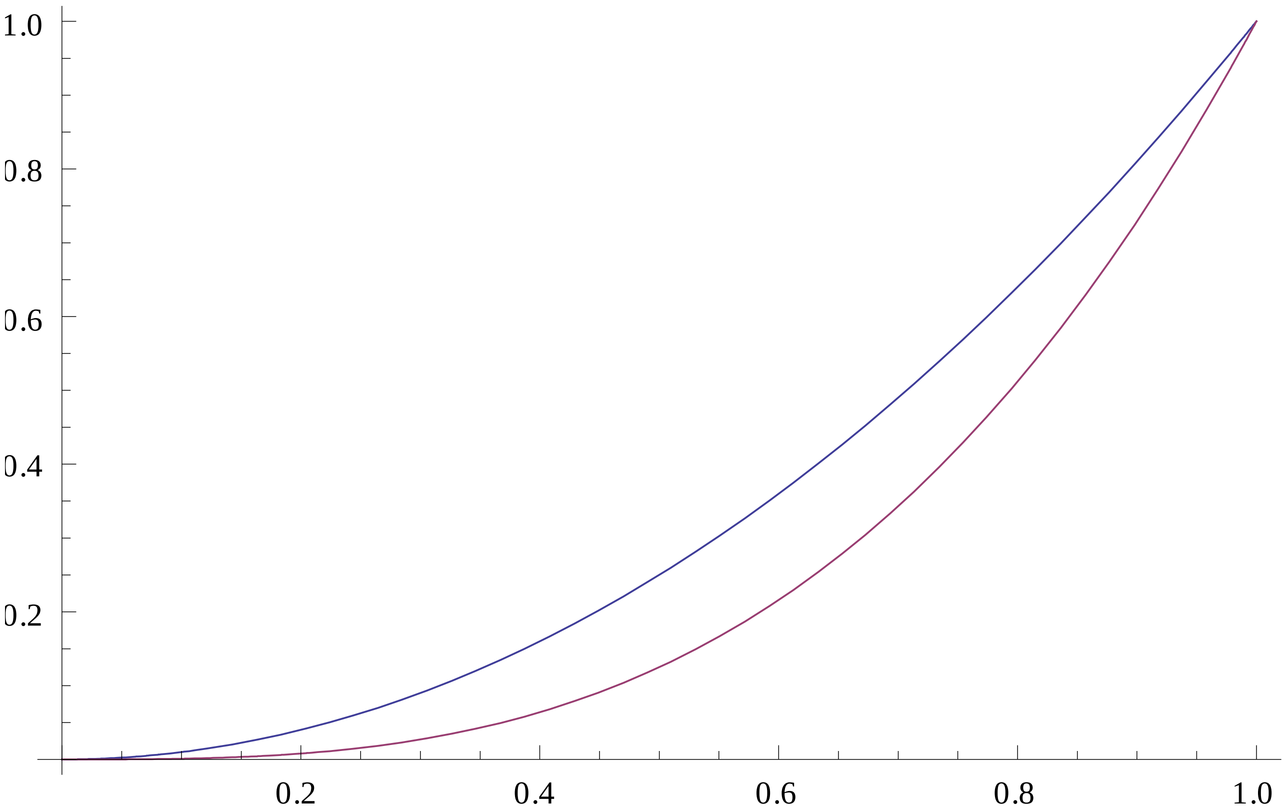
PROBLEMS
(solutions at end)



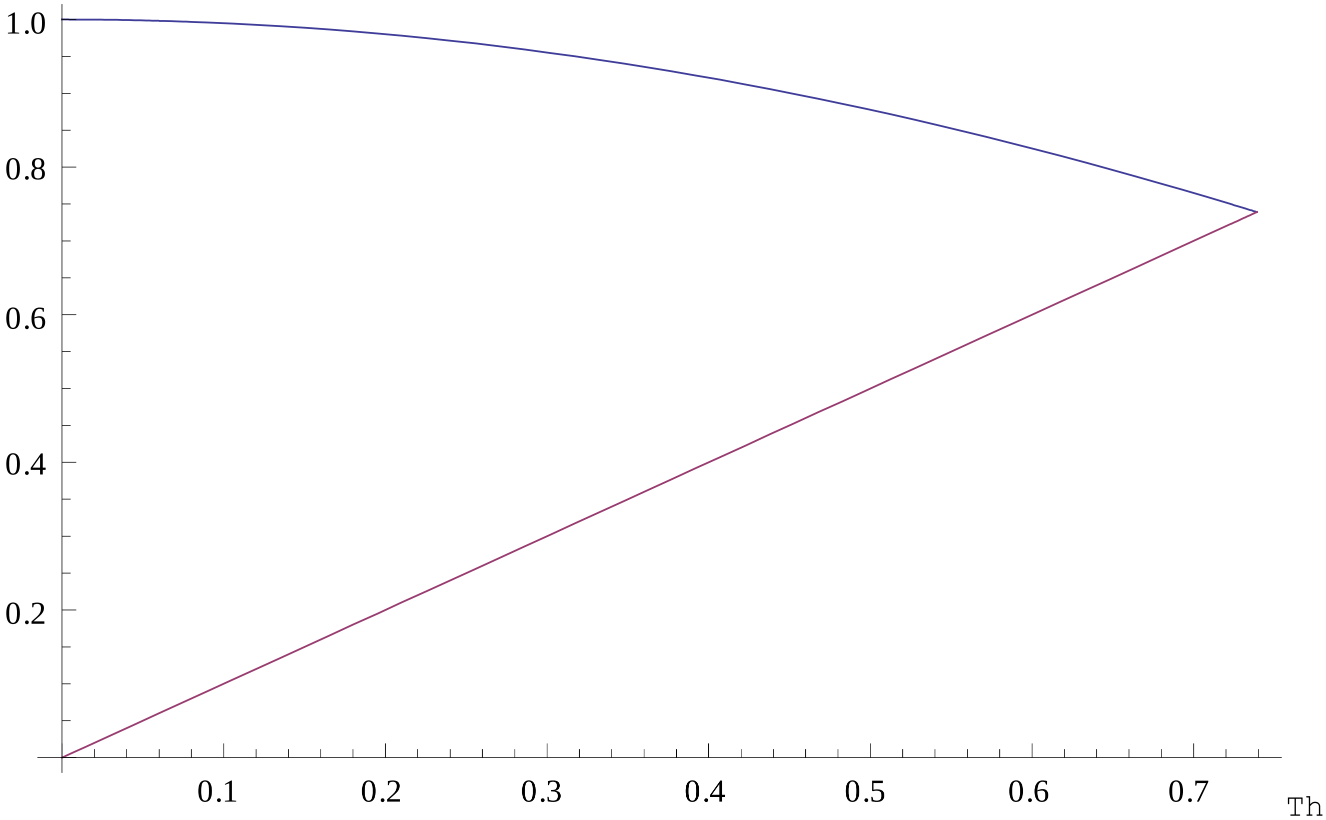
Problem #1

The blue curve is the function y = x^2, the purple is y = x^3.

Write down the double integral of a function f(x,y) over this region. Do this both ways:

(1) First integrate with respect to y then with respect to x.

(2) First integrate with respect to x then with respect to y.



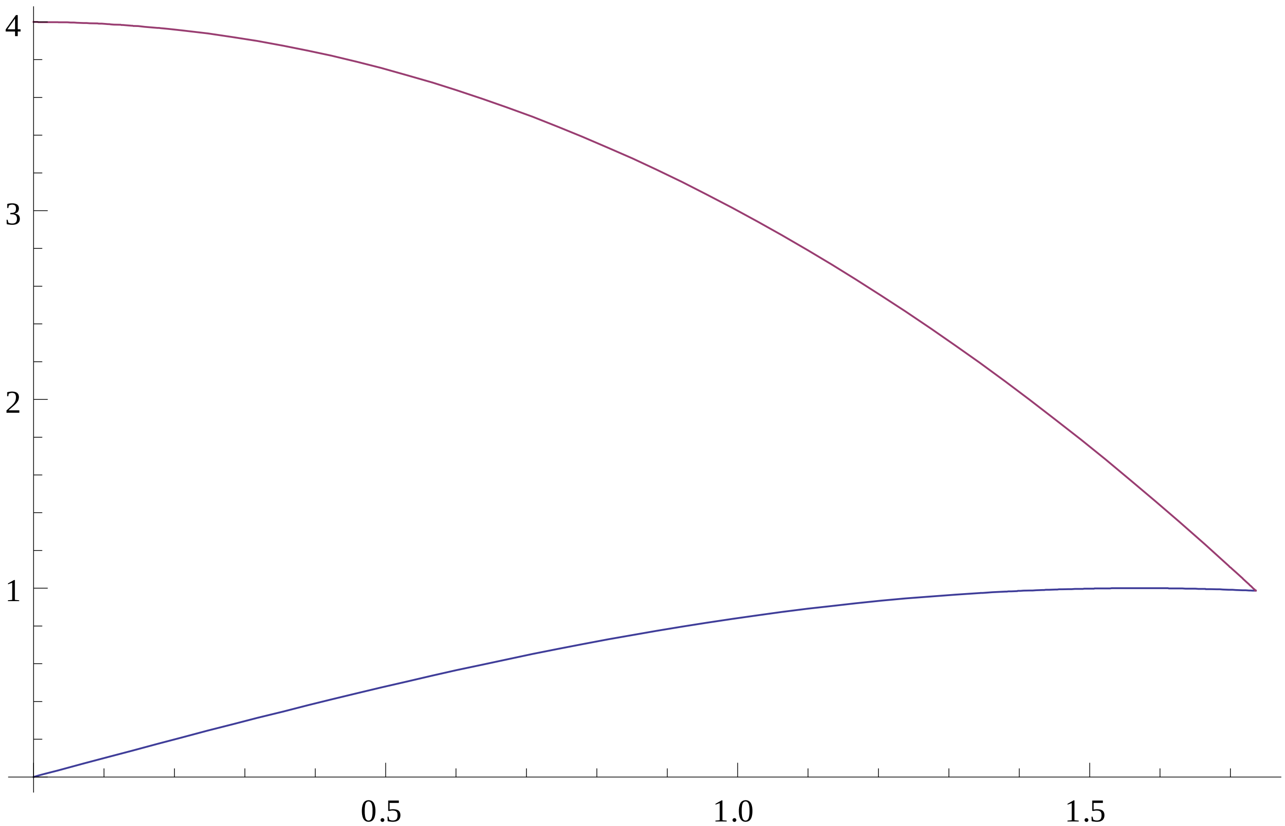
Problem #2

The blue function (top) is y = Cos(x).
The purple function (bottom) is y = x.
The two functions intersect at approximately x = 0.739.
Call this point of intersection x_0.

Write down the double integral of a function f(x,y) over this region. Do this both ways:

(1) First integrate with respect to y then with respect to x.

(2) First integrate with respect to x then with respect to y.



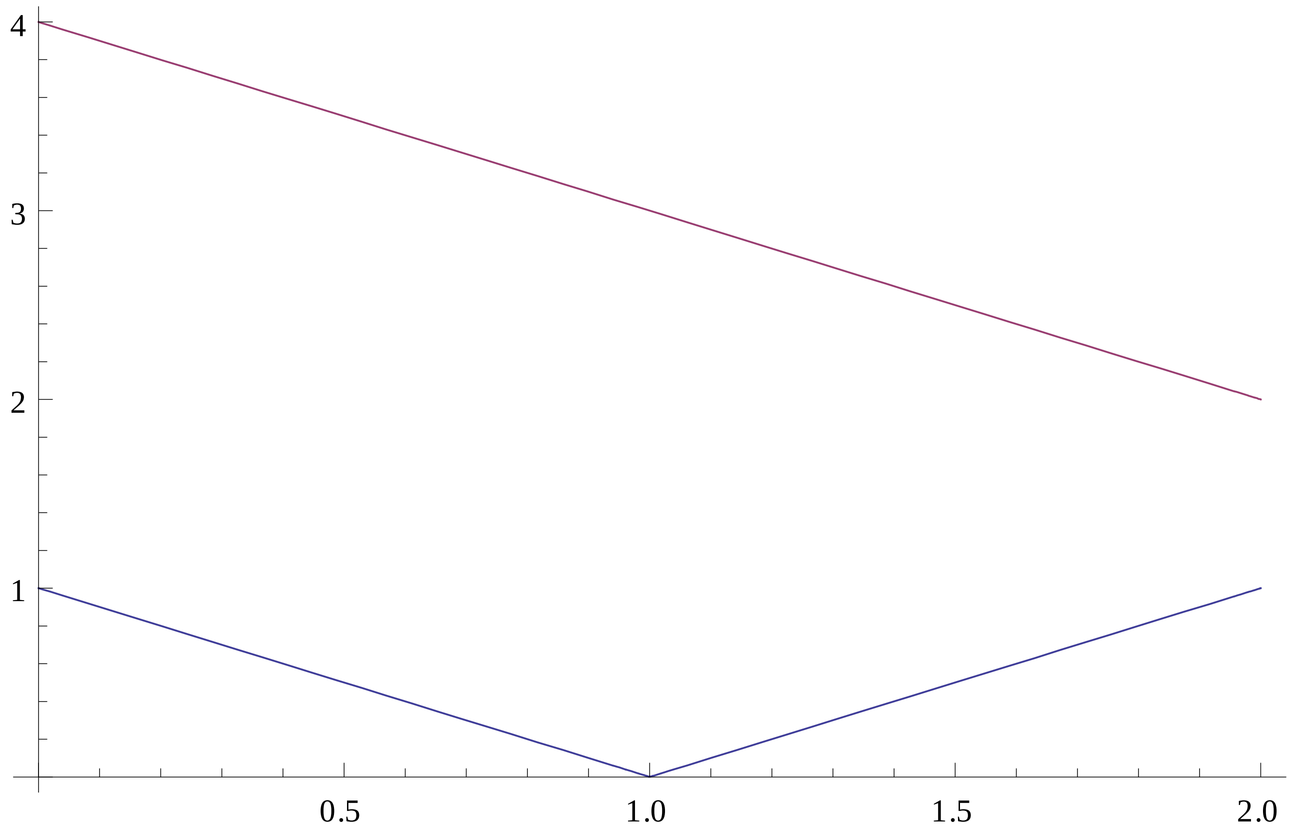
Problem #3

The blue function (bottom) is y = Sin(x).
The purple function (top) is y = 4 - x^2.
The two functions intersect at approximately x = 1.73598.
Call this point of intersection x_0.

Write down the double integral of a function f(x,y) over this region. Do this both ways:

(1) First integrate with respect to y then with respect to x.

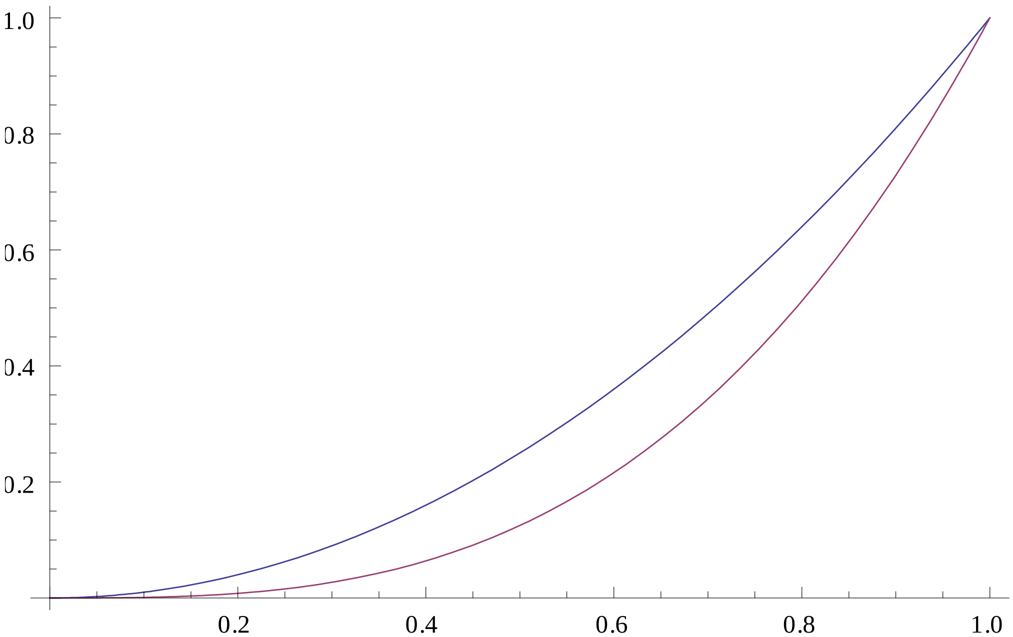
(2) First integrate with respect to x then with respect to y.



The top function (purple) is $y = 4 - x$.
The bottom function (blue) is $y = |1 - x|$.

Is this region x-simple? Is it y-simple? Set up the integration if it is x-simple. Set up the integration if it is y-simple.

SOLUTIONS



Problem #1

The blue curve is the function y = x^2, the purple is y = x^3.

Write down the double integral of a function f(x,y) over this region. Do this both ways:

(1) First integrate with respect to y then with respect to x.

Ans: Int_{x = 0 to 1} Int_{y = x^3 to x^2} f(x,y) dy dx.

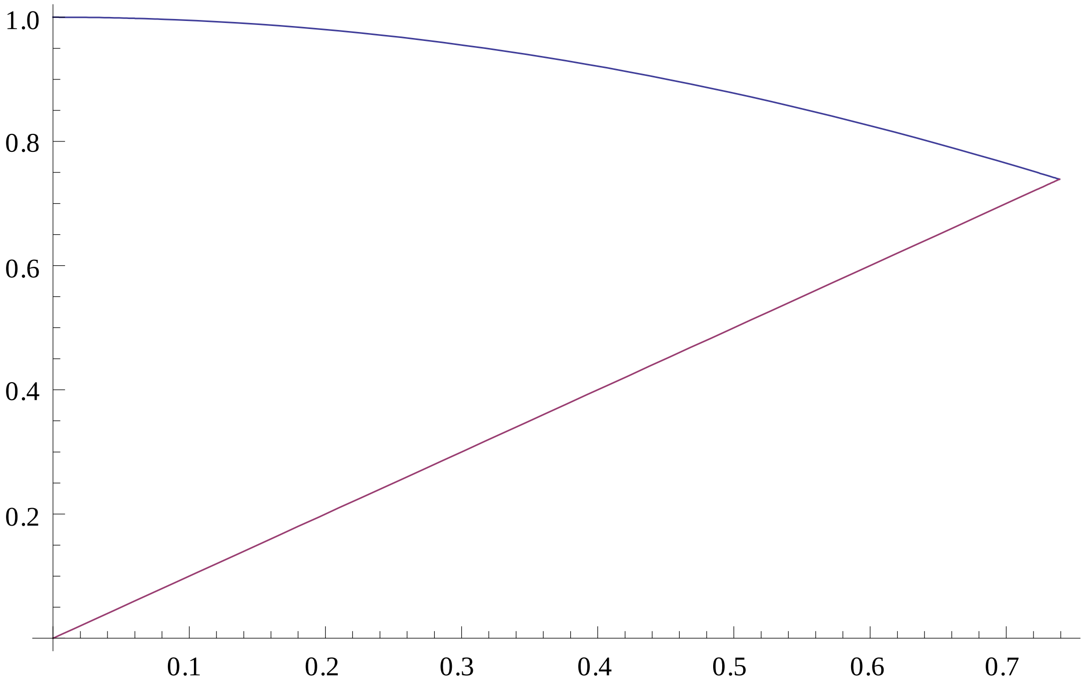
Here phi_1(x) = x^3 (the bottom) and phi_2(x) = x^2 (the top)

(2) First integrate with respect to x then with respect to y.

Ans: Int_{y = 0 to 1} Int_{x = y^1/2 to y^1/3} f(x,y) dx dy

Here psi_1(y) = x^1/2 and psi_2(y) = x^1/3.

Note the blue curve is y = x^2, so if we are given a value of y the corresponding value of x is sqrt(x) or x^1/2.



Problem #2

The blue function (top) is y = Cos(x).
The purple function (bottom) is y = x.
The two functions intersect at approximately x = 0.739.
Call this point of intersection x_0.

Write down the double integral of a function f(x,y) over this region. Do this both ways:

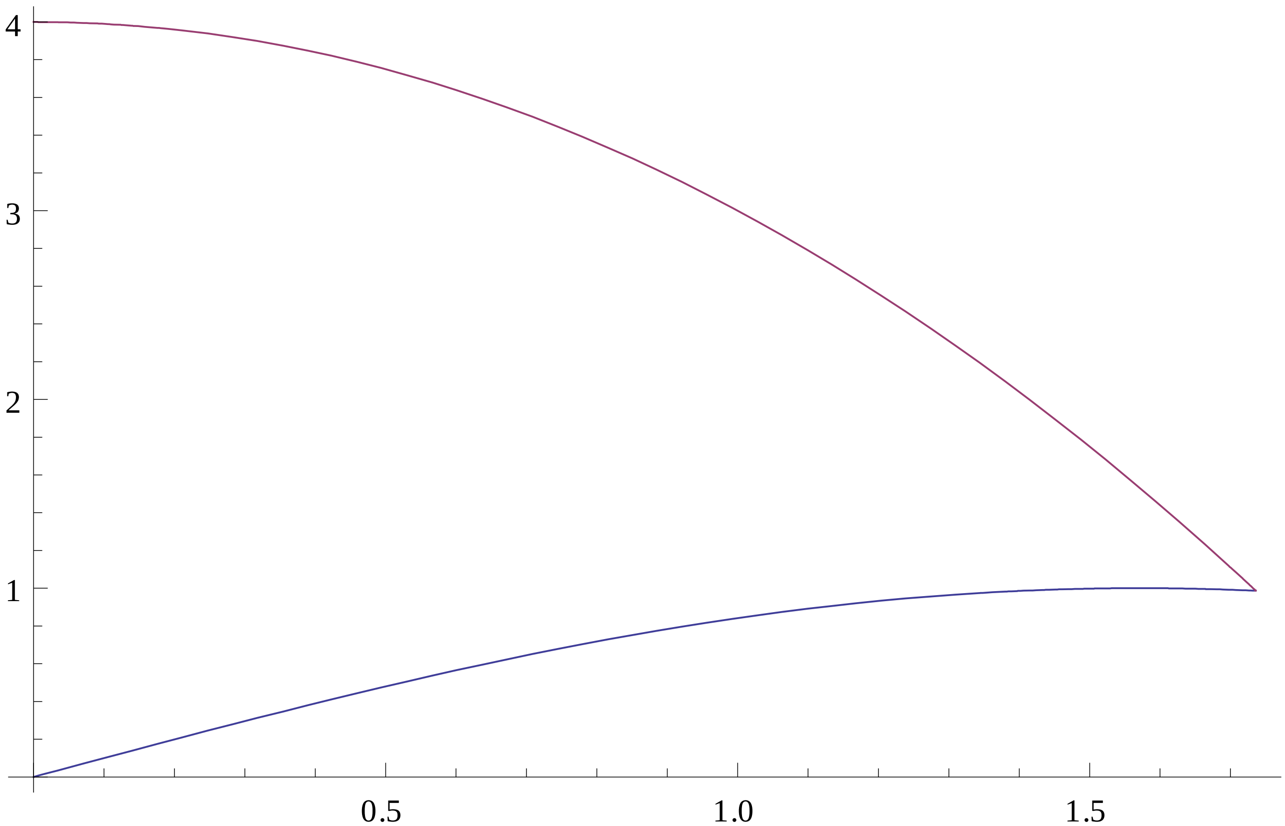
(1) First integrate with respect to y then with respect to x.

Ans: Int_{x = 0 to x_0} Int_{y = x to Cos(x)} f(x,y) dy dx

(2) First integrate with respect to x then with respect to y.

Ans: Int_{y = 0 to 1} Int_{x = 0 to Psi_2(y)} f(x,y) dy dx

Where Psi_2(y) = y if 0 <= y <= .739 and Psi_2(y) = ArcCos(y) if .739 <= y <= 1.



Problem #3

The blue function (bottom) is y = Sin(x).
The purple function (top) is y = 4 - x^2.
The two functions intersect at approximately x = 1.73598.
Call this point of intersection x_0.

Write down the double integral of a function f(x,y) over this region. Do this both ways:

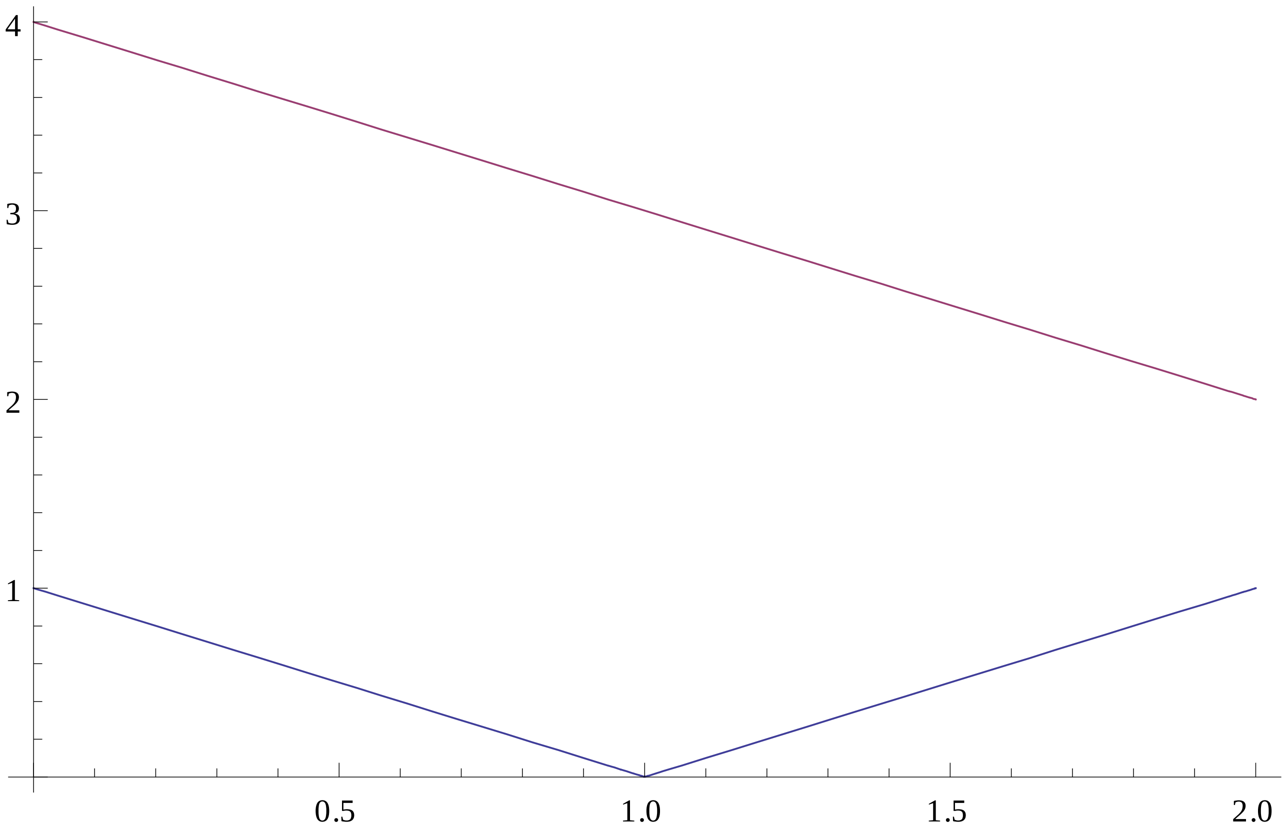
(1) First integrate with respect to y then with respect to x.

Ans: Int_{x = 0 to x_0} Int_{y = Sin(x) to 4 - x^2} f(x,y) dy dx

(2) First integrate with respect to x then with respect to y.

Ans: Int_{y = 0 to 4} Int_{x = 0 to Psi_2(y)} f(x,y) dy dx

Where Psi_2(y) = ArcSin(y) for 0 <= y <= Sin(1.73598) and
Psi_2(y) = Sqrt(4-y) if Sin(1.73598) <= y <= 4.

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The top function (purple) is y = 4 - x.
The bottom function (blue) is y = |1 - x|.

Is this region x-simple? Is it y-simple? Set up the integration if it is x-
simple. Set up the integration if it is y-simple.
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Ans: The region is both x-simple and y-simple. The y-simple is the easiest, and leads to

Int_{x = 0 to 2} Int_{y = |1-x| to 4 – x} f(x,y) dy dx.

For the x-simple, the bounds depend on what value y takes on. I will just do what happens for 0 <= y <= 1. In this region, for a given y we have x ranges from 1-y to 1+y. To see this, if 0 <= x <= 1 then |1-x| = 1-x, and hence if y = 1-x then x = 1-y. If instead 1 <= x <= 2 then |1-x| = x-1, and now y = x-1 or x = 1+y.

Thus for 0 <= y <= 1 we have Psi_1(y) = 1-y and Psi_2(y) = 1+y.