

MATH 406: ANALYSIS AND NUMBER THEORY

INTRODUCTION

↳ Explain my background

basics of class

↳ several levels: broad overview + details

↳ options to read more

↳ options to do research/explore

↳ emphasize techniques and universalities

QUESTION

↳ What do you think of when hear "analysis" or "number theory"

[

Move to a brief description of topics:

$\{n^k \alpha\}$ and Poissonian behavior
(FA, P, NT)

Random matrices and L -fns
(P, L, NT)

Spacings b/w events

↳ Fourier Analysis (FA)

Probability (P)

Linear Algebra (L)

Number Theory (NT)

Berford's Law (P, FA)

Circle Method (FA, NT)

PNT (FA, NT)

MSTD (P)

COUNTING EVENTS

↳ Fourier Analysis (FA)

Probability (P)

Number Theory (NT)

Other: $3X+1$, algebraic structure of numbers (Champernowne)

RESULT	NTG and Poisson Sum	RMT	Circle Method	Benford	PS) and L-fns
BASIC FOURIER SERIES	try functions to help analyze pattern		try functions are the generating fns for problem, get coeffs from Complex Analysis	Fourier series and transform to study probability distn	useful expansions, complex analysis to get coefficients
↳ Poisson Summation			↳ Modulo 1 CLT products converge to Benford behavior	↳ Many processes are Benford	↳ Functional Equations
↳ Fejér/Dirichlet's Theorem	↳ approximate step fns with try polynomials		↳ Many type problems/ understanding sizes of exponential sums	Rates of convergence (Lehmer and euclid distribution)	Special values of L-fns encode arithmetically interesting information
Algebraic Structure of numbers	Approximating irrationals with rationals to understand dynamics			↳ certain density sets are Benford	
↳ Kronecker-Weyl	↳ equidistribution				
Combinatorics		moments (matchings and Catalan numbers), Toeplitz matrices, ...	Counting solutions to Diophantine Eqs		Unique factorization and the Euler-Product, coeff of L-fns encode arithmetic
Philosophy of Square-root cancellation		Scale for studying the eigenvalues	Estimating size of exponential sums		"Random Primes": The CLT is equiv to the Riemann hypothesis
Probability	distributions, moments	distributions moments	heuristic models	distributions moments	heuristic models
PROBLEMS	Evidistribution	Semicircle Law/ McKay's Law, ...	Weyl, Goldbach, Germain, ...	Digit frequencies (order statistics and Benford's Law)	Prime number theory, rate of elliptic curve, class number, ...
↳ Counting Evid	Poissonian Behavior	GOE Spacings	Germain, Twin Primes.		Zeros \Rightarrow class number, Riemann, ...
↳ Spacings b/w Evid					