A Modern Application of the Traveling Salesman Problem: Understanding Wal-Mart

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1 Introduction

1.1 The Traveling Salesman Problem

The Traveling Salesman Problem (TSP) is one of the most widely studied problems in computational mathematics. The goal of this problem is to minimize the total distance a salesman must travel to go to n cities. Studying the TSP is important because it can be extended and applied to many fields of interest including logistics, manufacturing, genetics, and telecommunications. While there is no immediate solution to this problem, it has provided a good foundation for developing and modeling techniques and tools to solve computational problems.

The TSP can be applied to more complex real world problems, one being distribution strategies. Since their creation in the 1950s, their popularity has drastically increased, accounting for about 88% of total sales in the market by 1989 (Jia, 2007). This paper focuses on the unique case of Wal-Mart and the distribution strategies that made the company successful. In examining its ascendancy to one of the world's most popular companies, it helps to compare Wal-Mart's strategies to Kmart, another discount retail giant that dominated the industry for the majority of the late 20th century. Comparing their strategies and seeing the different decisions each company made in order to expand their empire emphasize the importance of Wal-Mart's distribution networks.

The goal of any business is to maximize profit and minimize costs. The strategies chosen to achieve said goals are critical to the success of the company. Wal-Mart's main strategy is heavily focused on the supply end and can be best described as a hub-and-spoke model (Moore, 1993). By establishing distribution centers and building stores relatively near these

warehouses, Wal-Mart significantly cuts down shipping and transportation costs from 5%for normal customers to 3%(Chandran, 2003). These savings, coupled with their innovative use of technological advancements to organize and manage information, have allowed Wal-Mart to be the company it is today. In its initial rise to power, Wal-Mart entered the market by focusing on rural areas with relatively little competition and building their empire around these centers (Moore, 1993). Kmart, on the other hand, focused on expanding by establishing stores in urban areas and regions around the country (Graff, 1998). Eventually problems would arise with competition and store performance, forcing Kmart to constantly switch strategies and relinquish their top spot to Wal-Mart as America's leading discount retailer.

In examining Wal-Mart's expansion, the distinction between economies of scale and economies of density becomes more significant. An economist, Thomas Holmes, notes, "An economy of density is a kind of economy of scale" (Holmes, 2011). The difference lies in the methods of expansion when seeking to minimize costs. Scale economies pursue cost advantages by expanding geographically in size and increasing the reaches of their industry. Density economies, on the other hand, yield lower costs by increasing the amount of store units in a given area. In this way, Wal-Mart focuses on expanding its empire by adding store locations in the same areas near distribution centers. Kmart, on the other hand, established store locations in heavily populated areas with no coherent network of expansion. While effective in the short term, the long term left Kmart with many rundown and poorly located discount stores that were no longer as profitable as originally projected (Graff, 1998). As a result, Kmart was forced to focus on refurbishing old stores and taking other measures to avoid bankruptcy, while Wal-Mart continued to build upon its existing network of distribution centers and store locations to reduce costs. As Wal-Mart continues to expand, this paper seeks to analyze the supply strategies that Wal-Mart uses to place new stores in the market in order to find a model that will maximize the total profit.

1.2 Supply Chain Management

Supply chain management involves the processes for moving materials and goods to different firms. When firms utilize the supply chain in order to "enhance key outcomes" that improve a firm's performance, it becomes a matter of strategic supply chain management (Hult et al., 2004). Best value supply chains seek to solve problems by focusing on agility, adaptability, and alignment, as well as their ability to handle different competitive priorities that fall in line with the firm's goals. These supply chains have proven to be useful strategic weapons for many of today's major firms such as Toyota, Dell, and Wal-Mart (Ketchen and Hult, 2007). In researching supply chain management, it is important to consider some of the main theoretical perspectives on best value supply chain management. Two of the main theories that Wal-Mart's distribution strategies utilize would be resource dependence theory and network theory. In addition, it is important to discuss the difference between

contagious and hierarchical diffusion in order to understand the expansion of these large firms.

Resource dependence theory deals with a firm's relationship with other companies in order to gather goods and materials needed for supplies. As multiple firms work closely together, they become more dependent on each other. Consequentially, a firm realizes the danger with being overly dependent on one company and takes measures to make sure that the companies it depends on do not exploit it. At the same time, however, it realizes the importance of having other companies depend on them and the power that comes with the dependency. In order to benefit all parties, companies soon realize that cooperation yields the best results financially, making relationships between these firms based on mutual trust and need. The relationship between Wal-Mart and Procter & Gamble (P&G) is an example. Using innovative information technology systems, both P&G and Wal-Mart are able to cooperate with one another in order to efficiently get supplies to the stores at lower costs (Chandran, 2003). Because of their size and market power, both companies are better off working together than trying to compete with each other.

Network theory is similar in that it operates on the basis of strong and weak ties in the supply chain. The firms tied together by strong bonds are more reliable, whereas weaker linked firms have more flexibility. In order to maximize supply chain performance, a combination of strong relationships and weak relationships is needed (Ketchen and Hult, 2007). Wal-Mart forges these relationships by investing a lot of time meeting with vendors and establishing long-term relationships with them. They focus on researching their cost structures to ensure that they get the best deal possible (Chandran, 2003).

When studying the habits of an expanding firm, it is important to have an understanding about spatial or expansion diffusion, particularly the differences between contagious and hierarchical diffusion. Contagious diffusion explains the trends of entities that have expanded or spread from one central location to many others in a relatively small amount of time (Graff, 1998). In contrast, hierarchical diffusion explains what happens when an idea or entity begins in a densely populated area and slowly spreads to other areas starting with more urban areas and eventually ending in more rural areas (Graff, 1998). Assuming more suppliers are located near larger metropolitan areas, a firm should be implementing hierarchical diffusion strategies in order to maintain close relationships with these suppliers. This is because the capacity of the distribution centers and warehouses can become efficiently utilized over multiple outlets, effectively and continuously dividing advertising and distribution costs for the firm (Graff, 1998). When the goal is maximizing sales volume, a firm with substantial capital will implement methods of hierarchical diffusion to achieve it. However, Kmart has shown that this method is met with varying degrees of success at different times (Graff, 1998). Wal-Mart has chosen to apply a different approach to expanding

its empire by incorporating elements of contagious diffusion into their infrastructure. They expanded small, town bases into metropolitan markets and acted accordingly to supply their retail stores surrounding their distribution centers. By starting off in more rural areas, they experienced little competition, and, as a result, allowed the company to focus on cutting costs in their supply chain. In addition, this strategy of contagious diffusion allows the company to efficiently allocate resources by "utiliz[ing] existing infrastructure" (Graff, 1998). This allows some flexibility with the firm's financial resources to establish locations where necessary.

2 Wal-Mart Background -History, Market, Expansion, and Competition

Wal-Mart, currently ranked second on Fortune's top 500 companies, is the world's largest retailer with revenues reaching a staggering $447 billion in 2011 (CNN). However, this giant retail discounter started as a single store in rural Arkansas a mere fifty years ago. In 1962, Sam Walton opened the first Wal-Mart in Rogers, Arkansas. While successful early in its life, Walton maintained a cautious, planned strategy for expanding Wal-Mart. Rather than leaving the market that he knew, Walton slowly built up Wal-Mart's presence first in Arkansas, then in surrounding states, and thereafter in the rest of the rural South (eventually expanding to the entire United States and abroad after Walton passed away). After a decade in business, there were only 24 Wal-Marts established solely around Arkansas (Wal-Mart Website). However, after 35 years in business, Wal-Mart had 2,362 stores with at least one store in every state (Jia, 2007).

Walton's business strategy was unique and risky in many ways as it initially seems counter intuitive. The strategy was two part: 1) establishing Wal-Marts in rural areas, and 2) locating Wal-Marts next to each other before expanding to surrounding areas. This two-part strategy was risky. The first part was so because, by being in rural areas, Wal-Mart would be limiting the size of potential customers per store. The second part was also a gamble because Wal-Mart ran the risk of considerable store cannibalization. In other words, as Wal-Mart places stores close together, the new Wal-Mart takes away some sales from the older Wal-Mart(s) in the area. This is also known as inter-chain competition between stores (Holmes, 2011).

2.1 Strategy, Part I: Targeting the Rural Market

Rather than placing stores in urban areas to reach a large number of customers, Walton placed his stores in rural areas just off of highway exits for easy access. He would open them near towns that had the population size of approximately 5,000 people (Moore, 1993). Walton placed stores where "the customer population wasn't large enough to maintain two

rival discounters. Thus, once Wal-Mart established a store in a particular area and had beaten back weak local retailers, it was seldom threatened with future local competition from other discounters, like Kmart" (Moore, 1993). Walton made it his goal to establish monopolies in local rural markets in order to inspire customer loyalty. With strong customer loyalty, Wal-Mart stores were able to maintain adequate revenues (even compared to urban retailers) in sparsely populated areas.

In contrast, Kmart started out differently and targeted a different market. Both Wal-Mart and Kmart were established in the 1960's, but Kmart was built upon the S.S. Kresge franchise of variety stores (Moore, 1993). Kmart started in the Detroit suburbs and targeted metropolitan areas all over the country (Graff, 1998). It chose to place its stores near existing malls and towns with a population of more than 50,000 people (Moore, 1993). However, by establishing Kmarts in more metropolitan areas, Kmart faced stiffer competition than Wal-Mart and was serving a customer that usually did not have as much loyalty to a single firm. As Holmes states in his recent paper, there are many substitutes available in a large market and distance becomes less important in an urban (and even a suburban) setting than it does in the rural setting. Holmes illustrates this through a model that shows:

> "A rural consumer right next to a Wal-Mart shops there with a probability that is essentially 1. At a population density of 50,000, this falls to .72 and falls to only .24 at 250,000" (Holmes, 2011).

As we can observe from Holmes' model, the more populous the area, the less important distance to the store is. A rural customer's primary determinate of where he/she goes is distance, and variety and price come as a close second. A customer is not going to travel an extra twenty miles to a Kmart instead of a Wal-Mart unless Kmart has a much better inventory and significantly lower prices. Whereas, those in the city have the luxury of being able to choose among a variety of retailers, all of whom are relatively close.

2.2 Strategy, Part II: Clustering Stores

The second part of Walton's strategy for expansion was concentrating stores in a particular area before expanding to adjacent regions. It is this strategy that helped Wal-Mart become known for having the most efficient supply chain in the world. According to Captain Vernon Beatty, aide-de-camp to the commander at the Defense Supply Chain, "No one does that [supply chain management] better than Wal-Mart" (Chandran, 2003). As Walton built up the number of Wal-Marts, he placed them closely together and set up "Wal-Mart's trademark hub-and-spoke distribution system," whereby distribution centers serve a group of stores that are no more than 48 hours away from it (Moore, 1993). In order for this model to be efficient (worth the cost of maintaining a large warehouse), the

distribution center needs to serve many stores. Thus, having a large number of stores in an area is a requirement.

Kmart once again chose an alternate strategy in determining where to place its stores. Instead of clustering its stores to take advantage of supply chain savings, Kmart scattered its stores across the country. Because of its strategy of targeting metropolitan areas, Kmart began expanding by placing stores in cities across the nation. Kmart expanded all over the country much quicker than Wal-Mart, but its store concentration in these areas was significantly less (Graff, 1998). This strategy meant that logistics were more complicated from the start as stores were not coordinating with a local, Kmart owned distribution center. In addition, shipment times were longer as stores were not placed close together.

By using its own distribution center and trucks, Wal-Mart does not need to outsource its logistics, which helps to save money. Wal-Mart uses its own warehouses to supply 85% of its inventory compared to its competitors who only source 50-65% of their inventory by their own warehouses (Chandran, 2003). Wal-Mart is also able to run these distribution centers more efficiently since it solely serves Wal-Marts in a relatively small radius. Each individual Wal-Mart can restock quicker each distribution center is able to ship items more efficiently than their competitors. There are over 3,500 company-owned Wal-Mart trucks that drive between the suppliers, distribution centers, and retail stores. These trucks are responsible for delivering the goods to a store within 48 hours of the request, and replenishing the store shelves every two weeks. The delivery system is strictly coordinated, so that the drivers are required to report their hours daily and are dispatched to a new task as soon as his previous task is completed within his working hours. A driver is permitted to unload his truck trailer only at scheduled unloading times, which are coordinated with the particular Wal-Mart store. The unloading of goods all occur during the night, and each trailer is allocated two hours to unload (Chandran, 2003). In order to maximize the efficiency of the employees, each distribution center is equipped with facilities such as fitness centers showers, dining area, and an area to rest. This efficiency pays off as Wal-Mart is able to supply inventory in two days compared to five days for its competitors and shipping costs are 3% of sales for Wal-Mart versus 5%for its competition (Chandran, 2003). As Holmes estimates:

> "If all 5,000 Wal-Mart stores (here, supercenters are counted as two stores) were each 100 miles farther from their distribution centers, Wal-Mart's costs would increase by almost $2 billion per year" (Holmes, 2011).

While these differences seem small, considering the number of goods it carries and the amount of revenue Wal-Mart earns per year −$447 billion in 2011 − the savings are significant and enable Wal-Mart to continue to be the discount retailer with the lowest price (CNN).

2.3 Expansion and the Advent of the Supercenter

Realizing its superior ability to dominate its target market and the savings associated with the economies of density created by its hub-and-spoke distribution strategy, Wal-Mart began to aggressively expand in the 1980's. It averaged 140 store openings per year in the 1980's and surpassed Kmart as the largest discount retailer in 1991 (Jia, 2007). In an effort to maintain their position and continue to expand, Wal-Mart launched the supercenter, a concept that was being marketed by Kmart at the same time (Graff, 1998). The basic idea was to combine discount retail and groceries, so that stores like Wal-Mart supercenters and Super Kmarts would be one-stop shops (Graff, 1998). Retailers would be able to increase their revenue per customer. In introducing this new format, Wal-Mart continued the strategy that brought it to the top, targeting the rural South first and locating stores near each other to create a highly efficient distribution network. According to Graff, Wal-Mart planned to "supply 100 supercenters within a radius of 300 miles from each of its grocery distribution centers" (Graff, 1998). Many of the supercenters Wal-Mart established were not new, but rather Wal-Mart converted regular Wal-Marts to supercenters to take advantage of customer loyalty in regions they had already infiltrated. Likewise, Kmart applied its strategy for regular Kmarts to Super Kmarts. According to Graff, by 1996, Wal-Mart had over 50% of its supercenters in counties with fewer than 50,000 people. The average county population size was 105,000 (Graff, 1998). On the other hand, Super Kmarts' average county population size was over 700,000, and 81 of the 96 were located in counties classified as metropolitan by the census (note, median data not provided)(Graff, 1998).

Wal-Mart further strengthened Walton's hub-and-spoke model beginning in the 1980's through the adoption of information and communications technologies into their system. The company utilized the bar coding and radio frequency technologies to form a Point-of-Sales (POS) system. By providing hand-held computers to the employees at the stores, the employees are able to keep track of the inventory in stores, as well as the deliveries and backup merchandise in stock at the respective distribution centers (Chandran, 2003). They could track the location of a product as it was being delivered from a distribution center. In addition, Wal-Mart developed an algorithm system that forecasts the exact quantities of each item to be delivered to a store based on its inventories. Wal-Mart also built a retail link system that monitors the sales of their goods at all stores and replenishes the inventories. The details of the daily transactions were available to all Wal-Mart stores, updated on a daily basis. The great level of information customized to each store regarding its inventory allowed the stores to have authority over how it managed its inventory, whether that involved managing the price markdowns or the pack sizes of bulk goods best suited for the specific store's needs (Chandran, 2003).

Along with improving the technology within the Wal-Mart stores, Wal-Mart also developed various systems in collaboration with its suppliers to further increase the efficiency of the inventory management. For instance, Wal-Mart created an automated reordering system in collaboration with Procter and Gamble (P&G), one of its main suppliers. The system would identify a product low on stock at a Wal-Mart store, then send a message to P&G so that P&G could deliver the product to the distribution center closest to the store, or directly to the store (Chandran 2003). This system improved the coordination between the two companies and lowered the transaction costs on both sides. It benefited Wal-Mart's cross-docking distribution strategy, used along with the hub-and-spoke model, where goods were distributed directly from the supplier to the stores. There were five methods of cross-docking, catered to different situations and suppliers. Through the different methods, informal cooperation arose amongst the various stores, distribution centers, and the suppliers (Chandran, 2003).

2.4 Wal-Mart's Results

Sam Walton's simple, but innovative business strategy grew Wal-Mart into one of the top grossing companies in the world and wiped out the competition in the discount retailing sector. As of 2007, Wal-Mart was the largest private employer in the United States, employing 1.3 million Americans in its 3,400 US Wal-Mart's and Sam's Clubs. In 2005, Wal-Mart had more revenue than the next five U.S. retailers combined (Home Depot, Kroger, Sears/Kmart, Costco, and Target). According to the Pew Research Center, 120 million Americans shop at Wal-Mart every week and "84% of Americans shopped at Wal-Mart at least once during 2005" (Basker, 2007). This success was possible because Wal-Mart is able to deliver on its promise of the lowest prices and greatest variety, both of which are fueled by Wal-Mart's organized and efficient distribution center that allow it to deliver goods cheaply and quickly.

3 Modeling Wal-Mart

As with many cases, it can be difficult to apply theoretical concepts and methods of optimizing to a real world problem for a variety of reasons. In searching for a way to quantify the benefit Wal-Mart derives from its logistics system, one runs into numerous issues. One of the main issues is the availability of information. Wal-Mart is known for being highly secretive. Thus, no information is released on logistics costs and those interested in Wal-Mart are left to find estimates and use information provided in Wal-Mart's annual report (Holmes, 2011, 256). Another difficulty is the sheer number of factors that can and need to be taken into account. As seen below, the most comprehensive model published to date, Holmes (2011), considers many variables and factors, but even it leaves out some important factors that would make it more realistic. The sheer number of possible

store-opening sequences both in terms of location and type of store make conventional approaches in theory very hard to apply (Holmes, 2011).

In the following section, we will look at Holmes' model in particular. We look at the variables he takes into account and how he uses the data available to find numbers for such variables. We will discuss where Holmes makes simplifications and what Holmes' model leaves out.

3.1 Holmes' Model

Introduction

Holmes has published several articles about Wal-Mart, and in his most recent work, entitled "The Diffusion of Wal-Mart and Economies of Density," he builds on earlier works as he addresses the question, "Where should the new Wal-Marts and supercenters be put?" (Holmes, 2011). He answers this question by looking at the past and doing a historical analysis of how and when Wal-Mart added new stores and distribution centers in its fifty year history (Holmes, 2011). Particularly salient to this paper and looking at Wal-mart's decision making is analyzing how Wal-mart balances economies of density savings and sales cannibalization. Holmes explores this issue by setting up a model that looks at Wal-mart's past decisions. He runs possible scenarios to observe if Wal-mart chose a different sequence in rolling out its stores, how much this would affect Wal-mart's sales and profit. Holmes creates a model that estimates demand based on population and store characteristics and then assumes consumers choose among the Wal-Mart's in their general area (Holmes, 201). Holmes also creates a cost model based on store-level data and distances to distribution centers. After creating this model (explained below), he uses linear programming to create an equation for profit based on a given vector that represents the sequencing of store openings. By changing this vector to reflect different strategies for store opening sequencing, Holmes can compare the results of different strategies. Through using moment inequalities (a new economics concept), he limits the number of different scenarios he looks at to make this comparison. He specifically builds on the approach to moment inequalities outlined by Pakes, Porter, Ho and Ishii (Pakes, 2006). In the end, he concludes that while Wal-mart sacrificed some sales dollars in its strategy, the cost savings associated with its strategy outweighed the losses (Holmes, 2011).

Note: Holmes uses store-level sales estimates from ACNielsen, demographic data from U.S. Censuses, and company data from Wal-Mart's annual reports (Holmes, 2011).

[Note: The following is based on Holmes' article mentioned above, therefore we will not cite Holmes in every section but make a general statement that the credit for this model and its ideas are due to Thomas Holmes.]

Model Set-Up

Within a market, there is a finite set of locations, denoted as $l = 1, ..., L$. The distance (in miles) between l and l' can then be expressed as $d_{ll'}$. Let B_t^{Wal} be the subset of locations that have a Wal-Mart within a specified period of time t, and B_t^{Super} be a subset of B_t^{Wal} that have Wal-Mart supercenters. A store $j \in B_t^{Wal}$ is a regular Wal-Mart store and only sells general merchandise, expressed as g. If however $j \in B_t^{Super}$, then store j is a supercenter and sells both general merchandise g and food, expressed as f. To express merchandise segments in general, let $e \in [g, f]$. Holmes treats a supercenter as two stores, a general merchandise store and a food store. Hence, in his model, he counts supercenter twice, once within B_t^{Wal} and once within B_t^{Super}. Although the merchandise prices are not constant across all of the Wal-Mart stores, Holmes has decided to assume that there is no price variation across the stores and over time.

Dynamics

Holmes creates a model that addresses the openings of Wal-Mart stores and the distribution centers across time. He sets the period length to be one year, and sets the discount factor for a period to be $\beta = .95$. In the model, it is assumed that a Wal-Mart store B_t^{Wal} once opened never closes. Therefore, $B_t^{Wal} = B_{t-1}^{Wal} + A_t^{Wal}$ where A_t^{Wal} is the set of new stores opened over period t. A supercenter, which can open either as a new Wal-Mart store or as an expansion of an existing Wal-Mart store, is expressed as $B_t^{Super} = B_{t-1}^{Super} + A_t^{Super}$.

Holmes sets ρ_t as the growth factor of Wal-Mart's productivity over time t, meaning that if the number of stores were maintained constant and the population demographics of all areas remained the same, the company would grow by a factor of ρ_t. This growth factor addresses the growth that arose out of expansion of product lines such as drugs and eyeglasses.

The policy choice of where Wal-Mart opens its new stores is defined as vector $a = (A_1^{Wal}, A_1^{Super}, A_2^{Wal}, A_2^{Super}, ...)$, which specifies the location where a store is opened at time t. Vector a is feasible if the number of stores opened under the policy a is the same as the number of stores actually opened.

At time 0, Wal-Mart needs to choose a policy choice a that maximizes the objective function below:

$$\max_a \sum_{t=1}^{\infty} (\rho_t \beta)^{t-1} \left[\sum_{j \in B_t^{Wal}} \left[\pi_{jt}^g - c_{jt}^g - \tau d_{jt}^g \right] + \sum_{j \in B_t^{Super}} \left[\pi_{jt}^f - c_{jt}^f - \tau d_{jt}^f \right] \right],$$

where the operating profit π for merchandise segment of $e \in [g, f]$ at store j over time t is expressed as:

$$\pi^e_{jt} = \mu R^e_{jt} - Wage_{jt} Labor^e_{jt} - Rent_{jt} Land^e_{jt} - Other^e_{jt},$$

and where d^e_{jt} is the distance to the closest distribution center at time t for merchandise e.

Demand Components

In order to estimate profit, Holmes firhost starts out by detailing the components of demand. He employs a choice function where a consumer at location ℓ chooses between shopping at any Wal-Mart located within 25 miles or another "outside option." When the consumer chooses Wal-Mart, the choice set is defined as

$$\bar{B}^{Wal}_\ell = \{j, j \in B^{Wal} \text{ and Distance}_{\ell j} \leq 25\},$$

where $\text{Distance}_{\ell j}$ is the distance between the consumer's location ℓ and the location of Wal-Mart j. If the consumer does not choose to shop at Wal-Mart, the choice function defaults to 0. The utility functions for these choices incorporate variables for population density (Popden), block groups (LocationChar_ℓ), store characteristics of Wal-Mart ($\text{StoreChar}_{j\gamma}$), as well as a slack variable ε_0 that accounts for any logit error in the logistic regression.

$$u_0 = b(\text{Popden}_\ell) + \text{LocationChar}_\ell \alpha + \varepsilon_{\mathbf{0}}.$$

$$u_{\ell j} = -h(\text{Popden}_\ell)\text{Distance}_{\ell j} + \text{StoreChar}_j \gamma + \varepsilon_j.$$

These utility functions take into consideration more substitutes in larger markets, as well as the idea that newer stores acquire fewer sales than older locations.

Taking these utility functions into account, Holmes uses logistic regression in order to calculate the probability $p^g_{j\ell}$ that a consumer at location ℓ shops at store j from the standard logit formula. Logistic regression is a tool used to predict the outcome of a categorical dependent variable (Christensen, 1990). The probabilities modeled by the logistic function imply that the model for the predicted general merchandise revenue for store j is

$$R^g_j = \sum_{\{\ell | j \in \bar{B}^{Wal}_\ell\}} \lambda^g \times p^g_{j\ell} \times n_\ell,$$

where λ^g represents the spending per customer. With n_ℓ customers at location ℓ, only a fraction of them based on the probability $p^g_{j\ell}$ will be at a Wal-Mart store j.

The same logic applies when considering the demand of food, where instead we denote the spending as λ^f. In addition, it is also important to note that the choice set for food $\bar{B}^{Super}_\ell$ is not necessarily equivalent to the choice function for general merchandise $\bar{B}^{Wal}_\ell$.

Costs Accounted for in Operating Profit π

There are certain costs that vary across the stores based on the size of the sales and the number of costumers. These include the number of workers operating the store, the size of the parking lot, the size of the building. Although the sizes of a building and parking lot seem to be a fixed costs, Holmes treats them as variable costs due to the ability of Wal-Mart to frequently update its stores.

Let the variable costs at store $j$be all dependent on the sales volume R_j:

$$Labor_j = v^{Labor} R_j$$

$$Land_j = v^{Land} R_j$$

$$Other_j = v^{Other} R_j.$$

$Labor_j$ and $Land_j$ account for the varying prices of wages and land prices across the nation and time. While $Other_j$ accounts for the costs of renting structures and equipments necessary for the store.

Holmes assumes that required labor input is proportional to sales. This is a reasonable assumption. Sales are most likely greater at larger stores who have more customers, and thus these stores need more staff. From the data, Holmes calculates that Wal-Mart averages 3.61 store employees per million dollars of annual sales. Let this be the fixed labor coefficient $\nu^{labor} = 3.61$. Then to determine the cost of labor at a particular store, Holmes multiplies the fixed labor coefficient by the average retail wage in the county where the store is located. The median store in these estimations faces a labor cost of \$21,000 per work annually. Using the median store labor cost and the fixed labor coefficient, the cost of store labor is approximately 7.5% of sales.

In order to calculate land costs at each location, Holmes uses the U.S. Census to create a residential property value index for each store. Holmes defines the store's neighborhood as a two mile radius around each store. Then he aggregates the value of owner occupied property and added this to 100 times the monthly gross rents. He then divides this by the number of acres in the neighborhood. Finally, he uses the consumer price index to convert this property value to 2005 dollars. Holmes essentially creates a land value to sales ratio for each store in order to estimate the cost of land as it varies across location. Not surprisingly, the most expensive location was in California (Mountain View, CA) where land value is about 65% of annual sales. However, Holmes estimates the median annual land costs across locations is about half a percent of sales.

Some store variable costs are the same across locations. These kinds of variable costs include COGS and SG&A expenses. In looking at calculating COGS or cost of goods sold (all the costs incurred in producing goods), Holmes turns to Wal-Mart's annual reports. According to Holmes, Wal-Mart reports a gross margin usually between .22 and .26. Gross margin is sales minus cost of goods sold, and then dividing this by sales. Knowing sales and margin, one can calculate cost of goods sold easily. In this case, Holmes lets gross margin, μ, be .24. One can then calculate the COGS for any Wal-Mart store by assuming its margin is .24 and using historical store sales.

Selling, general, and administrative expenses (SG&A) is another category of expenses to look at. These include most of the other expenses incurred that are not directly associated to the goods sold. For example, the store-level labor costs that Holmes calculated earlier are part of SG&A because they are not directly associated with the manufacturing of goods. Thus, Holmes' model already counted this part of SG&A. Holmes also does not want to include other variable costs that are not associated with an individual store, but are associated with the greater Wal-Mart organization. This includes the cost of the distribution system and the cost of running the central administration. Holmes sets the residual variable cost parameter ν^{other} to .07 to reflect the costs (expressed as a fraction of total sales) that he does not want to include in his cost for each individual store. Thus, the net margin is $\mu - \nu^{other}$ and using Holmes' numbers this is $.24 - .07 = .17$.

Costs

In addition to the costs accounted for within the operating profit, Holmes addresses another component of a store's costs that can vary across locations. A store's costs can vary based on how urban or rural the store's location is. Holmes points out that in an urban setting, the typical Wal-Mart store off a highway exit is not applicable. Hence, the assumptions made about the costs incurred by a typical Wal-Mart store in a rural setting, on which Holmes' model is based, would not hold for the urban Wal-Mart stores. In order to capture the differences that arise out of different population landscape, Holmes expresses the fixed cost of store j as a function of $c(Popden_j)$, which is the function of the population density of the store j location.

$$c(Popden_j) = \omega_0 + \omega_1 \ln(Popden_j) + \omega_2 \ln(Popden_j)^2.$$

Distribution Costs

Distribution costs come out of transferring goods from the closest distribution center to store j. If we are to assume that there are two separate distribution centers, one for food and another for the general merchandise, then there are two sets of distances, d^g_{jt}

the distance from store j to the closest distribution center for general merchandise at time t, and d_{jt}^f the distance from the store to the closest food distribution center. Let τ be the parameter for cost per mile per period per merchandise segment of servicing store j. Hence, the distribution cost for a general Wa-Mart at time t would be

$$\tau d_{jt}^g,$$

whereas for a super center, it would be

$$\tau d_{jt}^g + \tau d_{jt}^f.$$

Holmes assumes that the distribution cost is fixed and not dependent on the volume of the store sales or the size of the store. This is based on the fact that in order to maintain the inventories for every store, there is a similar frequency of good deliveries across all stores.

Extending to Other Years

The above explained the demand model for 2005, but the key to Holmes' analysis is looking at the data historically. In order to get an operating profit for earlier years, Holmes assumes that the demand is the same except for the multiplicative scaling factor ρ_t.

For example, a specific store using the 2005 demand model with no rescaling predicted that the store would have had $31.5 million in sales in 1971. However, Holmes knows from actual data that the actual store sales for 1971 (in 2005 dollars) is $7.4 million. This is a large gap, but by adding the scale factor, Holmes' model adjusts demand proportionately so that the model's predicted sales, if available, for the past matches actual sales data for that year.

Model's Conclusion

From running his model, Holmes concludes that while there are possible deviations from Wal-Mart's actual policy that could have yielded a higher operating profit per store, Wal-mart's savings on distribution costs as a result of its policy outweighed this possible benefit. Looking at a conservative estimate of distribution costs alone, Holmes estimates that Wal-Mart saves $350,000 per store for having a regional distribution center closer by 100 miles. To think about this in another way, this means that if all 5,000 Wal-Mart stores (supercenters counted as two stores) were 100 miles further from distribution centers, Wal-Mart's cost would increase by $2 billion per year (Holmes, 2011). This savings is significant as $2 billion dollars is approximately 13% of Wal-Mart's net income as of January 30,2012 (Yahoo). This does not even take into account the effect of population density (and

competition) on customer loyalty and sales nor the other benefits of density like savings on advertising and personnel. Holmes' model confirms Wal-mart's risky strategy in that the savings from economies of density is worth the small amount of sales cannibalization that comes with placing stores close together.

Constraints and Assumptions

Although Holmes' model addresses a great number of issues, there is always a limit to how many points a model can address. The major limitation is that Holmes barely takes competition into account. He assumes that a Wal-Mart store's sales and revenues are not affected by other businesses. The only way in which he addresses competition is by acknowledging the differences between stores in rural and urban areas, expressed as $c(Popden)$. Here, he assumes urban locations have more competitors. Therefore, the larger the population density of the store location, the greater the costs associated with competitors. However, one can assume that there are several other factors to be considered. For instance, there is the question of how first-mover's advantage, which is the advantage of being the first business in a location, affects a Wal-Mart store's sales. Additionally, it is likely that there are differences in competing against existing local small businesses and similar retail chain stores like Target and Kmart. If individual Wal-Mart stores have the power to manage their own stocks and set prices as Chandran says, there may be significant variation between stores on how they deal with their competitors.

Not only does Holmes assume that competition is negligible, but also assumes that there are no price variations across stores over time. These assumptions may create a great difference in the model and the result should they have been taken into account. There are geographic differences in preferences and demands over goods due to different climates and lifestyles. Snow shovels and snow chains for tires are not going to have high demand at the stores in Florida, or be always in stock. In addition to geographic differences, there are also variations in demands over goods over a period of time. Changes in the economy can certainly affect both the supply and demand sides of the pricing. Consumer trends, such as the current trend to buy locally, can also affect the demand level for goods. The demand can also be affected by factors like negative press image.

Holmes assumes that the distribution costs are fixed, but he only addresses the distribution costs that are incurred out of transferring goods between the distribution center and the stores. He does not address the distribution cost that comes out of the deliveries from the suppliers to the distribution centers. Furthermore, by assuming that distribution costs are fixed, he does not take into account the cross-docking strategy, which is Wal-Mart's strategy to have goods delivered directly from the suppliers to the individual stores. In regards to the Wal-Mart supercenters which sell food goods, there must have been a point

at which there was a transition of grocery distribution center from being outsourced to inhouse. However, Holmes' model assumes from the beginning that the grocery distribution has always been inhouse.

In addition to the change over time in the grocery distribution methods and its effect on the distribution costs, there are other factors which have changed or can change dramatically over a period of time, and therefore should be treated differently between different time periods. First, it is possible to divide the time up into the pre-chain store period and the post-chain store period. The market economy was very different between these two periods, and these differences should be taken into account. Additionally, there is a question of how a population growth or a decrease would benefit different types of chain stores differently. Therefore, the population change could affect Wal-Mart in one manner, and could affect Wal-Mart's competitors in another manner. Holmes assumes there are no major changes over time, despite the fact that there has been significant changes over the period that have affected how and in what ways the Wal-Mart stores have been successful within their area.

Within the model, Holmes places productivity growth within ρ_t, and comments that the growth has been due to the expansion of product lines, such as drugs and glasses. It is questionable as to whether Holmes has oversimplified the growth. Not all stores have the same set of product lines since, for instance, only certain stores have gas stations. It is not clear whether Holmes addresses the effects of partner stores upon the Wal-Mart sales revenues. Within some of the Wal-Mart stores, there are partner stores like Dunkin' Donuts, McDonalds, and local hair salons that are set up, certainly affecting the Wal-Mart stores' sales revenues to a certain extent.

3.2 Other Literature

Holmes analyzes one perspective of how Wal-Mart rose to power. Other literature we have examined have taken different approaches and consider different variables when analyzing Wal-Mart's success. The most relevant work in the field is a paper by Panle Jia (2007), which develops a model that investigates the impact of decisions of discount retailers. It also quantifies the size of the scale economies within a chain and its effect on profitability. Jia takes into consideration competition, allowing flexible patterns across all retailers, as well as scale economies of multiple markets (Jia, 2007). She begins developing her model by analyzing a market structure in which only small firms compete against each other. Afterwards, she introduces Wal-Mart and Kmart, assuming they make decisions that would maximize their profitability in all markets. Finally, she studies the habits of other firms in relation to these large discount chains and the decisions they make to stay or enter a market where they must compete with big name companies like Wal-Mart and Kmart (Jia, 2007). The introduction of Wal-Mart and Kmart into these markets further

emphasizes the conclusion that when these large firms come to town, smaller firms become unprofitable and, as a result, are driven out of the market (Jia, 2007). Jia extends this paper to apply to different firms whose economies of scale are significant to their expansion. In addition, examining the benefits of these chain strategies helps companies strategically plan for the future.

4 Conclusion

By increasing the number of store locations in a given area, Wal-Mart is able to achieve economies of density. In order to achieve these economies of density, Wal-Mart is willing to accept store cannibalization rates between 1-2% of total sales (Holmes 2011). This sacrifice is worth the reduction of costs on the supply front because it also permits the development of a brand name commitment, and this becomes quite clear when comparing Wal-Mart to Kmart. Sam Walton's unique business strategy meant that he sought to minimize costs, especially transportation costs, in order to give his customers the lowest prices. In contrast, Kmart sought to maximize each individual store profit (in Holmes' model, this was π), but this meant Kmarts were placed far away so transportation costs were high andthe ability to quickly restock hindered. Because of Wal-Mart's success, other businesses have also began adopting these distribution strategies in order to cut costs and build a strong network of stores. One important example of another large firm that utilizes this strategy is Starbucks. By increasing the number of locations on one street alone, consumers are unable to ignore the impact it has on the market.

While this model incorporates many constraints, there are still many variables that Holmes does not take into account. For example, competition plays a huge factor in the market and his model does not capture it. Furthermore, additional research can be conducted to optimize other aspects of Wal-Mart's business structure in a way that incorporates other benefits the company might be interested in. These can range anywhere from advantages in shared advertising to training costs. In order to improve on Holmes' model, it is important to better quantify and incorporate some of the assumptions that Holmes fails to take into consideration. Unfortunately, the difficulty of capturing multiple constraints in a real world application stems from the boundless possibilities of extending and improving these models.

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