

# Math 150: Multivariable Calculus Fractals: From Khan to Frozen

(Shhh: Movie is a surprise for my daughter Kayla)

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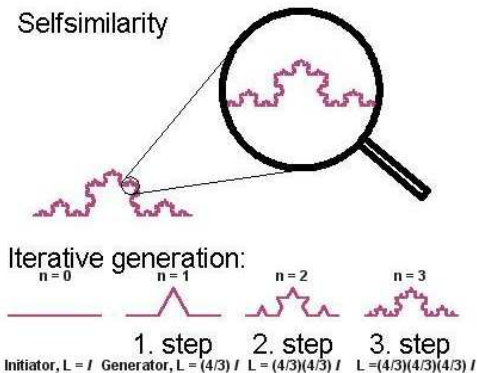
[http://www.williams.edu/Mathematics/sjmiller/public\\_html/150](http://www.williams.edu/Mathematics/sjmiller/public_html/150)

Bronfman Science Center  
Williams College, March 20, 2014

## Theory of Fractals

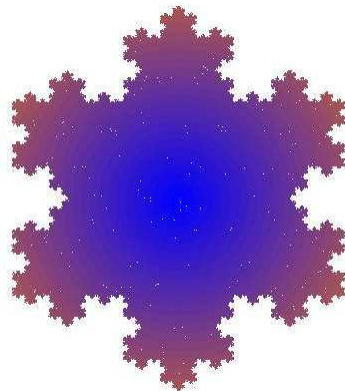
## Basics

Often simple rules leading to complex, repeating behavior.



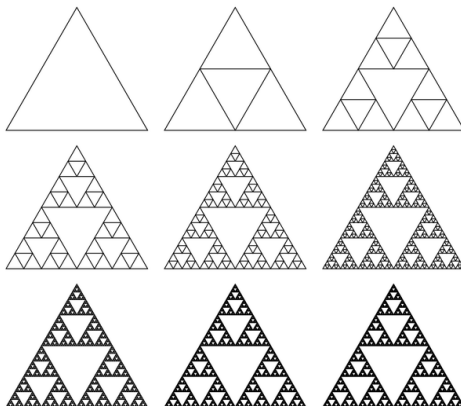
## Basics

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Often simple rules leading to complex, repeating behavior.



# Pascal's Triangle

```

      1
     1 1
    1 2 1
   1 3 3 1
  1 4 6 4 1
 1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 186 715 1287 1716 1716 1287 715 186 78 13 1

```

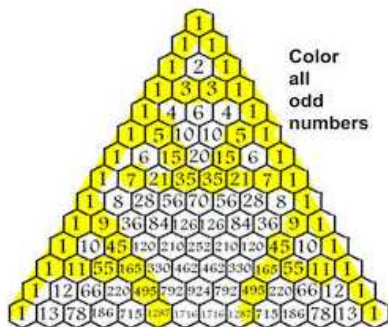
[http://www.youtube.com/watch?v=tt4\\_4YajqRM](http://www.youtube.com/watch?v=tt4_4YajqRM)

## Pascal's Triangle Modulo 2

```

      1
     1 1
    1 2 1
   1 3 3 1
  1 4 6 4 1
 1 5 10 10 5 1
1 6 15 20 15 6 1
1 7 21 35 35 21 7 1
1 8 28 56 70 56 28 8 1
1 9 36 84 126 126 84 36 9 1
1 10 45 120 210 252 210 120 45 10 1
1 11 55 165 330 462 462 330 165 55 11 1
1 12 66 220 495 792 924 792 495 220 66 12 1
1 13 78 286 715 1287 1716 1716 1287 715 286 78 13 1

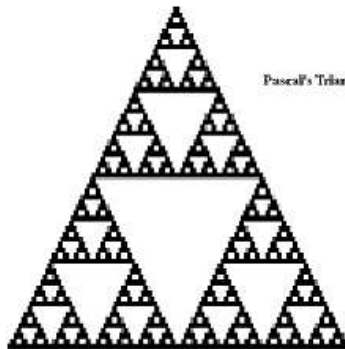
```



[http://www.youtube.com/watch?v=tt4\\_4YajqRM](http://www.youtube.com/watch?v=tt4_4YajqRM)

## Pascal's Triangle Modulo 2

FIGURE 2  
© 1990 by  
Jim Nugent

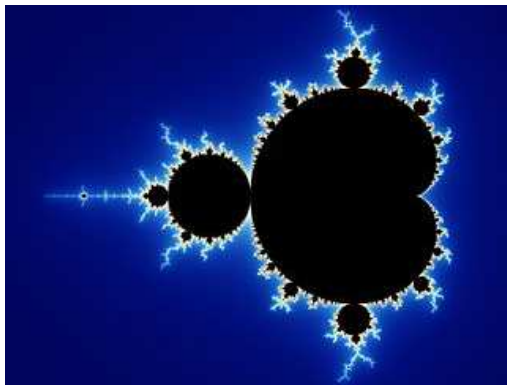


[http://www.youtube.com/watch?v=tt4\\_4YajqRM](http://www.youtube.com/watch?v=tt4_4YajqRM)



## Mandelbrot Set

For each  $c \in \mathbb{C}$ :  $z_0 = c$ ,  $z_1 = z_0^2 + c$ ,  $z_2 = z_1^2 + c$ ,  $\dots$



<http://www.youtube.com/watch?v=gEw8xpb1aRA>

## Star Trek II: Wrath of Khan

## Genesis Torpedo



<http://www.youtube.com/watch?v=Tsr-XtuKuSw>

<http://www.youtube.com/watch?v=Qe9qSLYK5q4>

[Links / References](#)

## References: Slides online here: <http://web.williams.edu/Mathematics>

[/sjmiller/public\\_html/150/talks/Fractaltalk/lectures14b\\_fractals.pdf](http://web.williams.edu/Mathematics/sjmiller/public_html/150/talks/Fractaltalk/lectures14b_fractals.pdf) (Steven J Miller)

- *Fractal Geometry Class (Yale University)*, Michael Frame, Benoit Mandelbrot and Nial Nager.  
<http://classes.yale.edu/fractals>.
- *How Benoit Mandelbrot Discovered Fractals*, A Short Film by Errol Morris.  
<http://www.openculture.com/2013/11/how-benoit-mandelbrot-discovered-fractals-a-short-f>
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<http://www.openculture.com/2013/08/arthur-c-clarke-narrates-film-on-mandelbrots-fracta>
- *The Hidden Fractals in Jurassic Park*, Gizmodo.  
<http://gizmodo.com/5981726/the-hidden-fractals-in-jurassic-park>.
- *Fractals and the art of roughness: Benoit Mandelbrot on TED.com*.  
[http://blog.ted.com/2010/07/06/fractals\\_and\\_th/](http://blog.ted.com/2010/07/06/fractals_and_th/).
- **Math 150 Homepage, Steven J Miller, Spring 2014.**  
[http://web.williams.edu/Mathematics/sjmiller/public\\_html/150/](http://web.williams.edu/Mathematics/sjmiller/public_html/150/).
- **Math 150 Additional Comments (see Monday, March 10 for applications of fractals to economics), Steven J Miller, Spring 2014.**  
[http://web.williams.edu/Mathematics/sjmiller/public\\_html/150/addcomments.htm](http://web.williams.edu/Mathematics/sjmiller/public_html/150/addcomments.htm)
- **Teachers As Scholars: Pascal and Binomial Coefficients, Steven J. Miller, February 2014.**  
<http://www.youtube.com/watch?v=nrknXC8xmTU&feature=youtu.be>

## Feature Presentation



Math for Frozen:

<https://www.math.ucla.edu/~jteran/papers/SSCTS13.pdf>.