

[http://web.williams.edu/Mathematics/sjmiller/public\\_html/150/](http://web.williams.edu/Mathematics/sjmiller/public_html/150/)

## Math 150: Multivariable Calculus: 10-10:50am, 11-11:50am Bronfman 106

Professor Steven Miller ([sjm1@williams.edu](mailto:sjm1@williams.edu)), 202 Bronfman Science Center (413-597-3293).

My office hours: TBD and whenever in my office ([click here for my schedule](#)). [Click here for TA office hours](#)

**COURSE DESCRIPTION:** Applications of calculus in mathematics, science, economics, psychology, the social sciences, involve several variables. This course extends calculus to several variables: vectors, partial derivatives, multiple integrals. There is also a unit on infinite series, sometimes with applications to differential equations. This course is the right starting point for students who have seen differentiation and integration before. Students with the equivalent of advanced placement of AB 4, BC 3 or above should enroll in Mathematics 105. Prerequisites: Mathematics 104 or equivalent, such as satisfactory performance on an Advanced Placement Examination. *No enrollment limit (expected: 45). NOTE: We will be moving at a very fast pace. You should spend at least one if not two hours a day (every day!) on this course. I strongly encourage you to work in groups, and you should skim the reading before each class. **You must watch the videos before class.** We will not cover all the material in the book in class; you are responsible for reading the other examples at home.*

### • First Homework Assignment: Due Monday, Feb 10:

- Read: Section 11.1, 11.2. Use that time to read the material and make sure your Calc I/II is fresh. Feel free to check out the review videos: [part 1](#) [part 2](#).
- Video of the week: [Fibonacci numbers](#) ([click here for more on Fibonacci numbers](#)).
- [Slides on the course mechanics](#).
- HW problems: (1) What is wrong with the following argument (from [Mathematical Fallacies, Flaws, and Flimflam](#) - by Edward Barbeau): There is no point on the parabola  $16y = x^2$  closest to (0,5). This is because the distance-squared from (0,5) to a point (x,y) on the parabola is  $x^2 + (y-5)^2$ . As  $16y = x^2$  the distance-squared is  $f(y) = 16y + (y-5)^2$ . As  $df/dy = 2y+6$ , there is only one critical point, at  $y = -3$ ; however, there is no x such that (x,-3) is on the parabola. Thus there is no shortest distance! (2) Compute the derivative of  $\cos(\sin(3x^2 + 2x \ln x))$ . Note that if you can do this derivative correctly, your knowledge of derivatives should be fine for the course. (3) Let  $f(x) = x^2 + 8x + 16$  and  $g(x) = x^2 + 2x - 8$ . Compute the limits as x goes to 0, 3 and  $\infty$  of  $f(x)+g(x)$ ,  $f(x)g(x)$  and  $f(x)/g(x)$ .
- Extra Credit (due 2/12/14): Assume team A wins p percent of their games, and team B wins q percent of their games. Which formula do you think does a good job of predicting the probability that team A beats team B? Why? (a)  $(p+pq) / (p+q+2pq)$ , (b)  $(p+pq) / (p+q-2pq)$ , (c)  $(p-pq) / (p+q+2pq)$ , or (d)  $(p-pq) / (p+q-2pq)$ .

**GRADING / HW:** Homework 15%, Midterms 40% (there will be 2 or 3), Final 40%; being prepared for class is worth 5% of your grade; this means skimming the reading and watching any video. You are on the honor code to inform me if you fail to do so, you may miss two classes without penalty. Exams are [black tie optional](#). Homework is to be handed in on time, stapled and legible; there will be HW due each class. *Late, messy or unstapled homework will not be graded.* I encourage you to work in small groups, but everyone must submit their own homework assignment. All exams are cumulative, the lowest midterm grade will be dropped. There are also three other options, each individually worth 5%. Doing one of the options below reduces everything else to 95% of your grade, doing two reduces the above to 90%, and so on.

- Scribe: LaTeX a good, complete set of notes for one lecture; this may be done in a group of two.
- Quizzes: You may take the weekly quizzes at home. There is no partial credit, lowest quiz grade is dropped.
- Project:: You may explore a topic in multivariable calculus in great detail and write it up.

**SYLLABUS GENERAL:** The textbook is the seventh edition of Edwards and Penney: Calculus (Early Transcendentals). This should be the textbook used in Math 140. You may use either the 7th edition or the 6th; unfortunately, while the content is essentially the same, the page numbering and chapter labeling differ, and you are responsible for making sure you do the right problems (I'll try and make sure the problems are the same, but it is your responsibility to make sure you do the right ones). There will also be supplemental handouts. *Please read the relevant sections before class. This means you should be familiar with the definitions as well as what we are going to study; this does not mean you should be able to give the lecture.* You do not need a calculator for this class, though I strongly urge you to become familiar with either Matlab or Mathematica to plot some of the multi-dimensional objects. There are many good references on the web. You can access certain books online: [Calculus in Vector Spaces](#) (Lawrence J. Corwin, Robert Henry Szczarba) and [Multivariable Calculus](#) (Lawrence J. Corwin, Robert Henry Szczarba) are somewhat theoretical expositions. Another great source is [Cain and Herod's book on multivariable calculus \(which you can download in its entirety for free\)](#). If you have any concerns or suggestions for the course and would prefer to communicate them anonymously, you may email me by using the account [ephsmath@gmail.com](mailto:ephsmath@gmail.com) (the password is the first eight [Fibonacci numbers](#), 011235813).

## INFORMATION ON READING BEFORE CLASS

Below are some comments to help you prepare for each class' lecture. For each section in the book, I'll mention what you should have read for class. In other words,

what are the key points. When you come to class, you should have already read the section and have some sense of the definitions of the terms we'll study and the results we'll prove. This does not mean you should know the material well enough to give the lecture; it does mean that you should have a familiarity with the material so that when I lecture on the math, it won't be your first exposure to the terminology or results. Everyone processes and learns material in different ways; for me, I find it *very* hard to go to a lecture on a subject I'm unfamiliar with and get much out of it. I need to have some sense of what will happen, as otherwise I spend too much time absorbing the definitions, and then I fall behind. I'm hoping the bullet points below will help you in preparing for each lecture. If there is anything else I can do to assist, as always let me know (either email directly, or anonymously through [mathephys@gmail.com](mailto:mathephys@gmail.com), password 11235813).

Also, you may wish to look at some worked out examples before class that are similar to the HW. [Examples from when I taught the class in 2010 are available online here, though we used a different book then and covered slightly different material](#); I will do many of these problems in class. The reason I want to do these is precisely because I have written up the solution. This way you can sit back a bit more and follow the example without worrying about writing everything down.

## **CHAPTER 11: Vectors, Curves and Surfaces in Space**

- **Section 11.1: Vectors in the Plane**
  - Notation, definition of vectors and properties.
  - Proof of the Pythagorean formula (which is crucial in determining lengths).
- **Section 11.2: Three-Dimensional Vectors**
  - Know the definition of the dot product of two vectors, and the connection of that to the angle between two vectors.
- **Section 11.3: The Cross Product of Vectors**
  - Know the definition of determinants of  $2 \times 2$  and  $3 \times 3$  matrices, and how to compute these.
  - The determinant has much geometrical meaning, denoting the (signed) volume of the parallelepiped spanned by the rows (or columns).
  - Know the definition of the cross product and how to compute it, as well as some of its properties.
- **Section 11.4: Lines and Planes in Space**
  - Know the various formulas for writing the equation of a line.
  - There are several ways to write the equation of a plane; it's similar to writing the equation of a line: depending what information you are given, some ways are more convenient than others.

- One easy way to find the equation of a plane is to know the normal direction. This is a great application of the cross product, as  $\mathbf{v} \times \mathbf{w}$  is perpendicular to both  $\mathbf{v}$  and  $\mathbf{w}$ . Unfortunately we don't have the cross product in higher dimensions.
- **Section 11.8: Cylindrical and Spherical Coordinates**
  - Know the different formulas to convert from Cartesian to Cylindrical or Spherical coordinates.

## CHAPTER 12: Partial Differentiations

- **Section 12.1: Introduction**
  - Not much here except (what a surprise) that many functions in the real world depend on several variables.
- **Section 12.2: Functions of Several Variables**
  - First is when a function is defined on a domain (usually just making sure the denominator is non-zero).
  - The level set (of value  $c$ ) of a function are all inputs that are mapped to  $c$ . Think of this as all points on a mountain that are the same height, or on a weathermap all places with the same temperature.
- **Section 12.3: Limits and Continuity**
  - Know the definition and basic properties of limits.
  - Caveats: certain operations are not defined:  $\infty - \infty$ ,  $0 * \infty$ .
- **Section 12.4: Partial Derivatives**
  - Know the definition of how to take a partial derivative. Similar to one-variable calculus, we do not want to have to use this definition in practice, and thus want to modify our rules of one-variable differentiation to allow us to take derivatives here.
  - Know the formula for computing the tangent plane to  $z = f(x,y)$  at a given point, so long as the partial derivatives exist at that point.
  - Iterated Partial Derivatives:
    - The definition of mixed partial derivatives: Given a function  $f$ , we can compute its partial derivatives, such as  $\delta f / \delta x$  and  $\delta f / \delta y$ . We can then take the partial derivatives of the partial derivatives:  $\delta(\delta f / \delta x) / \delta y$  and  $\delta(\delta f / \delta y) / \delta x$ . In the first, we first take the derivative ...