Math 150: Calculus III: Multivariable Calculus

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Lecture 8: 2-23-2022: https://youtu.be/gz64jWipVSU

Class 08: http://youtu.be/np_Icg33Lug (February 19, 2014: Coordinate systems, level sets)

Plan for the day: Lecture 8: February 23, 2022:

Topics:

Coordinate Systems

Level Sets

Trying to find appropriate units / coordinates for the problem.

Lot of strange units: bruno, helen (milli-helen), smoot….

https://en.wikipedia.org/wiki/Gauss_%28unit%29#:~:text=The%20gauss%2C%20symbol%20G%20%28sometimes%20Gs%29%2C%20is%20a,mathematician%20and%20physicist%20Carl%20Friedrich%20Gauss%20in%201936.

The **gauss**, symbol G (sometimes Gs), is a unit of measurement of magnetic induction, also known as *magnetic flux density*. The unit is part of the Gaussian system of units, which inherited it from the older CGS-EMU system. It was named after the German mathematician and physicist Carl Friedrich Gauss in 1936. One gauss is defined as one maxwell per square centimetre.

As the cgs system has been superseded by the International System of Units (SI), the use of the gauss has been deprecated by the standards bodies, but is still regularly used in various subfields of science. The SI unit for magnetic flux density is the tesla (symbol T),[1] which corresponds to 10,000 gauss.

- **0.25–0.60 G** – the Earth's magnetic field at its surface
- **25 G** – the Earth's magnetic field in its core[4]
- **50 G** – a typical refrigerator magnet
- **100 G** – an iron magnet

How far is 25 trillion miles? Put in context….

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Earth’s radius is about 4000 miles.

Distance from the moon to the Earth is about 240,000 miles.

Distance from the sun to the Earth is about 93,000,000 miles.

How far is 25 trillion miles? Put in context….

Earth's radius is about 4000 miles.

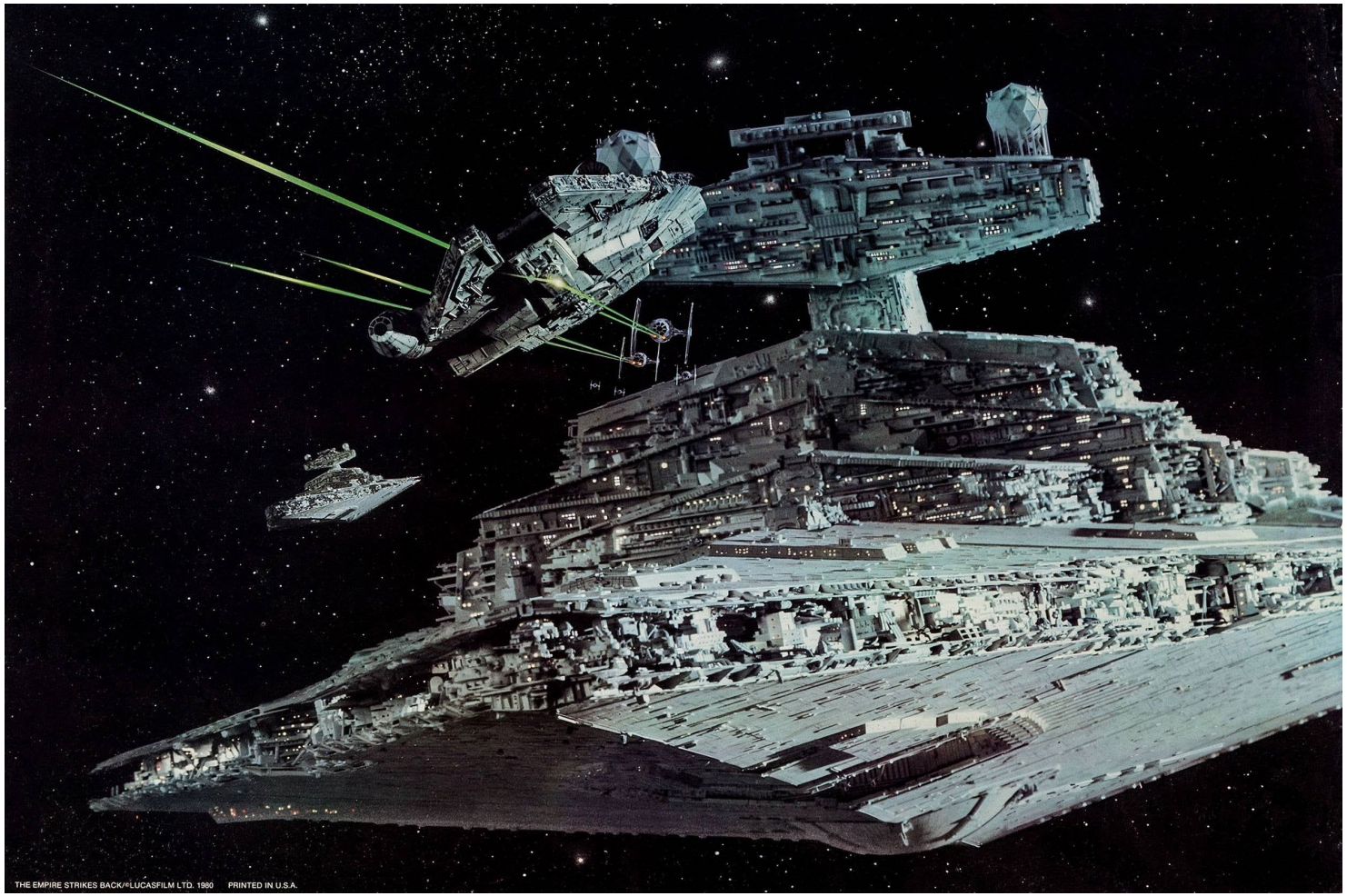
Distance from the moon to the Earth is about 240,000 miles.
Better: about 1.25 light-seconds.

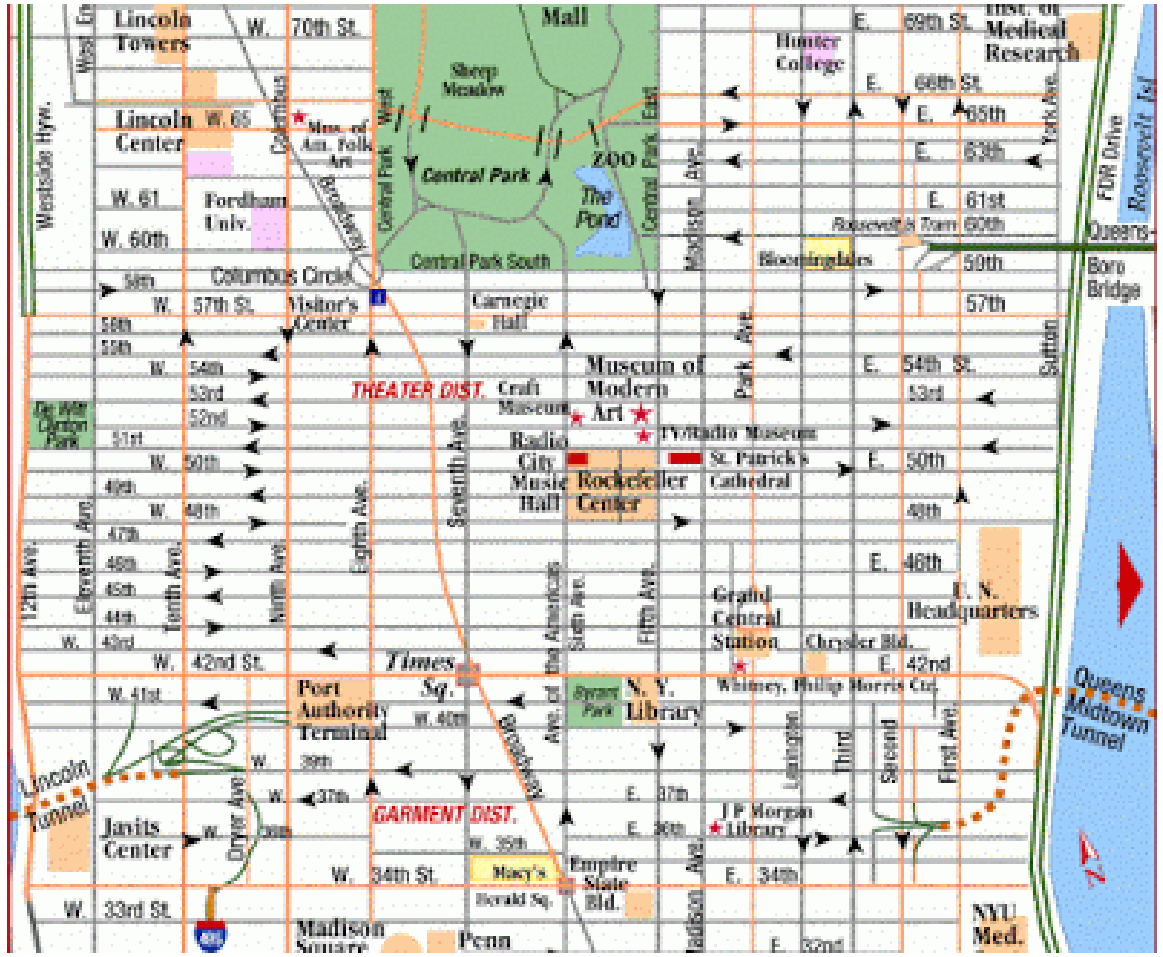
Distance from the sun to the Earth is about 93,000,000 miles.
Better: about 8.3 light-minutes.

25 trillion miles is about 4.3 light years, distance from the sun to Alpha Centauri.

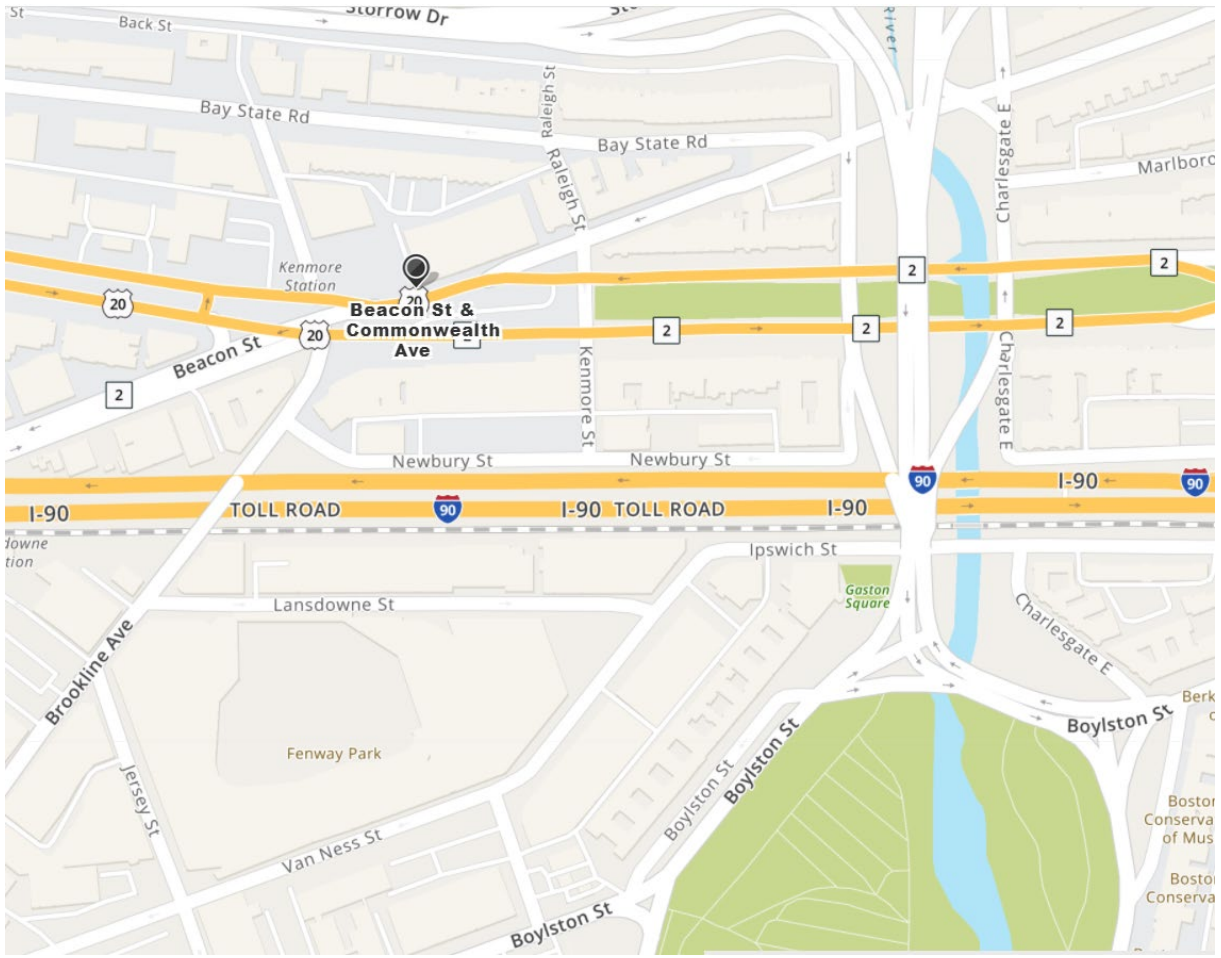
It’s the ship that made the Kessel Run in less than twelve parsecs!

It’s the ship that made the Kessel Run in less than twelve parsecs! (1 parsec is about 3.26 light-years)





https://www.joemygod.com/wp-content/uploads/2011/03/manhattan_midtown.jpg-400x330.gif



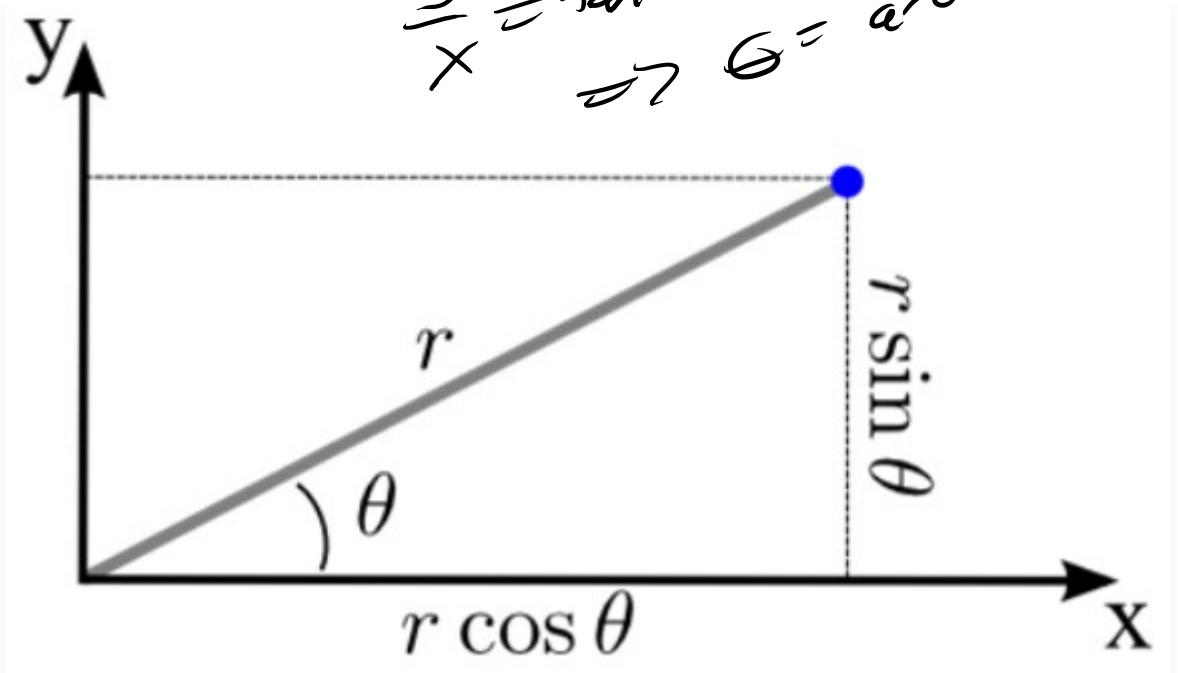
https://www.mapquest.com/us/ma/boston/02215-2601/569-newbury-st-42.348041,-71.095675?zoom=17

https://byjus.com/maths/polar-coordinates/

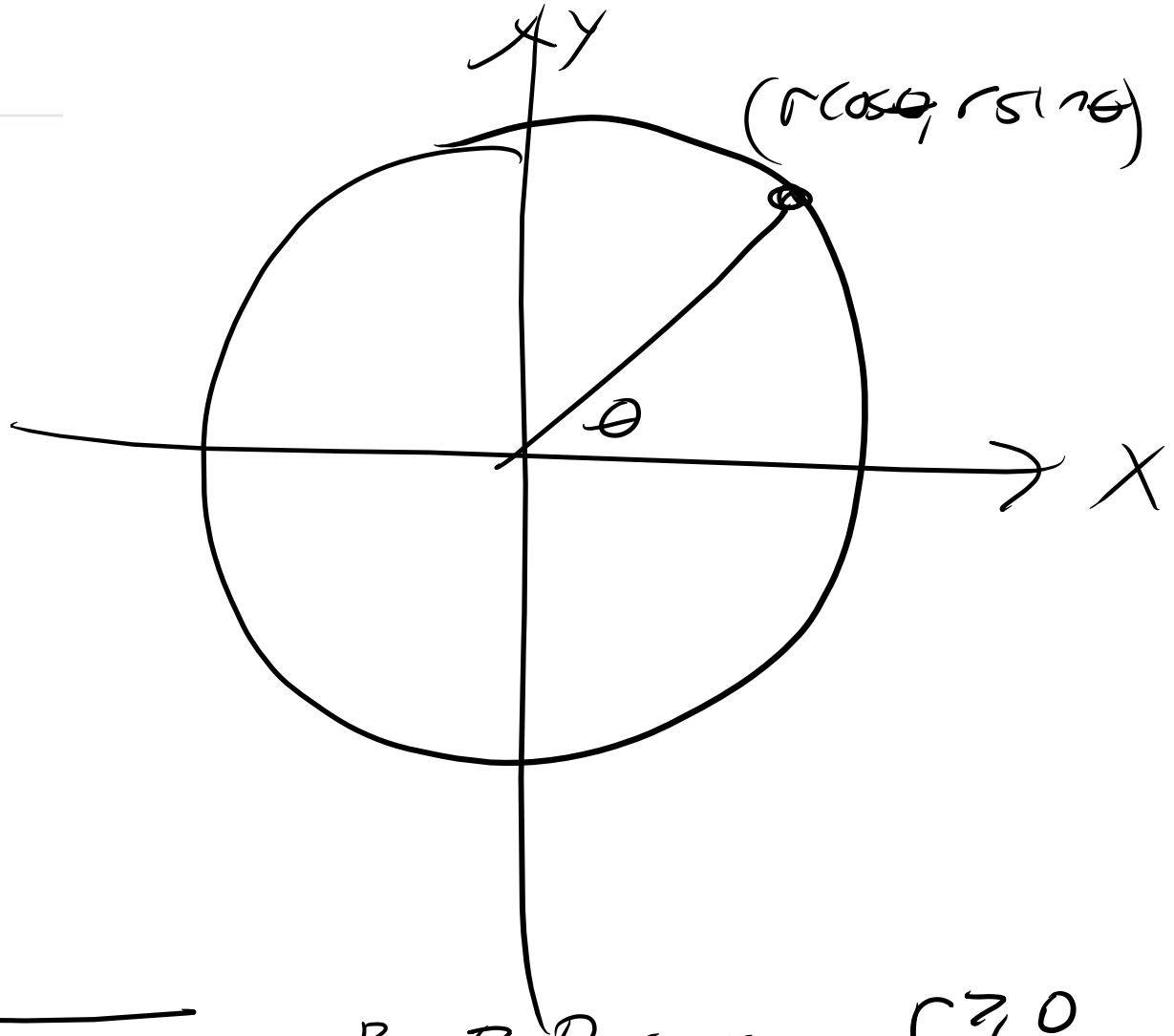
Cartesian to Polar Coordinates

$x = r \cos \theta$

$y = r \sin \theta$



$\frac{y}{x} = \tan\theta \Rightarrow \theta = \arctan(y/x)$



$X = r\cos\theta$

$y = r\sin\theta$

$r = \sqrt{x^2+y^2}$ By Pythagoras

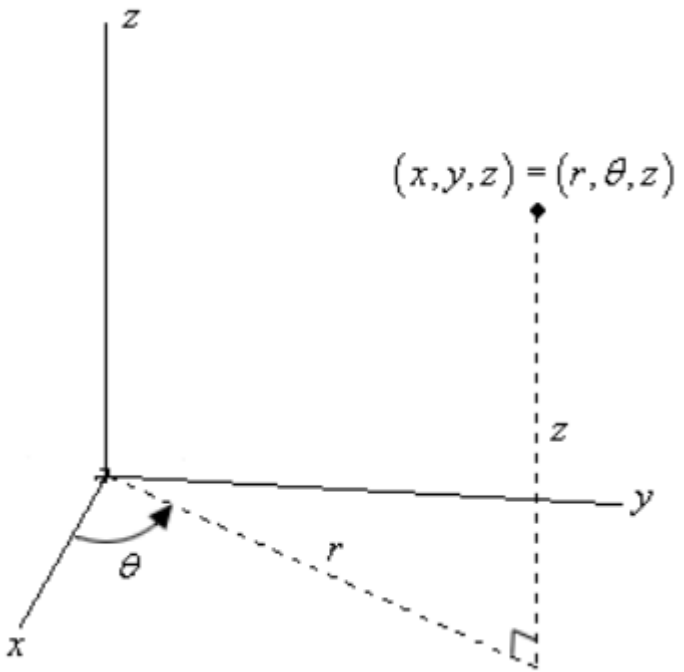
$\theta = \arctan(y/x)$

$r \geq 0$

$0 \leq \theta < 2\pi$

if $r = 0$ and θ anything get $(0,0)$

https://tutorial.math.lamar.edu/Classes/CalcIII/CylindricalCoords.aspx



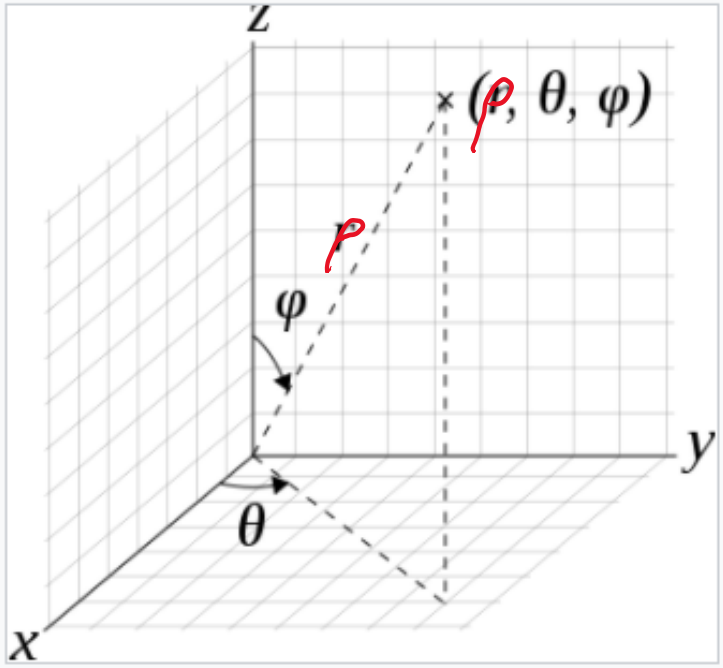
$$x = r\cos\theta$$
$$y = r\sin\theta$$
$$z = z$$

$$r = \sqrt{x^2 + y^2} \qquad \text{OR} \qquad r^2 = x^2 + y^2$$
$$\theta = \tan^{-1}\left(\frac{y}{x}\right) = \arctan(y/x)$$
$$z = z$$

$r \geq 0 \quad 0 \leq \theta < 2\pi$

Polar with $z = z$ okay for the ride

https://en.wikipedia.org/wiki/Spherical_coordinate_system#:~:text=In%20mathematics%2C%20a%20spherical%20coordinate%20system%20is%20a,from%20a%20fixed%20reference%20direction%20on%20that%20plane.



Spherical coordinates (r, θ, φ) as often used in ***mathematics***: radial distance r, azimuthal angle θ, and polar angle φ. The meanings of θ and φ have been swapped compared to the physics convention. As in physics, ρ (rho) is often used instead of r, to avoid confusion with the value r in cylindrical and 2D polar coordinates.

https://math.fandom.com/wiki/Spherical_coordinate_system

Cartesian coordinate system

The three spherical coordinates are converted to Cartesian coordinates by:

$$x = \rho \sin(\phi) \cos(\theta)$$
$$y = \rho \sin(\phi) \sin(\theta)$$
$$z = \rho \cos(\phi)$$

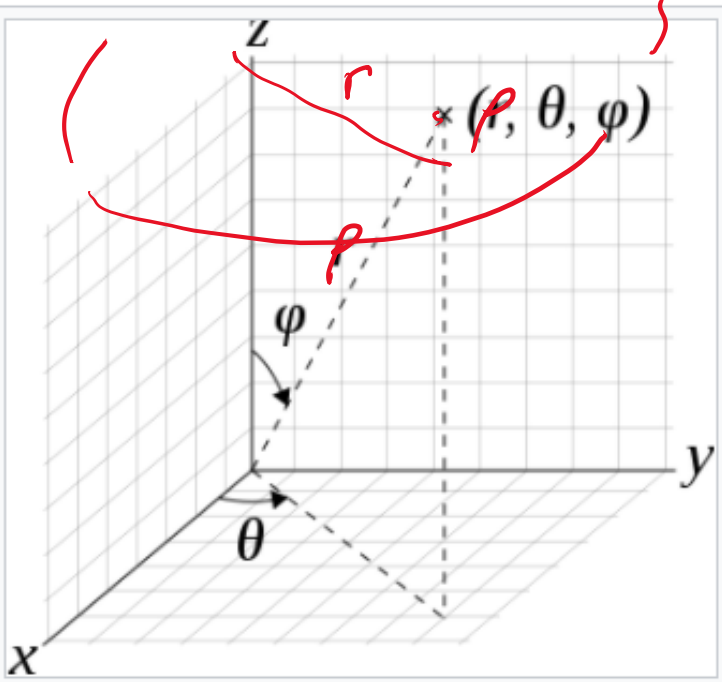
Conversely, Cartesian coordinates may be converted to spherical coordinates by:

$$\rho = \sqrt{x^2 + y^2 + z^2}$$
$$\phi = \arccos\left(\frac{z}{\sqrt{x^2 + y^2 + z^2}}\right)$$
$$\theta = \arctan\left(\frac{y}{x}\right)$$

ϕ: 0 to π or $0 \le \phi \le \pi$

θ: 0 to 2π or $0 \le \theta < 2\pi$

$\rho \ge 0$



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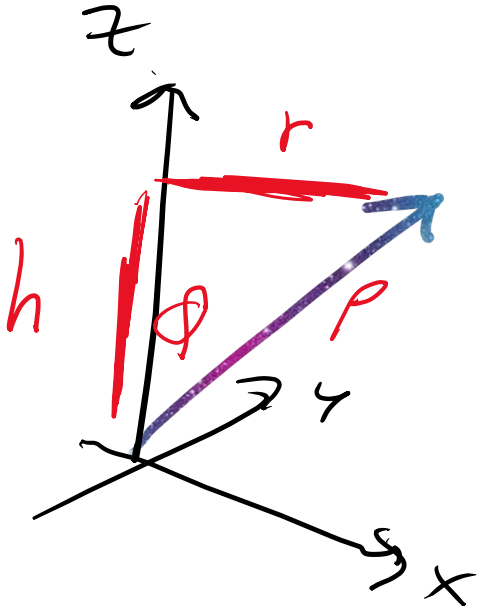


height $= z = \rho\cos\varphi$

$r = \rho\sin\varphi$

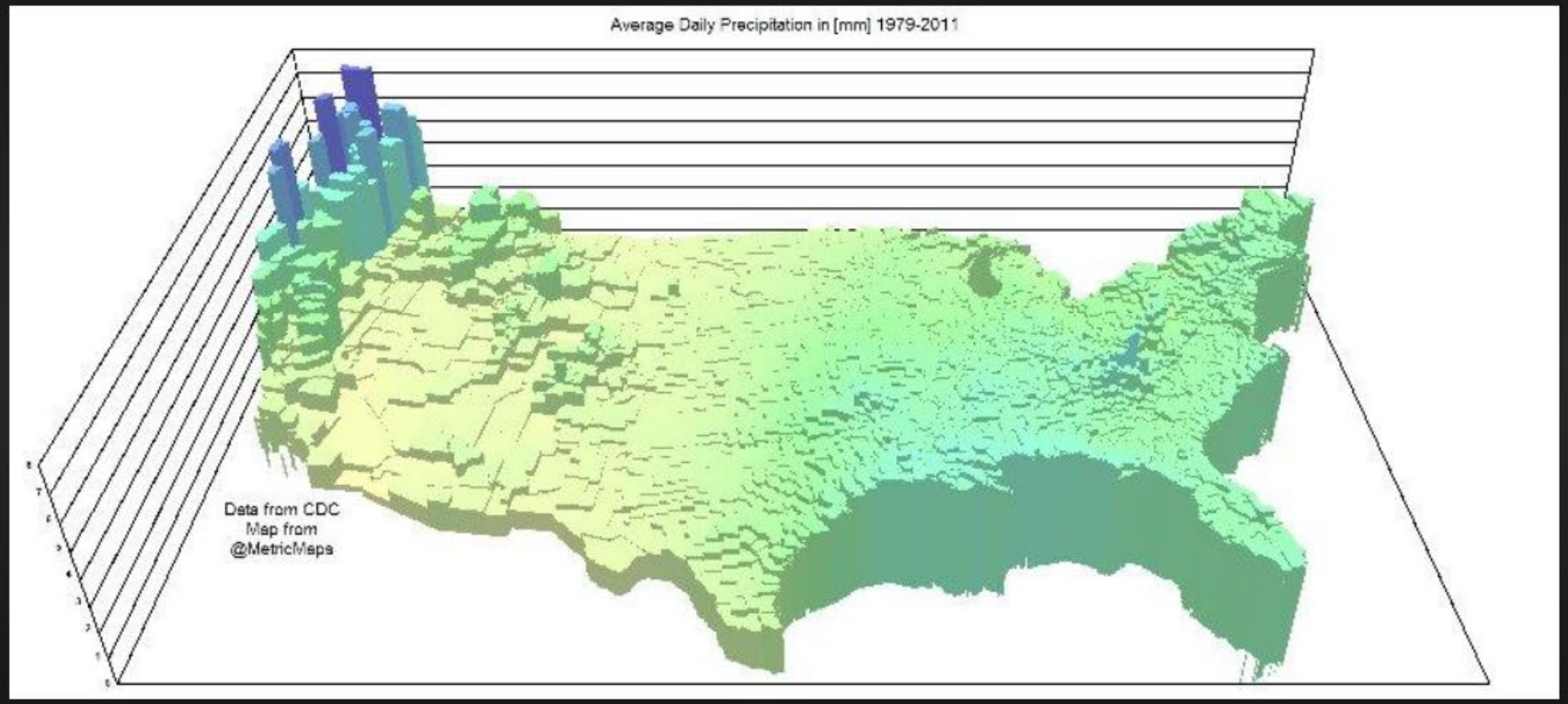
Feed into polar coords (r, θ) with θ given and $r = \rho\sin\varphi$

$x = r\cos\theta = \rho\sin\varphi\cos\theta$

$y = r\sin\theta = \rho\sin\varphi\sin\theta$

$z = \rho\cos\varphi$

$$\begin{aligned} x &= \rho\sin(\phi)\cos(\theta) \\ y &= \rho\sin(\phi)\sin(\theta) \\ z &= \rho\cos(\phi) \end{aligned}$$



Amazing Maps of the United States - BrowserCode Forums

forum.browsercode.com 868x388

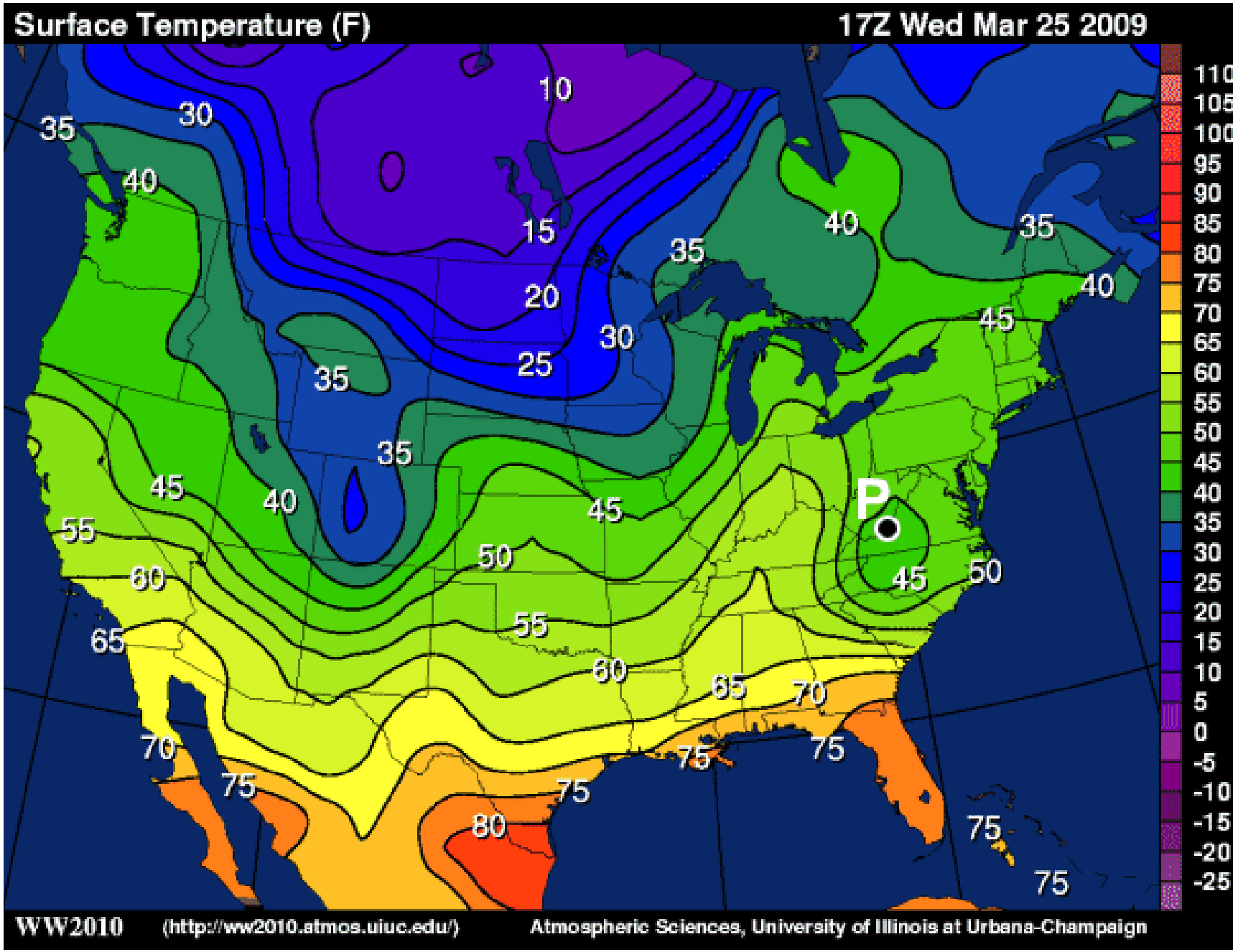
View Image View Page

https://pbs.twimg.com/media/BmBU5ufIcAAlQMO.jpg:large

https://en.wikipedia.org/wiki/Level_set#:~:text=In%20mathematics%2C%20a%20level%20set%20of%20a%20real,curve%2C%20also%20known%20as%20contour%20line%20or%20isoline.

In mathematics, a **level set** of a real-valued function *f* of *n* real variables is a set where the function takes on a given constant value *c*, that is:

$$L_c(f) = \{(x_1, \ldots, x_n) \mid f(x_1, \ldots, x_n) = c\} \ ,$$



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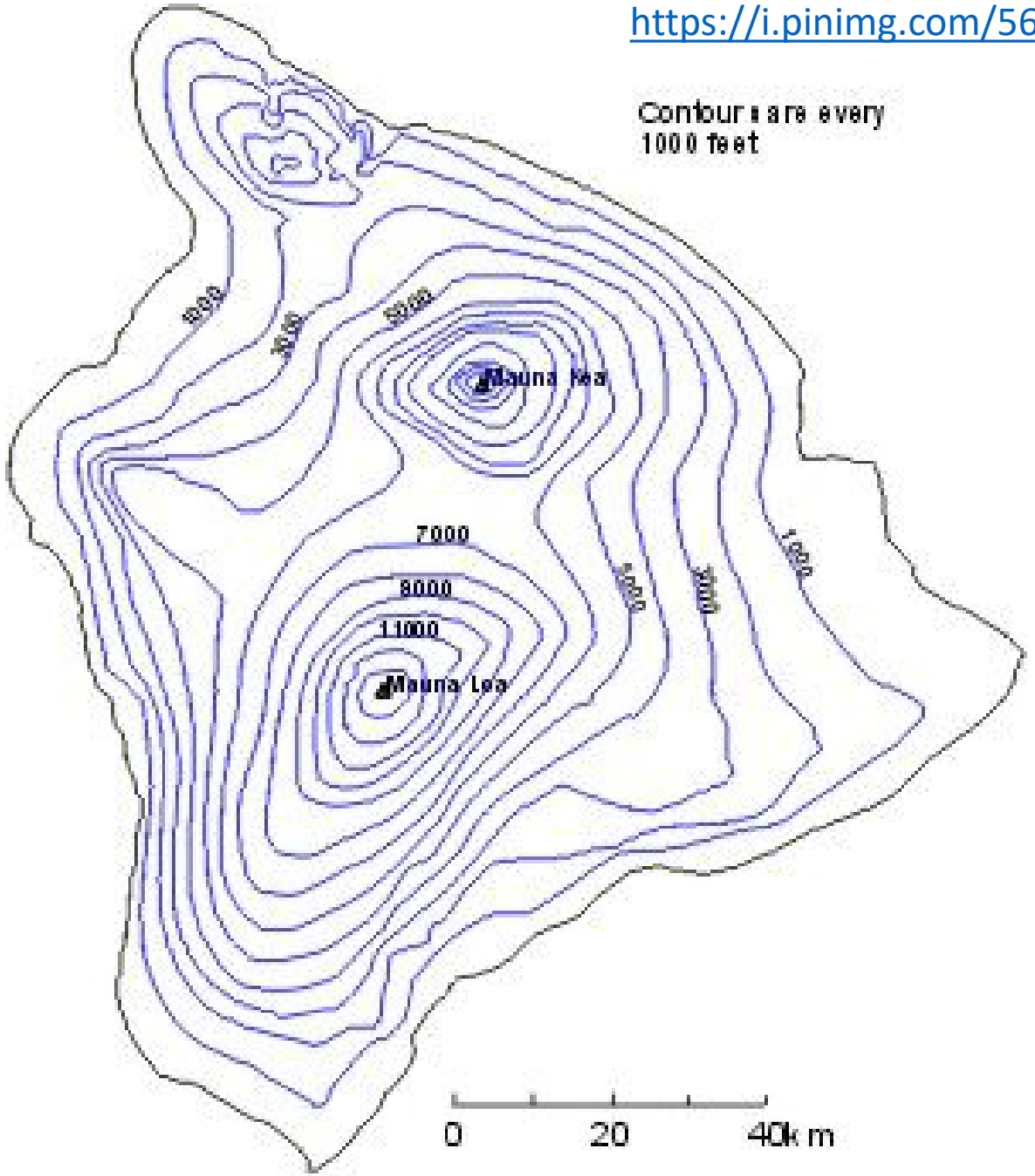
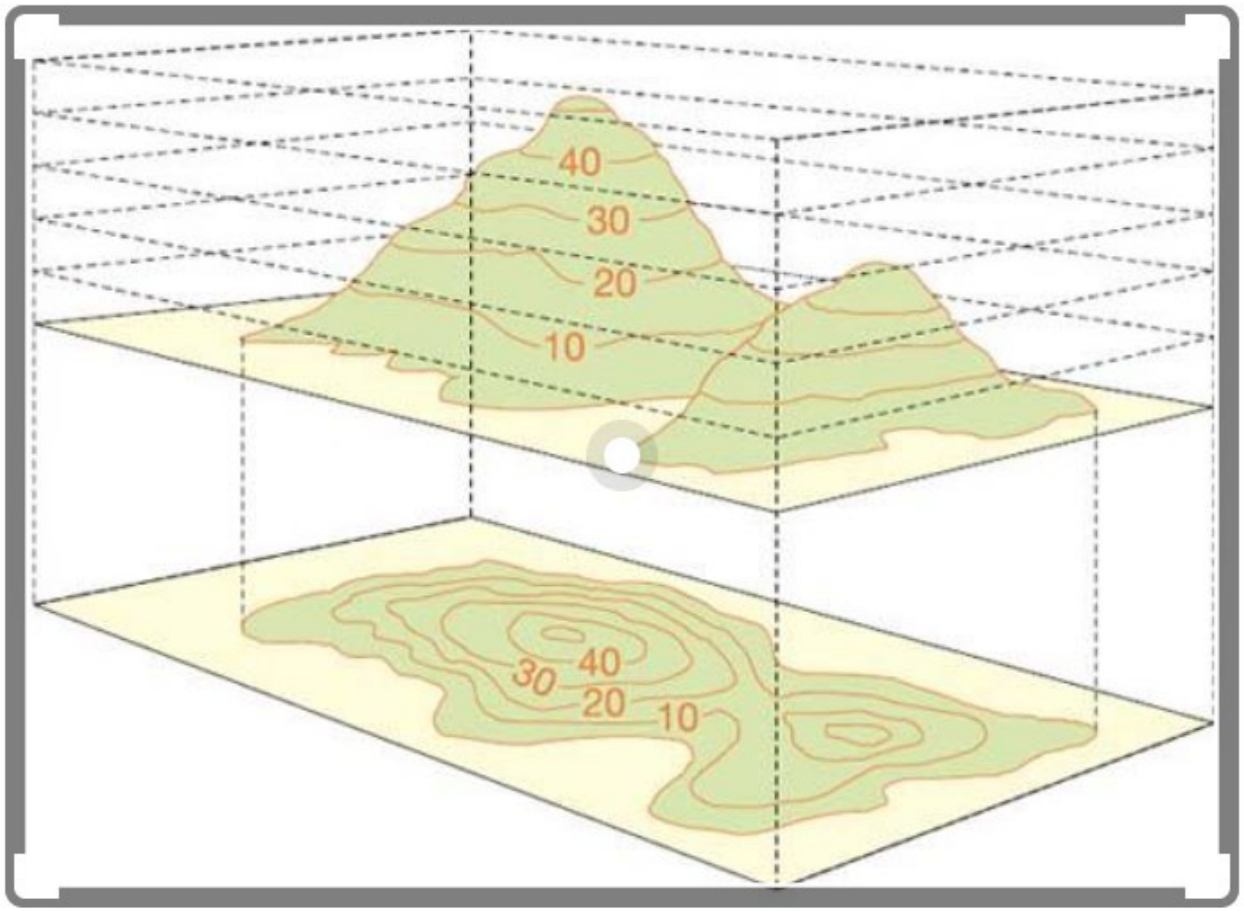
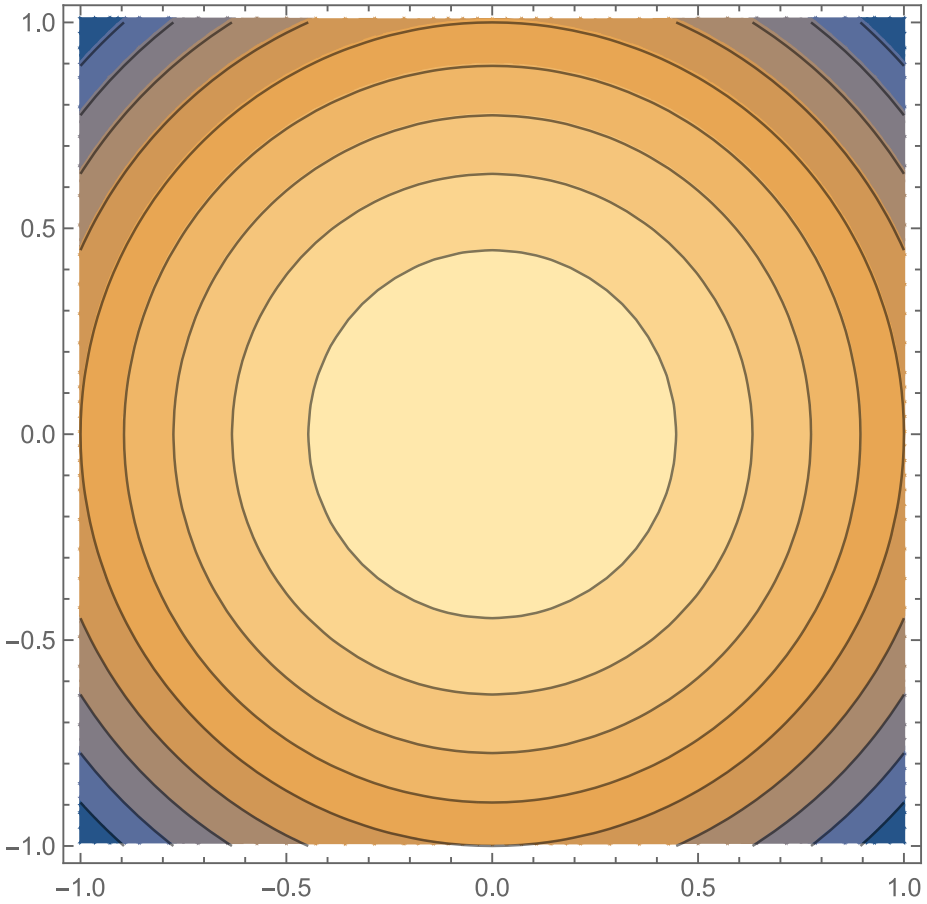




https://www.pinterest.com/pin/1477812354567473/visual-search/?x=10&y=10&w=530&h=382&cropSource=6&imageSignature=b08522e44b519087cff80bb2bcd6b042

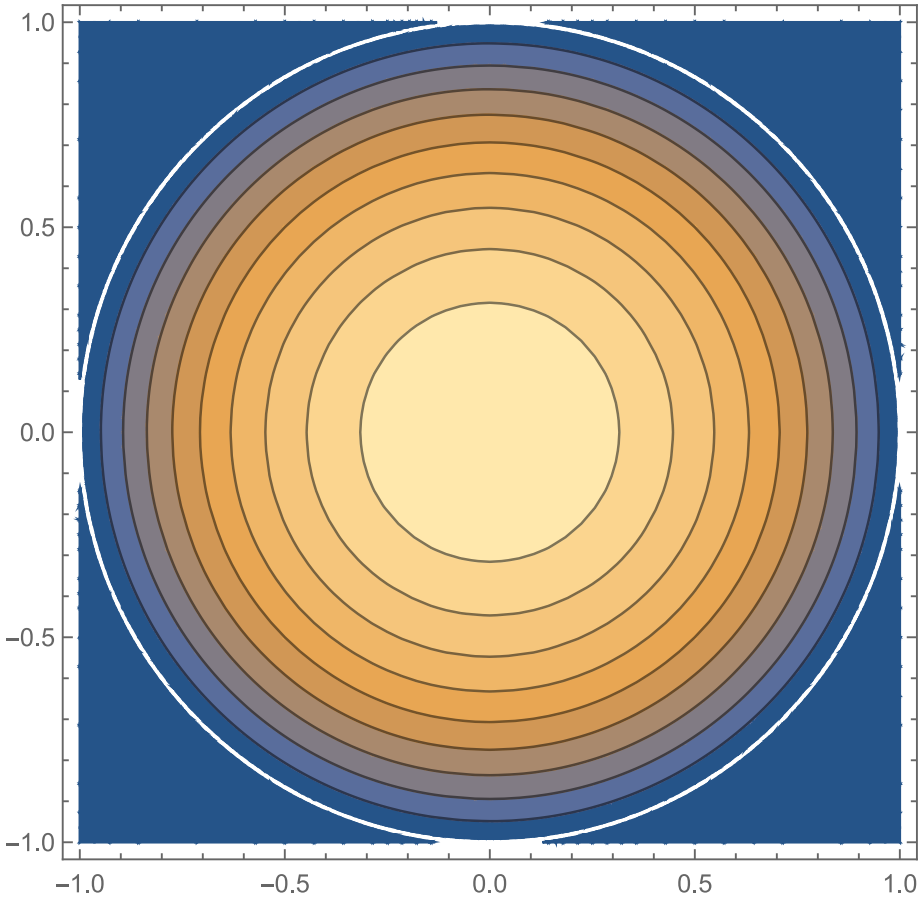
In[ ]:=
```
ContourPlot[1 - (x^2 + y^2), {x, -1, 1}, {y, -1, 1}]
```

Out[ ]=



In[ ]:=
```
ContourPlot[If[x^2 + y^2 ≤ 1, 1 - (x^2 + y^2), 0], {x, -1, 1}, {y, -1, 1}]
```

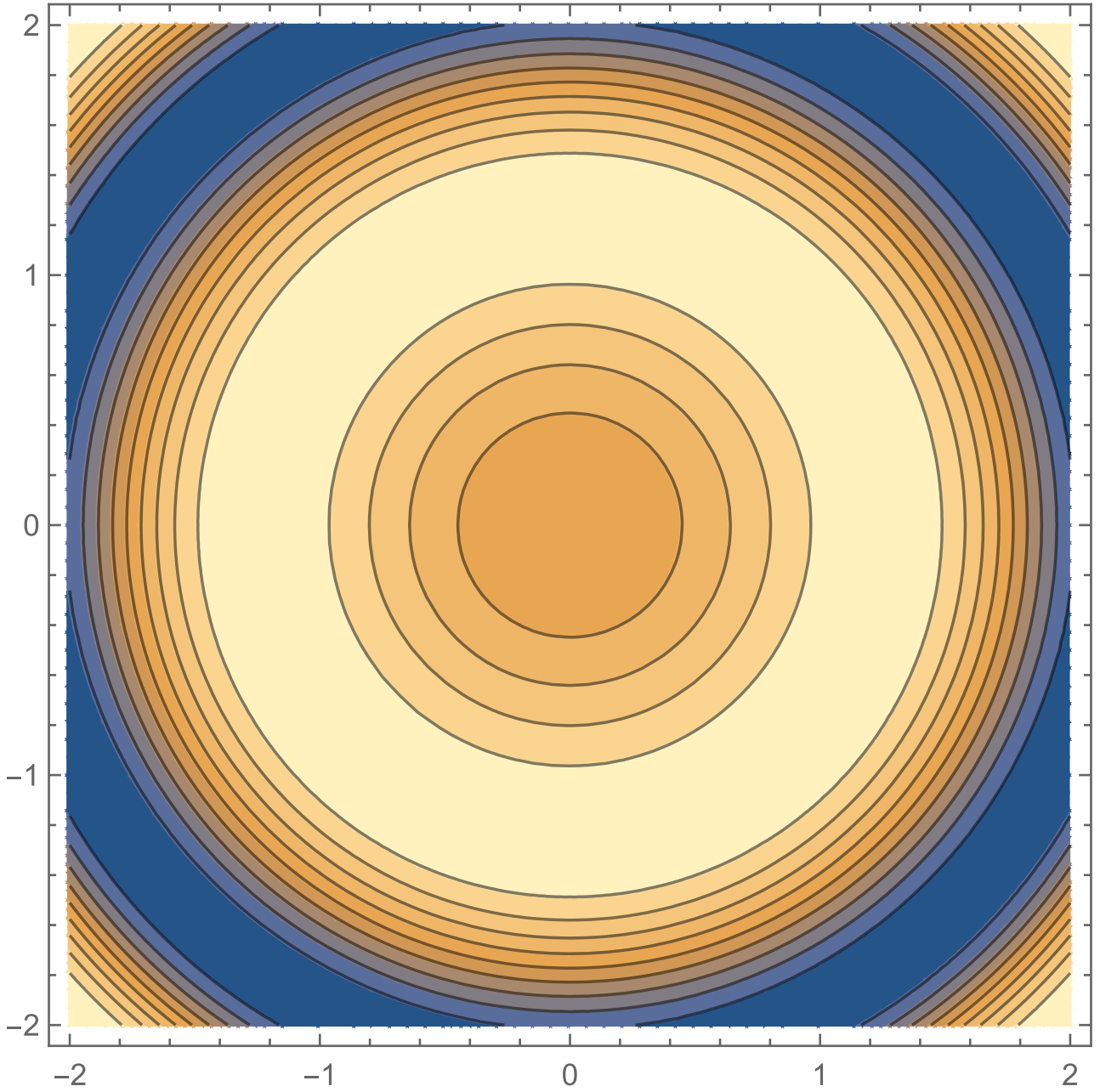
Out[ ]=



ContourPlot[Sin[x^2 + y^2], {x, −2,2}, {y, −2,2}]

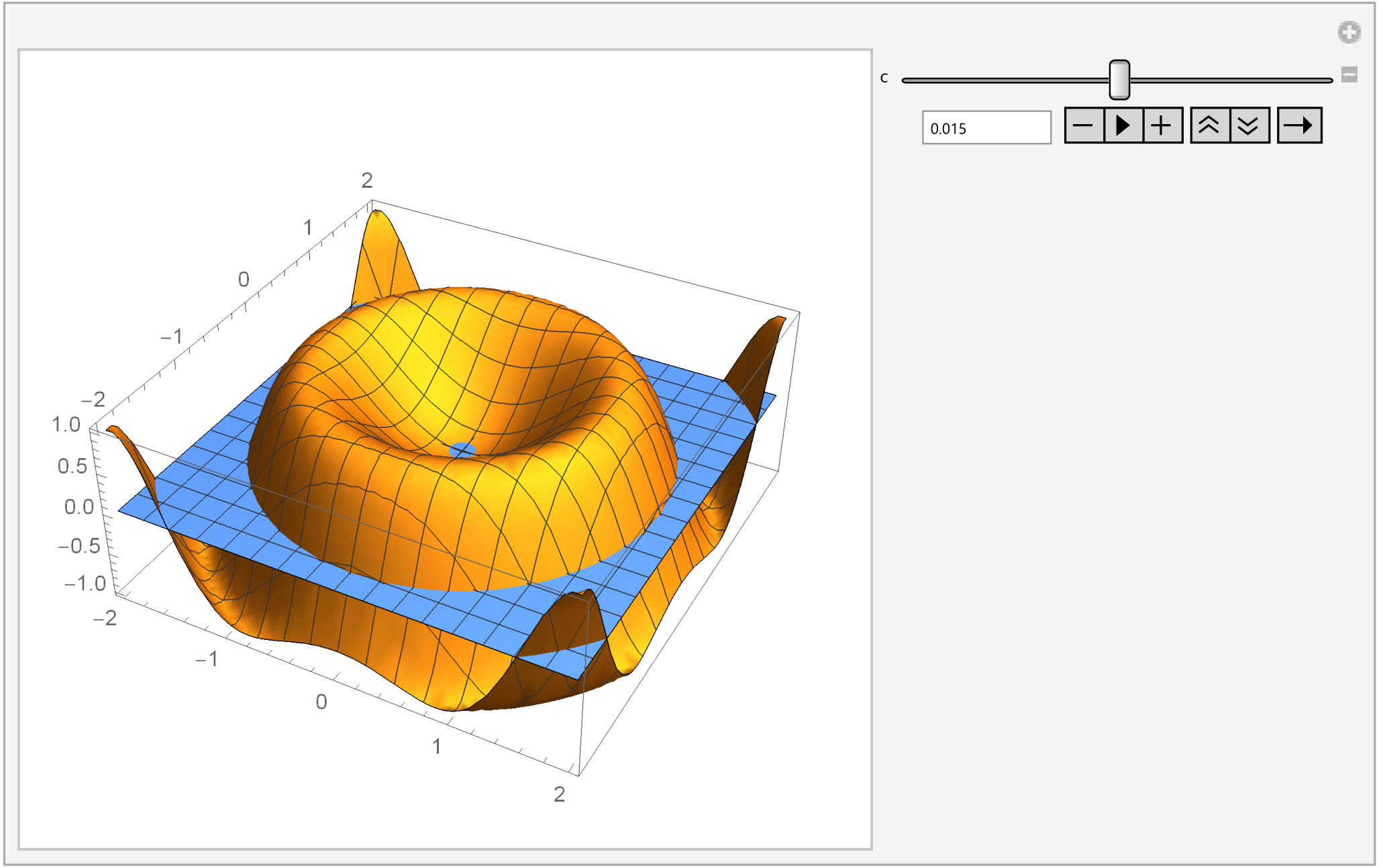
In[•]:=
```
ContourPlot[ Sin[x^2 + y^2], {x, -2, 2}, {y, -2, 2}]
```

Out[•]=



In[ ]:=
```
Manipulate[Plot3D[ {Sin[x^2 + y^2], c}, {x, -2, 2}, {y, -2, 2}], {c, -1.2, 1.2}]
```

Out[ ]=



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Manipulate[Plot3D[ {Sin[x^2 + y^2], c}, {x, -2, 2}, {y, -2, 2}], {c, -1.2, 1.2}]
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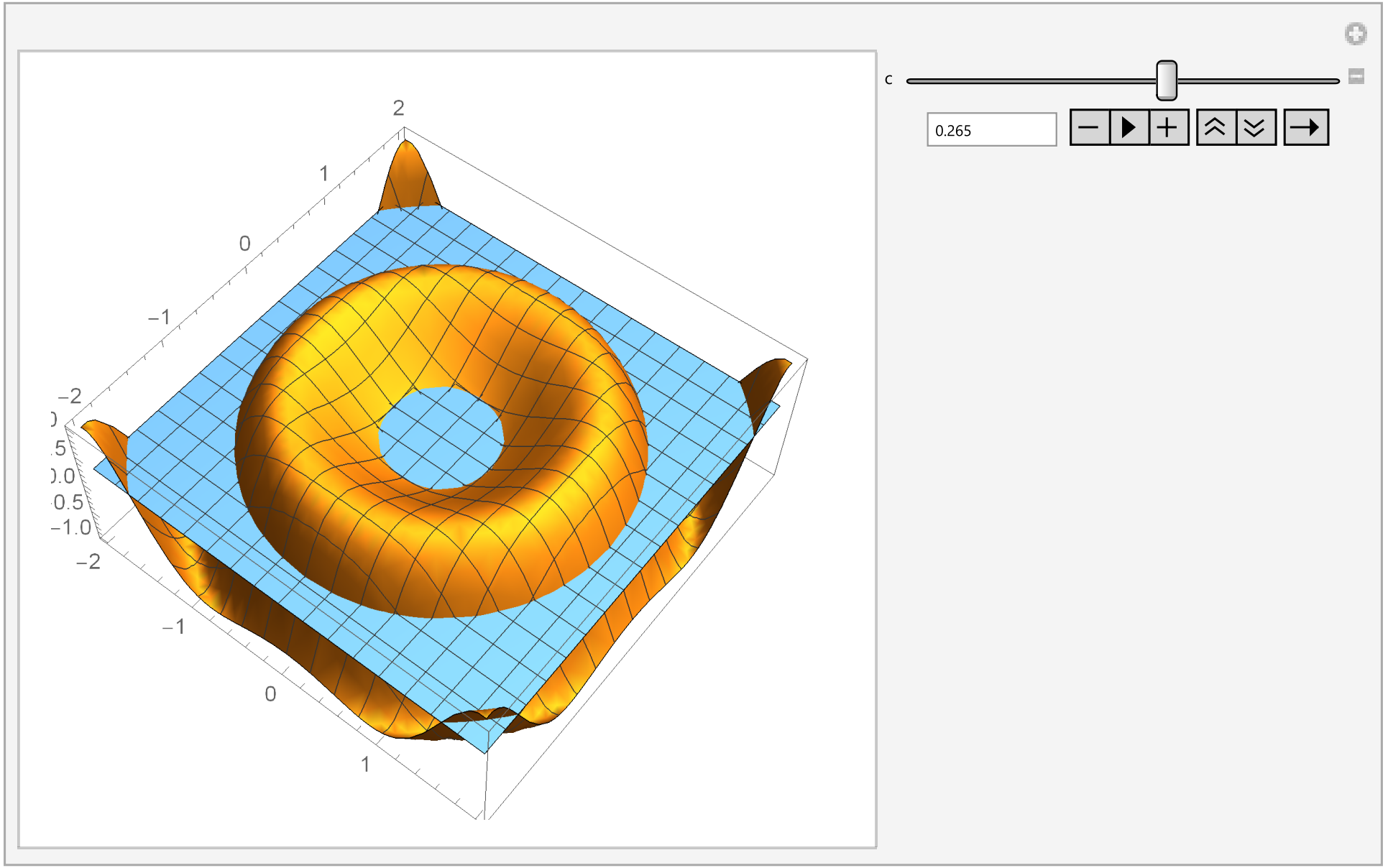
Out[ ]=



In[•]:=
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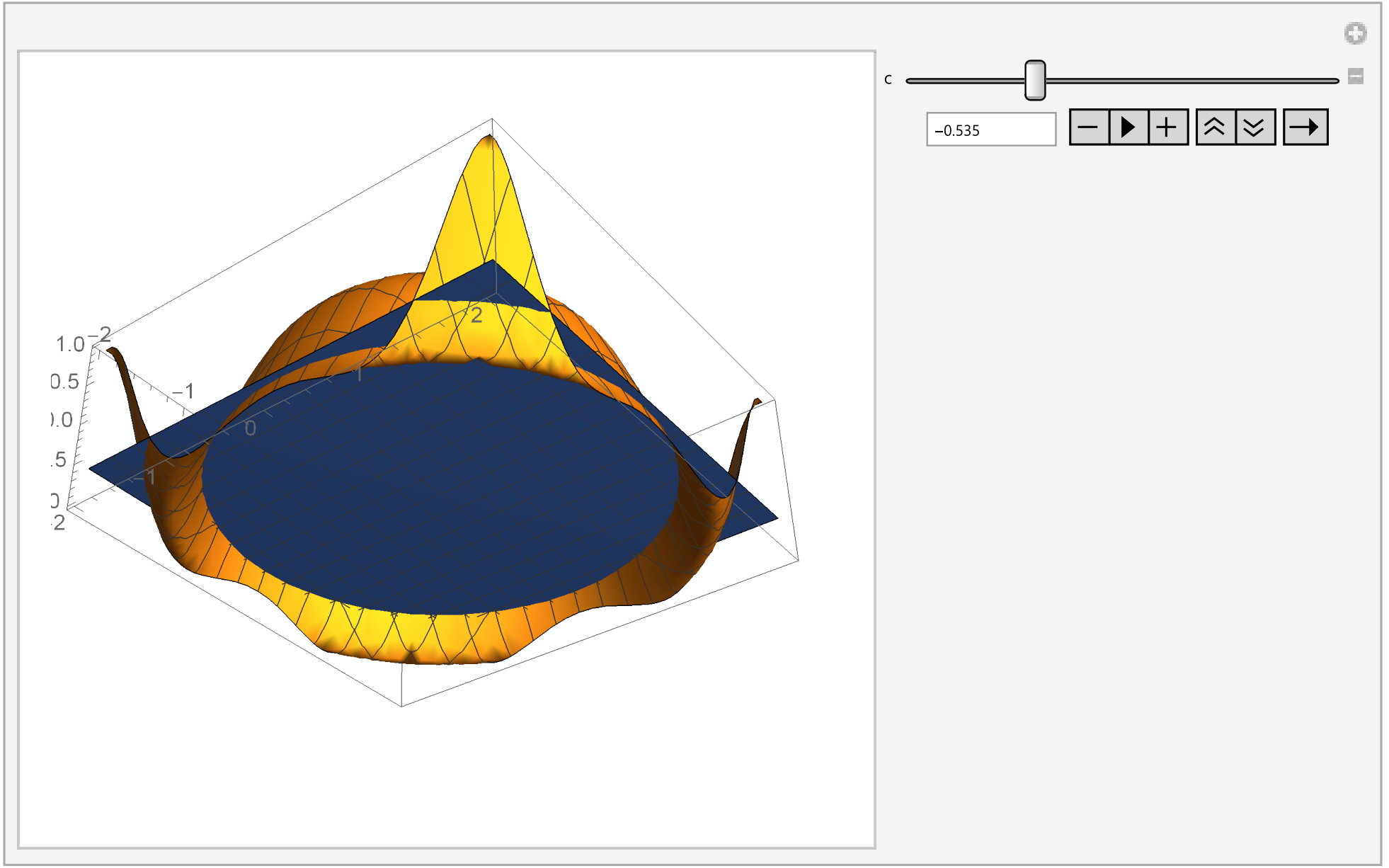
Manipulate[Plot3D[ {Sin[x^2 + y^2], c}, {x, -2, 2}, {y, -2, 2}], {c, -1.2, 1.2}]
```

Out[•]=



$f(x,y) = \sin(2x+3y)$

$L_c(f) = \{(x,y) : \sin(2x+3y) = c\}$

If $|c| > 1$ Then $L_c(f) = \emptyset$ empty set

↳ because $-1 \le \sin(\text{input}) \le 1$

Study $-1 \le c \le 1$

$f(x,y) = \sin(2x+3y)$

$-1 \le c \le 1$

$\sin(2x+3y) = c$

$\Longrightarrow 2x+3y = \arcsin(c)$

$\sin(2x+3y) = 1$

Then

$2x+3y = \frac{\pi}{2}, \frac{\pi}{2}+2\pi,$

$\frac{\pi}{2}-2\pi, \ldots$

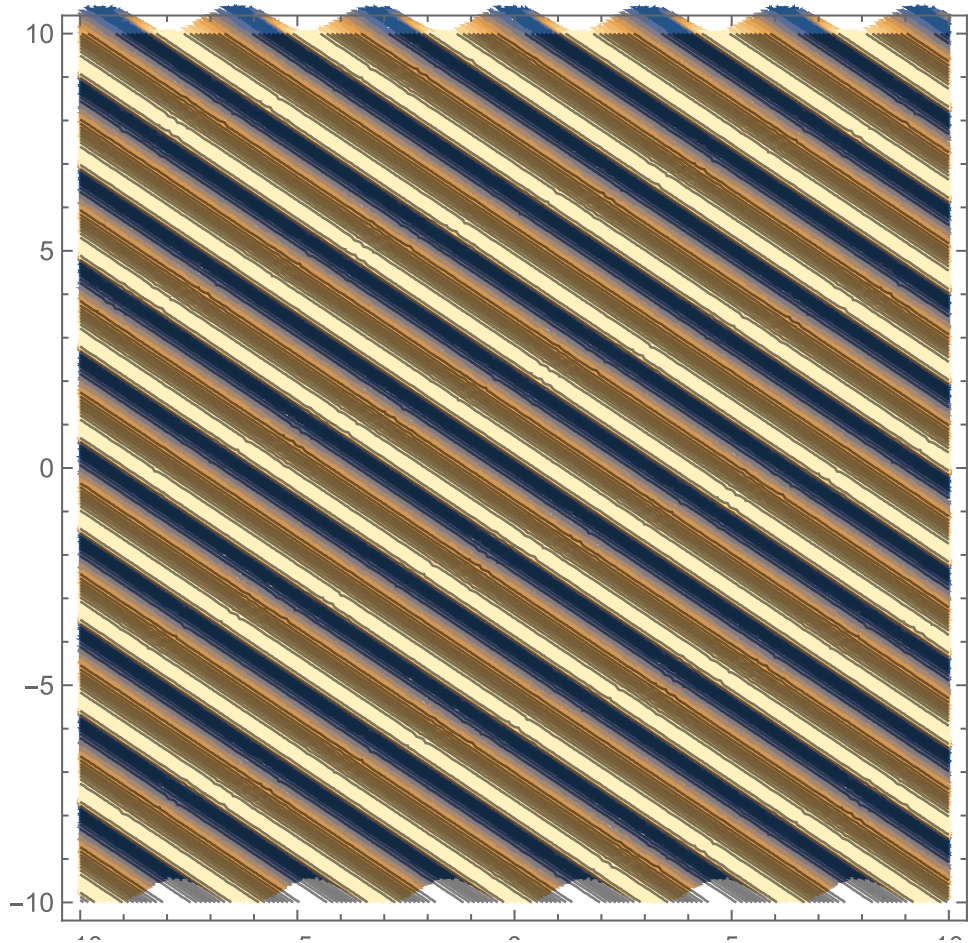
$2x+3y = \frac{\pi}{2}+n2\pi$

for n an integer

$y = -\frac{2}{3}x + \frac{1}{3}\left(\frac{\pi}{2}+2\pi n\right)$

```
In[ ]:= ContourPlot[Sin[2 x + 3 y], {x, -10, 10}, {y, -10, 10}]
```

Out[ ]=



In[ ]:=
```
Manipulate[Plot3D[{Sin[2 x + 3 y], c}, {x, -2, 2}, {y, -2, 2}], {c, -1.2, 1.2}]
```

In[ ]:=