Math 150: Calculus III: Spring '22 (Williams)

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Homepage: https://web.williams.edu/Mathematics/sjmiller/public_html/150Sp22/

Lecture 4: 2-11-22:

Plan for the day: Lecture 4: February 11, 2022:

General items.

Review Calc I/II

Discuss Applications as time permits

Discuss Class Policy

Prepare for Matrices and Vectors

Integration Review

u-substitution

Chain Rule: $(f(g(x)))' = f'(g(x))\, g'(x)$

$$\int_{x=a}^{b} [f(g(x))]' dx = \int_{x=a}^{b} f'(g(x)) g'(x)\, dx$$

Let $u = g(x)$ Then $\frac{du}{dx} = g'(x)$ so $du = g'(x)dx$

$$\int_{x=a}^{b} [f(g(x))]' dx = \int_{u=g(a)}^{g(b)} f'(u)\, du$$

$$f(g(x))\Big|_{x=a}^{b} = f(g(b)) - f(g(a)) = f(u)\Big|_{g(a)}^{g(b)} = f(g(b)) - f(g(a))$$

$$\int_{x=0}^{1} x^3 e^{2x^4}\,dx = \int_{x=0}^{1} e^{2x^4} \cdot x^3\,dx$$

$u = 2x^4$ $\qquad \left(\frac{du}{dx} = 8x^3\right)$ or $du = 8x^3\,dx$ $\qquad x: 0 \to 1$

$\frac{1}{8}\,du = x^3\,dx$ $\qquad u: 0 \to 2$

$$\int_{x=0}^{1} x^3 e^{2x^4}\,dx = \int_{u=0}^{2} e^u \tfrac{1}{8}\,du$$

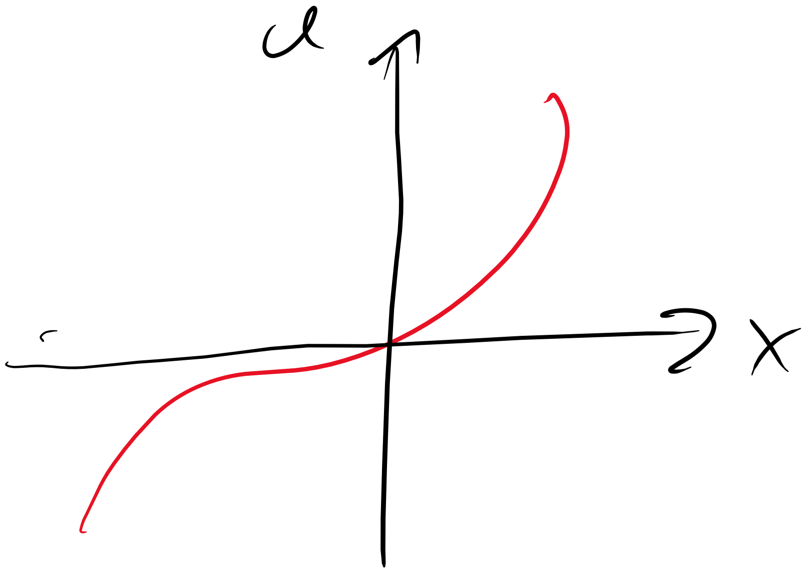
$$= \frac{1}{8}\int_{u=0}^{2} e^u\,du = \frac{1}{8} e^u \Big|_0^2$$

$$= \frac{1}{8}\left(e^2 - e^0\right) = \frac{e^2 - 1}{8}$$

$$\int_{-1}^{1} x^2 e^{x^3/3}\,dx \qquad x: -1 \to 1$$

$$u = x^3/3 \qquad \frac{du}{dx} = x^2 \qquad du = x^2 dx \qquad u: -\frac{1}{3} \to \frac{1}{3}$$

$$\int_{x=-1}^{1} x^2 e^{x^3/3}\,dx = \int_{u=-1/3}^{1/3} e^u\,du = e^u\Big|_{-1/3}^{1/3} = e^{1/3} - e^{-1/3}$$

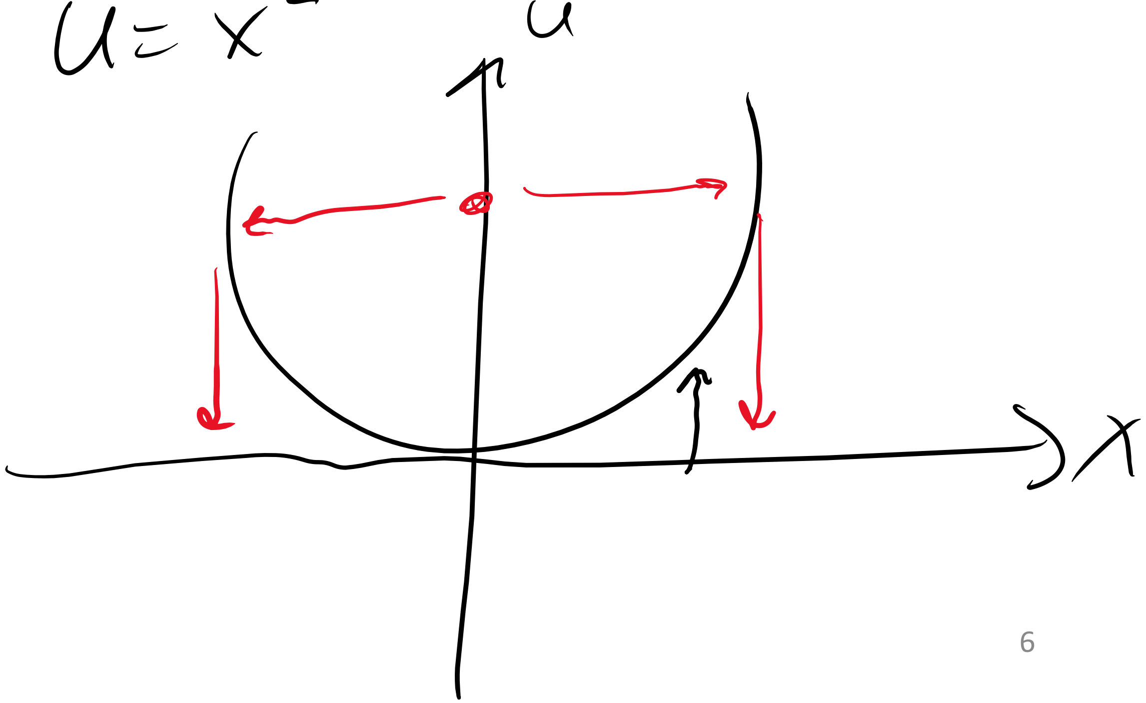


$$\int_{-1}^{1} x e^{x^2/2}\, dx$$

$u = x^2/2$ $\qquad \dfrac{du}{dx} = x$ so $du = x\,dx$ $\qquad x: -1 \to 1$ $\quad u: \frac{1}{2} \to \frac{1}{2}$

$$\underbrace{\int_{x=-1}^{1} x e^{x^2/2}\, dx}_{\text{This is zero odd function on a region symmetric about zero}} = \int_{u=1/2}^{1/2} e^{u}\, du = 0$$

$U = x^2$



Integration By Parts

$$[f(x)g(x)]' = f'(x)g(x) + f(x)g'(x)$$

$$f(x)\,g'(x) = [f(x)\,g(x)]' - f'(x)\,g(x)$$

$$\int_a^b f(x)\,g'(x)dx = \int_a^b [f(x)\,g(x)]'\,dx - \int_a^b f'(x)g(x)dx$$

$$\int_a^b f(x)\,g'(x)\,dx = f(x)g(x)\Big|_a^b - \int_a^b f'(x)g(x)dx$$

$u = f(x)$ $\quad v = g(x)$ $\quad dv = g'(x)dx$ $\quad du = f'(x)dx$

$$\int_a^b u\,dv = uv\Big|_a^b - \int_a^b v\,du$$

$u(b)\,v(b) - u(a)\,v(a)$

$$\int_0^{2\pi} x\cos(x)\,dx$$

$u = x$

$du = dx$

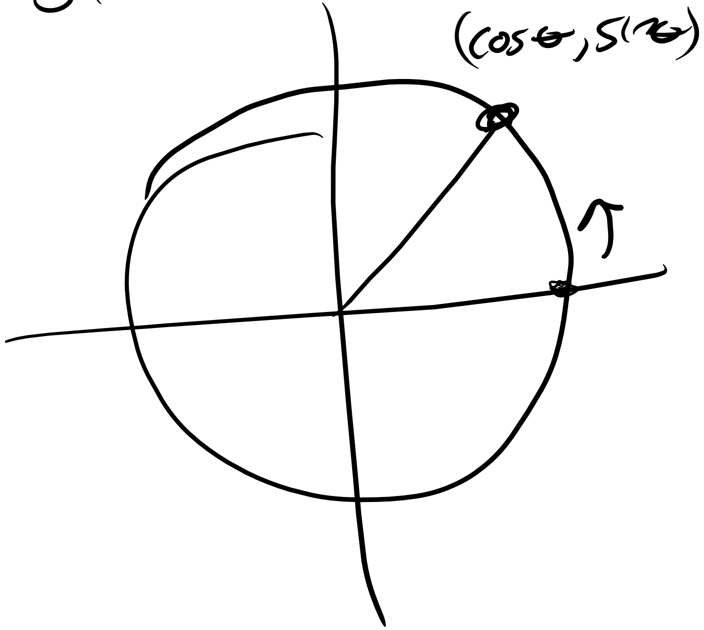
$du = \cos x$

$v = \sin x$

$\cos'(x) = -\sin(x)$

(minus sign)

$\sin'(x) = \cos x$



$$\int_0^{2\pi} x\cos x\,dx = x\sin x\Big|_0^{2\pi} - \int_0^{2\pi}\sin x\,dx$$

$$= -\int_0^{2\pi}\sin x\,dx$$

$$= \cos x\Big|_0^{2\pi}$$

$$= \cos 2\pi - \cos(0)$$

$$= 1 - 1 = 0$$

QWERTY

Calc Test

x^3	$x^{3/2}$	$x^{\sqrt{2}}$	fn
$3x^2$	$\frac{3}{2}x^{\frac{1}{2}}$	$\sqrt{2}\,x^{\sqrt{2}-1}$	deriv

Rule: $(x^r)' = r x^{r-1}$

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$ defn of the deriv

$$f(x) = x^n$$

Binomial Thm

$$f(x+h) = (x+h)^n = x^n + \binom{n}{1} x^{n-1} h + \binom{n}{2} x^{n-2} h^2 + \cdots$$

$$= x^n + n x^{n-1} h + h^2 * (\text{crap})$$

$$\frac{f(x+h) - f(x)}{h} = n x^{n-1} + \underbrace{h * (\text{crap})}_{\substack{\text{as } h \to 0 \\ \text{this goes to} \\ 0}}$$

So the deriv is $n x^{n-1}$

$$f(x) = x^{3/2}$$

$$f(x+h) = (x+h)^{3/2} = ?$$

$$\frac{(x+h)^{3/2} - x^{3/2}}{h} \quad \frac{(x+h)^{3/2} + x^{3/2}}{(x+h)^{3/2} + x^{3/2}}$$

$$= \frac{(x+h)^3 - x^3}{h\left((x+h)^{3/2} + x^{3/2}\right)}$$

$$f(x) = x^{3/2}$$

$$g(x) = f(x)^2 = x^3$$

$$g'(x) = 2f(x)f'(x) = 3x^2$$

$$f'(x) = \frac{3x^2}{2f(x)} = \frac{3x^2}{2x^{3/2}}$$

$$= \frac{3}{2}x^{1/2}$$

Ex: do $x^{p/q}$

$$f(x) = x^{\sqrt{2}} = e^{g(x)}$$

$$\ln\left(x^{\sqrt{2}}\right) = \ln\left(e^{g(x)}\right)$$

$$\sqrt{2}\ln(x) = g(x)\ln(e) = g(x)$$

$$x^{\sqrt{2}} = e^{\sqrt{2}\ln x}$$

$$\left(x^{\sqrt{2}}\right)' = e^{\sqrt{2}\ln x} \cdot \left(\sqrt{2}\ln x\right)'$$

$$= e^{\sqrt{2}\ln x} \cdot \sqrt{2}\,\frac{1}{x}$$

$$= x^{\sqrt{2}} \cdot \sqrt{2}\,x^{-1} = \sqrt{2}\,x^{\sqrt{2}-1}$$

$$e^x = 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots$$

$$= \sum_{n=0}^{\infty} \frac{x^n}{n!}$$