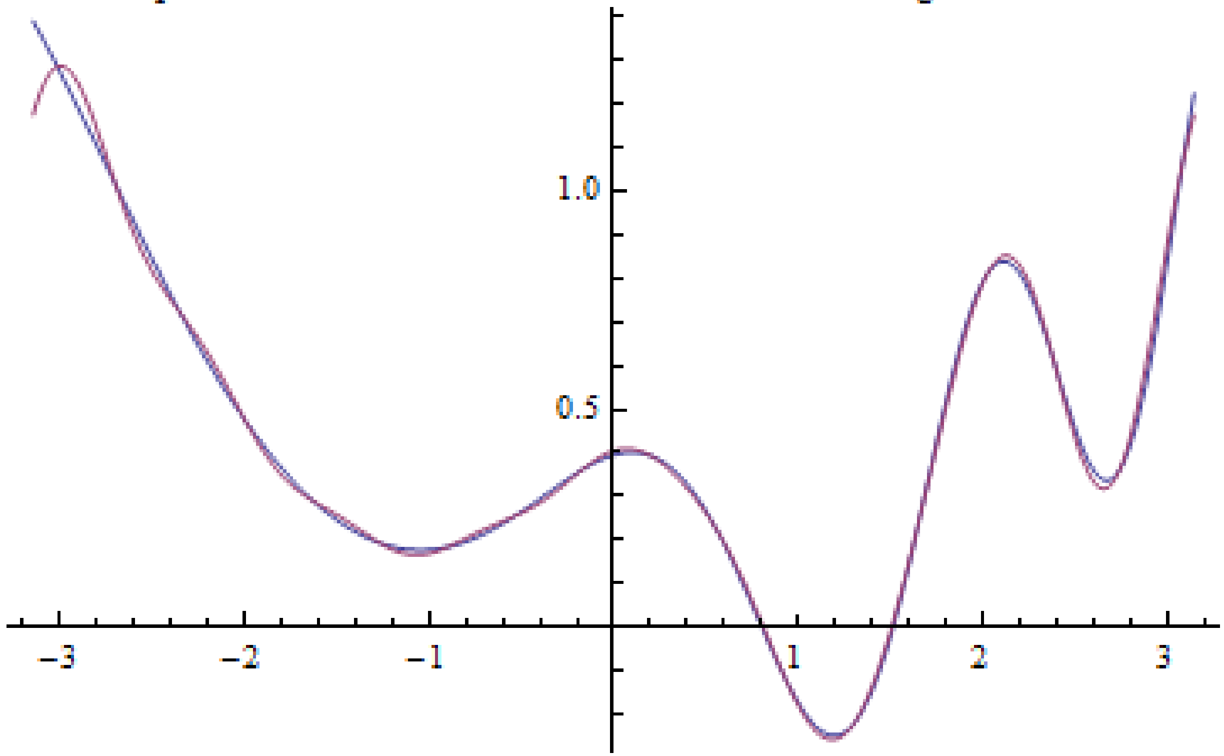
$$a_n = \int_{-\pi}^{\pi} f(x) \sin(nx)\, dx \,/\, \int_{-\pi}^{\pi} \sin^2(nx)\, dx = \int_{-\pi}^{\pi} f(x) \sin(nx)\, dx \,/\, \pi.$$


Plot of g[x] = f [x/π], where f [x] = x^2 + .4(Cos[x^3 − 14 Sin[x+2]])
1.0
0.5
−3
−2
−1
1
2
3

Comparison of Fourier Series with N=4 and the original function.



Comparison of Fourier Series with N=8 and the original function.



$$0.421988 - 0.357105\,\mathrm{Cos}[x] + 0.233479\,\mathrm{Cos}[2x] + 0.112953\,\mathrm{Cos}[3x] + 0.0436603\,\mathrm{Cos}[4x] - 0.103286\,\mathrm{Cos}[5x] + 0.0667661\,\mathrm{Cos}[6x] - 0.0381773\,\mathrm{Cos}[7x] + 0.0241078\,\mathrm{Cos}[8x] - 0.0917627\,\mathrm{Sin}[x] - 0.0122759\,\mathrm{Sin}[2x] - 0.0733134\,\mathrm{Sin}[3x] + 0.184999\,\mathrm{Sin}[4x] - 0.0706455\,\mathrm{Sin}[5x] + 0.0139808\,\mathrm{Sin}[6x] - 0.00305311\,\mathrm{Sin}[7x] + 0.00237302\,\mathrm{Sin}[8x]$$

g[x] = f[x/π], where f[x] = x^2 + .4(Cos[x^3 − 14 Sin[x+2]])