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11	.0414	45	0.91	0511	0500		525 6		0314		4	8 13	2		24 23	28	32 31	36 35
12	.0792	0828	0864	0899	0934	0060		1028	1077		4	71		14 1	9 22 8 21	25	30 28	33 32
13	.1139	1173	1206	1239	1271	1202	1225	1038	1072	1100	3	7 10		13 I 13 I	7 20 6 20	24	27	31
14	.1461	1492	1523	155		614	64	163	70	132		C		2 1	5 18	21	26 24	29 27
15 16 17 18 19	.1761 .2041 .2304 .2553 .2788	1790 2068 2330 2577 2810	1818 2095 2355 2601 2833	184 2122 2380 2625 2856	8 21-8 2405 2648 2878	1912 175 2430 2672 2900	901 2201 2455 2695 2923	1900 2227 2480 2718 2945	981 2253 2504 2742 2967	2014 2279 2529 2765 2989	1 3 2 2 2	5 8 5 1 5 1 4 0	3 1 3 1 7 1 7 1	1 I. 0 I. 0 I. 0 I. 9 I.	4 17 3 16 2 15 2 14 1 13	20 18 17 17 15	22 21 20 19 18	25 23 22 22 20
20 21 22 23 24	.3010 .3222 .3424 .3617 .3802	303¢ 3243 3444 3636 3820	3464 3655 3838	ver 3483 3674 385S	13M 3592 3692	ille 1 (a	er, \)W	Wil illia	lia m	ms s.e	C dı	ol I	le	ge	13 12 11 11 11	15 14 13 13	17 16 15 14 14	19 18 17 16 16
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- Discuss objects across many orders of magnitude.
- Linearize many non-linear functions (calculus becomes available).

Plot of 100 most populous cities





Definition of Logarithms

- If $x = b^y$ then $\log_b x = y$.
- Read as the logarithm of x base b is y.
- Often use base 10, and some authors suppress the subscript 10.
- Other popular bases are 2 for computers, and *e* for calculus; many sources write ln *x* for the natural logarithm of *x*, which is its logarithm base *e* (*e* is approximately 2.71828).

• Examples: $\log_b x = y$ means we need y powers of b to get x.

- $100 = 10^2$ becomes $\log_{10} 100 = 2$. In base *e* it is about 4.6.
- $1 = 10^{0}$ becomes $\log_{10} 1 = 0$. In base *e* it is still 0.
- $.001 = 10^{-3}$ becomes $\log_{10} .001 = -3$. In base *e* it is about -6.9.

Order of Magnitude of some	Lengths			Length of Lake Erie		Distance to farthest
LENGTH	meters		Diameter of red blood	NI		galaxy
radius of proton	10 ⁻¹⁵	Diameter of nuclear	corpuscie	4	Radius of first star	
radius of atom	10 ⁻¹⁰	particles		-STERO SENIO	(a) sun 4	
radius of virus	10 ⁻⁷	Diameter	ň	Concepto add	9 8	A Banker
radius of amoeba	10 ⁻⁴	of atom	M	a. Marile	the property of the	Allemy C d-
height of human being	10 ⁰	-15 -13 -11 -9	-7 -5 -3 -1 0+1	3 5 7 1	8 11 13 15 17 19	21 23 25 27
radius of earth	10 ⁷					
radius of sun	109	the great strain gallony	1 cm 1 m	Line and sport	and a manufacture of the	d ministration will
earth-sun distance	10 ¹¹	Vivv Wavelength	1 mm		1 light your	Merent State
radius of solar system	10 ¹³	of X ray	1µm C	pth of		and the second
distance of sun to nearest star	10 ¹⁶		welength	ale (2)	AN ANT THE SA	1 Decor
radius of milky way galaxy	10 ²¹		oright	D	Diam	eter of
radius of visible Universe	10 ²⁶			Radius of earth	our Mi Ga	ky Way laxy 4

Earthquake frequency and destructive power

The left side of the chart shows the magnitude of the earthquake and the right side represents the amount of high explosive required to produce the energy released by the earthquake. The middle of the chart shows the relative frequencies.

Mag	nitude	Notable earthquakes	Fnergy equivalents	(equivalent of explosive)
- and the		notable cal inquinco	Lifer By equivalence	 123 trillion lb.
10 -		Chile (1960)		(56 trillion kg)
Sec.		Alaska (1964)		4 trillion lb.
9 -	Creat earthquake: pear total	Japan (2011) 🜱		(1.8 trillion kg)
	destruction, messions loss of life	New Medda Me (1010)	Krakatoa volcanic eruption	4.15.10.101
	destruction, massive loss of me	New Madrid, Mo. (1812)	World's largest nuclear test (USSR)	123 billion lb.
8 -	Malor oorthousko: cousts oos	San Francisco (1906)	Maunt St. Helene eruntion	(56 billion ka)
	nomic impact large loss of life		T mount at. neiens eruption	(
77.00	nonne impact, large loss of me	Lema Prieta, Calif. (1989)	0	4 billion lb.
	Strong earthquake: damage	Kobe, Japan (1995) ず 🕹	5	(1.8 billion kg)
	(\$ billione) loss of life	Northridge, Calif. (1994)	The set has a family based	Concernance and
e -	(a bimbila), 1033 01 mc	La	A Hirosauna atomic bomb	123 million lb.
0	Moderate earthquake:		~ ~	(56 million kg)
	property damage	Long Island, N.Y. (1884)		a minimum th
5 -		L 2.0	00 <u> </u>	4 million 10.
- M	Light earthquake;	/	Average tornado	(1.8 million kg)
	some property damage			12 200 lb
4 -		- 12,0	000 000	(56 000 km)
	Minor earthquake;			(50,000 kg)
	felt by humans		Large lightning bolt	4 000 lb
3 -		100,	000 Oklahoma City bombing	(1.800 kg)
			Moderate lightning	polt (1,000 kg)
		-	and a state of the	123 lb.
2 -		1,000	000,	(56 kg)
				(+
		Number of earthquake	s ner vear (worldwide)	
	<	manioer or car inquakes	per Jour (normine)	< 5 S
Source	e-11S Contoninal Survey			MCT

1.0.000.0000000000



The pH Scale



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Recall: Definition of Logarithms

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Plots of Exponentiation and Logarithms

- If $x = b^y$ then $\log_b x = y$.
- Read as the logarithm of x base b is y.



- Discuss objects across many orders of magnitude.
- Linearize many non-linear functions (calculus becomes available).





• Linearize many non-linear functions (calculus becomes available).



Notice that even on a small range, from 1 to 10, the polynomial of highest degree drowns out the others and can barely see.

• Linearize many non-linear functions (calculus becomes available).



Left: Semi-log plot: $y = \log x^r$. Right: log-log plot: $\log y = \log x^r$. Note that we can now see the four functions on one plot, and the log-log plot now has linear relations.

Review: Exponent Laws

Laws

- $b^m b^n = b^{m+n}$
- $b^m / b^n = b^{m-n}$
- $(b^m)^n = b^{mn}$

Examples

- $10^3 10^2 = (10 * 10 * 10) * (10 * 10) = 10^5$
- $10^3/10^2 = (10 * 10 * 10)/(10 * 10) = 10^1$
- $(10^3)^2 = 10^3 * 10^3 = (10 * 10 * 10) * (10 * 10 * 10) = 10^6$

Logarithm Laws

Parts of a Slide Rule



- Remember if $x = b^y$ then $\log_b x = y$.
- Below assume $\log_b x_1 = y_1$ and $\log_b x_2 = y_2$.
- These allow us to simplify computations with logarithms.

THEOREM

• $\log_b(x^n) = n \log_b x$. • $\log_b(x_1 x_2) = \log_b(x_1) + \log_b(x_2)$. Log of a product is the sum of the logs. • $\log_b(x_1/x_2) = \log_b(x_1) - \log_b(x_2)$. Log of a quotient is the difference of the logs. • $\log_b x = \log_c x/\log_c b$. If know logs in one base, know in all.

OPTIONAL – PROOFS OF THE LOG LAWS



Remember if $x = b^y$ then $\log_b x = y$.

Below assume $\log_b x_1 = y_1$ and $\log_b x_2 = y_2$.

• $\log_b(x^n) = n \log_b x$, Log of a power is that power times the log.

•
$$\log_b x = y$$
 means $x = b^y$.

Remember if $x = b^y$ then $\log_b x = y$.

Below assume $\log_b x_1 = y_1$ and $\log_b x_2 = y_2$.

• $\log_b(x^n) = n \log_b x$, Log of a power is that power times the log.

Proof:

•
$$\log_b x = y$$
 means $x = b^y$.

• Thus $x^n = (b^y)^n = b^{ny}$.

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- Thus $x^n = (b^y)^n = b^{ny}$.
- Taking logarithms: $\log_b(xn) = ny = n \log_b x$.

- Remember if $x = b^{y}$ then $\log_{b} x = y$.
- Below assume $\log_b x_1 = y_1$ and $\log_b x_2 = y_2$.
- $\log_b(x_1 x_2) = \log_b(x_1) + \log_b(x_2)$, Log of a product is the sum of the logs.

Proof:

• As $\log_b x_1 = y_1$ and $\log_b x_2 = y_2$, we have $x_1 = b^{y_1}$ and $x_2 = b^{y_2}$.

$$\label{eq:hyperbolic} \begin{split} & \operatorname{Homorely}_{(1,1)}(v)_{(1,$$

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- Thus $x_1 x_2 = b^{y_1} b^{y_2} = b^{y_1 + y_2}$.

$$\begin{split} & \text{Restind} & (+ 1^{2}) \text{Resting} (+ \gamma) & \text{Resting} (+ \gamma) \\ & \text{Resting} (+ \gamma) & \text{Resting} (+ \gamma) \\ & - \frac{1}{2} (\beta _{1} + \beta _{2} \beta _{2} \beta _{2} \beta _{3} \beta _$$

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- Thus $x_1 x_2 = b^{y_1} b^{y_2} = b^{y_1 + y_2}$.
- Therefore $\log_b(x_1 x_2) = y_1 + y_2 = \log_b x_1 + \log_b x_2$.

Remember if $x = b^y$ then $\log_b x = y$.

Below assume $\log_c x = u$ (so $x = c^u$) and $\log_c b = v$ (so $b = c^v$).

• $\log_b x = \log_c x / \log_c b$. Know logs in one base, know in all.

• As
$$\log_b x = y$$
 have $x = b^y$. Similarly $x = c^u$ and $b = c^v$.

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- Thus $x = b^y = (c^v)^y = c^{vy}$.
- As also have $x = c^u$ we have u = vy or y = u/v.
- Substituting gives $\log_b x = \log_c x / \log_c b$.

```
Example: Factorial Function:
Number ways to order n objects when order matters:
n! = n * (n - 1) * \cdots * 3 * 2 * 1.
```

```
list = {}; semiloglist = {}; logloglist = {};
For[n = 1, n <= 200, n++,
```

```
list = AppendTo[list, {n, n!}];
```

```
semiloglist = AppendTo[semiloglist, {n, Log[n!]}];
```

```
logloglist = AppendTo[logloglist, {Log[n], Log[n!]}];
```

}];

```
Print[ListPlot[list]]; Print[ListPlot[semiloglist]]; Print[ListPlot[logloglist]];
```

Example: Factorial Function: Number ways to order *n* objects when order matters: $n! = n * (n - 1) * \cdots * 3 * 2 * 1.$

