From UPC Symbols to Target
IBM punchcard. From Gwern via Wikimedia Commons.
Hamming (7,4) Code

1111111, 0010110, 1010101, 0111100,
0110011, 1011010, 0011001, 1110000,
0001111, 1100110, 0100101, 1001100,
1000011, 0101010, 1101001, 0000000;
Figure 1: The plot of $y = x^2$ from -4 to 4.
Figure 2: Approximating the plot of $y = x^2$ by sampling the function 100 times in each interval of length 1 from -4 to 4, with the 800 points equally spaced.
Figure 3: Approximating the plot of $y = x^2$ by sampling the function 10 times in each interval of length 1 from -4 to 4, with the 800 points equally spaced.
Figure 4: Two plots of the same function, sampled 365 times from -3 to 3.
Figure 5: Plot of $y = \sin(x)$ and $y = x - x^3/6 + x^5/120$. 
Figure 6: Plot of $y = \sin(x)$ and $y = x - x^3/6 + x^5/120 - x^7/5040$. 
Figure 7: Plot of $y = \sin(x)$ and $y = x - x^3/6 + x^5/120 - x^7/5040 + x^9/362880$. 
Taylor Series: write a function as a linear combination of $1, x, x^2, x^3, x^4, \ldots$.

$$f(x) = a_0 + a_1 x + a_2 x^2 + a_3 x^3 + \ldots$$

Fourier Series: write a function as a linear combination of $1, \sin(x), \cos(x), \sin(2x), \cos(2x), \sin(3x), \cos(3x), \ldots$.

$$f(x) = a_0 + a_1 \sin(x) + b_1 \cos(x) + a_2 \sin(2x) + b_2 \cos(2x) + a_3 \sin(3x) + b_3 \cos(3x) + \ldots$$
Advantage of Fourier series over Taylor series:

Taylor series require differentiation, Fourier only requires integrability.

We have for example:
\[ a_n = \frac{\int_{-\pi}^{\pi} f(x) \sin(nx) \, dx}{\int_{-\pi}^{\pi} \sin^2(nx) \, dx} = \frac{\int_{-\pi}^{\pi} f(x) \sin(nx) \, dx}{\pi}. \]
Plot of $g(x) = f(x/\pi)$, where $f(x) = x^2 + .4(Cos[x^3 - 14 Sin[x+2]])$.
Comparison of Fourier Series with $N=4$ and the original function.
Comparison of Fourier Series with $N=8$ and the original function.

$$0.421988 - 0.357105 \cos(x) + 0.233479 \cos(2x) + 0.112953 \cos(3x) + 0.0436603 \cos(4x) - 0.103286 \cos(5x) + 0.0667661 \cos(6x) - 0.0381773 \cos(7x) + 0.0241078 \cos(8x) - 0.0917627 \sin(x) - 0.0122759 \sin(2x) - 0.0733134 \sin(3x) + 0.184999 \sin(4x) - 0.0706455 \sin(5x) + 0.0139808 \sin(6x) - 0.00305311 \sin(7x) + 0.00237302 \sin(8x)$$

$$g(x) = f(x/\pi), \text{ where } f(x) = x^2 + 0.4(\cos(x^3 - 14 \sin(x+2))]$$