Bridge Hands : Getting all cards

Math 331 : Fall 2024 : sjm1@williams.edu

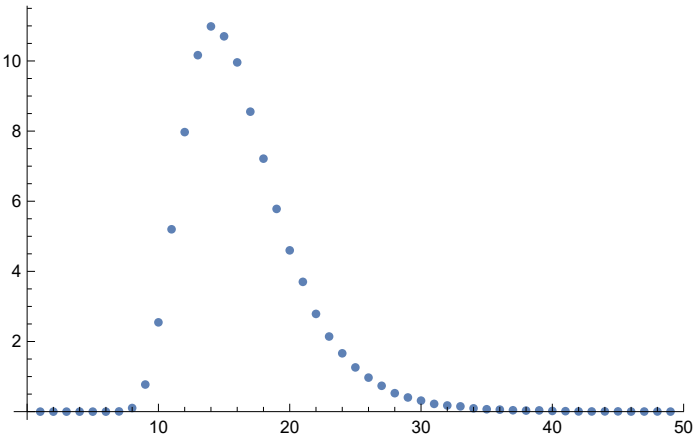
In bridge the 52 cards are dealt 13 to a player. How many deals are needed before a player expects to see each card at least once?

(Question from Kayla Miller, a few hours after a hand where she had 6 trump in opposition to Cameron Miller's 6!)

In[ ]:=
```
dealstillseeall[numcards_, numdo_] := Module[{},
  For[d = 1,
   d ≤ 80 * numcards * Log[8 * numcards] , d++, numdeals[d] = 0];
  (* numdeals records how many deals of length
   d were there before a given person saw everything.
    estimating that do not need to go
   further than numcards * Log[4*numcards] * 4 *)
  deck = {};
  For[d = 1, d ≤ 4 * numcards, d++, deck = AppendTo[deck, d]];
  (* creates deck *)
  For[n = 1, n ≤ numdo, n++,
   {
    For[d = 1, d ≤ 4 * numcards, d++, seen[d] = 0];
    (* initialize to have nothing *)
    currnumdeals = 0; (* start with no deals *)
    haveall = 0; (* check - do not have all *)
    While[haveall == 0,
     {
      hand = RandomSample[deck, numcards]; (* creates random hand *)
      currnumdeals = currnumdeals + 1; (* dealt another hand *)
      For[c = 1, c ≤ numcards, c++, seen[hand[[c]]] = 1];
      If[
       Sum[seen[i], {i, 1, 4 * numcards}] == 4 * numcards, haveall = 1];
      If[currnumdeals == 1000 * 4 * numcards,
       {
        haveall = 1; Print["Exiting - waiting too long."]
       }]; (* end of exit if statement *)
     }]; (* end of while loop on haveall = 0 *)
    numdeals[currnumdeals] = numdeals[currnumdeals] + 1;
   }]; (* end of n loop *)
```

```
  max = 1;
  For[d = 1, d ≤ 8 * numcards * Log[8 * numcards] , d++,
   If[numdeals[d] > 0, max = d]]; (* finds max observed *)
  For[d = 1, d ≤ max, d++, numdeals[d] = numdeals[d] * 100.0 / numdo];
  (* finds percentage *)
  numdealslist = {}; (* list to store values *)
  For[d = 1, d ≤ max, d++, numdealslist =
    AppendTo[numdealslist, {d, numdeals[d]}]]; (* creates list *)
  Print[ListPlot[numdealslist]];
  Print[numdealslist];
  mean = Sum[ numdeals[d] * d / 100.0, {d, 1, max}];
  stdev = Sqrt[Sum[ numdeals[d] * (d - mean)^2 / 100.0, {d, 1, max}]];
  Print["Mean = ", mean, " and StDev = ", stdev, "."] ;
 ]
```

In[•]:= `Timing[dealstillseeall[13, 100 000]]`

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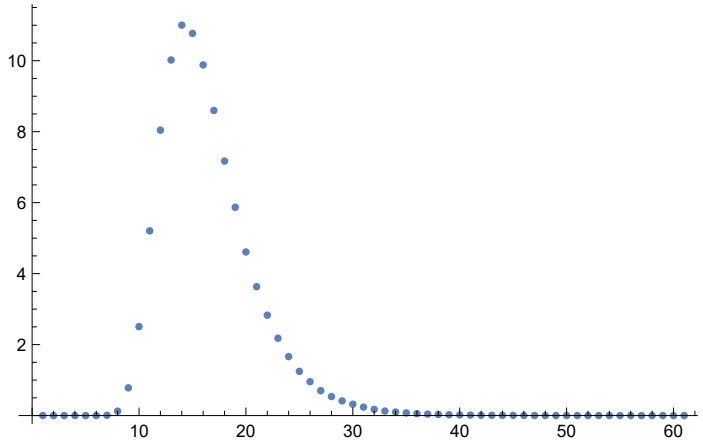
 {16, 9.959}, {17, 8.554}, {18, 7.214}, {19, 5.781}, {20, 4.599}, {21, 3.701}, {22, 2.787},
 {23, 2.142}, {24, 1.663}, {25, 1.261}, {26, 0.97}, {27, 0.737}, {28, 0.526}, {29, 0.404},
 {30, 0.314}, {31, 0.221}, {32, 0.176}, {33, 0.15}, {34, 0.094}, {35, 0.068}, {36, 0.057},
 {37, 0.039}, {38, 0.03}, {39, 0.035}, {40, 0.02}, {41, 0.013}, {42, 0.006}, {43, 0.005},
 {44, 0.006}, {45, 0.007}, {46, 0.002}, {47, 0.003}, {48, 0.004}, {49, 0.001}}

Mean = 16.4162 and StDev = 4.3567.
```

Out[•]=

```
{77.7969, Null}
```

In[•]:= `Timing[dealstillseeall[13, 1 000 000]]`

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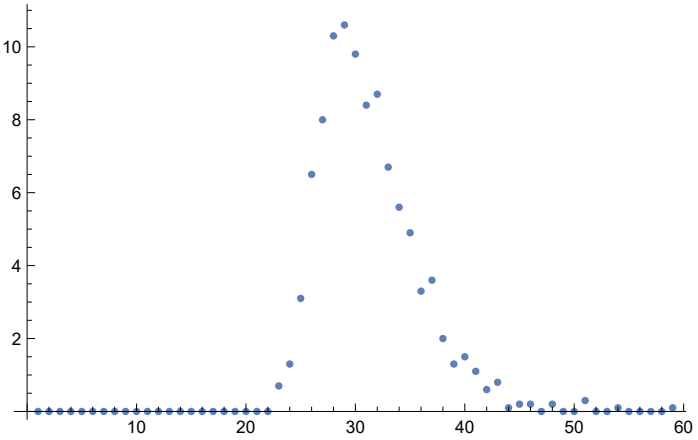
 {16, 9.8814}, {17, 8.5987}, {18, 7.1714}, {19, 5.8688}, {20, 4.61}, {21, 3.6326}, {22, 2.8283},
 {23, 2.1793}, {24, 1.6625}, {25, 1.2464}, {26, 0.954}, {27, 0.7016}, {28, 0.5362}, {29, 0.4136},
 {30, 0.3196}, {31, 0.238}, {32, 0.1743}, {33, 0.1269}, {34, 0.0997}, {35, 0.0748}, {36, 0.0543},
 {37, 0.0414}, {38, 0.0314}, {39, 0.0235}, {40, 0.0185}, {41, 0.013}, {42, 0.0094},
 {43, 0.0074}, {44, 0.0054}, {45, 0.0051}, {46, 0.0031}, {47, 0.002}, {48, 0.0013},
 {49, 0.0009}, {50, 0.0009}, {51, 0.0011}, {52, 0.0003}, {53, 0.0009}, {54, 0.0003},
 {55, 0.0002}, {56, 0.0002}, {57, 0.}, {58, 0.0004}, {59, 0.}, {60, 0.}, {61, 0.0001}}
Mean = 16.4165 and StDev = 4.35662.
```

Out[•]=

```
{646.172, Null}
```

In[•]:=

```
Timing[dealstillseeall[1000, 1000]]
```

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 {20, 0.}, {21, 0.}, {22, 0.}, {23, 0.7}, {24, 1.3}, {25, 3.1}, {26, 6.5}, {27, 8.},
 {28, 10.3}, {29, 10.6}, {30, 9.8}, {31, 8.4}, {32, 8.7}, {33, 6.7}, {34, 5.6}, {35, 4.9},
 {36, 3.3}, {37, 3.6}, {38, 2.}, {39, 1.3}, {40, 1.5}, {41, 1.1}, {42, 0.6}, {43, 0.8},
 {44, 0.1}, {45, 0.2}, {46, 0.2}, {47, 0.}, {48, 0.2}, {49, 0.}, {50, 0.}, {51, 0.3},
 {52, 0.}, {53, 0.}, {54, 0.1}, {55, 0.}, {56, 0.}, {57, 0.}, {58, 0.}, {59, 0.1}}

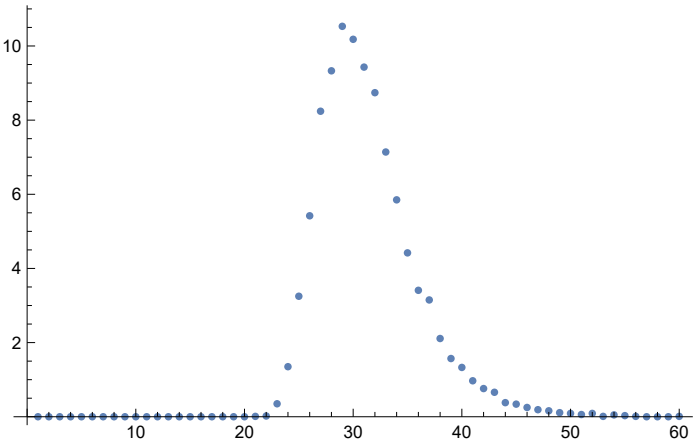
Mean = 31.236 and StDev = 4.51335.
```

Out[•]=

```
{126.234, Null}
```

In[•]:=

```
Timing[dealstillseeall[1000, 10000]]
```

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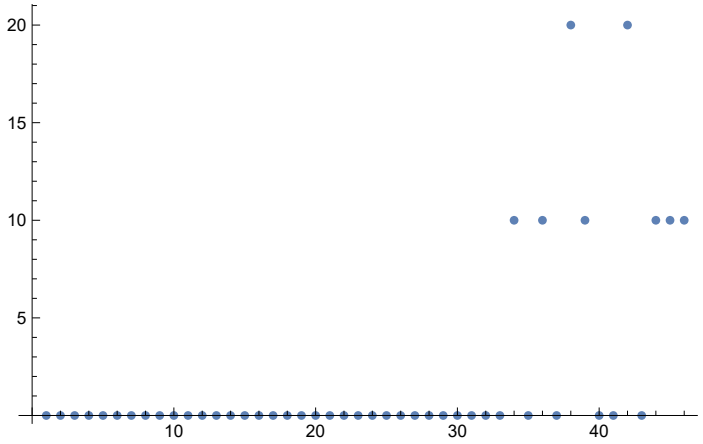
 {21, 0.01}, {22, 0.02}, {23, 0.35}, {24, 1.35}, {25, 3.25}, {26, 5.42}, {27, 8.24}, {28, 9.33},
 {29, 10.53}, {30, 10.18}, {31, 9.43}, {32, 8.74}, {33, 7.14}, {34, 5.85}, {35, 4.42}, {36, 3.41},
 {37, 3.15}, {38, 2.11}, {39, 1.57}, {40, 1.33}, {41, 0.97}, {42, 0.76}, {43, 0.66}, {44, 0.38},
 {45, 0.34}, {46, 0.25}, {47, 0.19}, {48, 0.16}, {49, 0.11}, {50, 0.09}, {51, 0.06}, {52, 0.09},
 {53, 0.01}, {54, 0.05}, {55, 0.03}, {56, 0.01}, {57, 0.}, {58, 0.}, {59, 0.}, {60, 0.01}}

Mean = 31.3678 and StDev = 4.50579.
```

Out[•]=

```
{1076.53, Null}
```

In[•]:= **Timing[dealstillseeall[10000, 10]]**



{{1, 0.}, {2, 0.}, {3, 0.}, {4, 0.}, {5, 0.}, {6, 0.}, {7, 0.}, {8, 0.}, {9, 0.}, {10, 0.},
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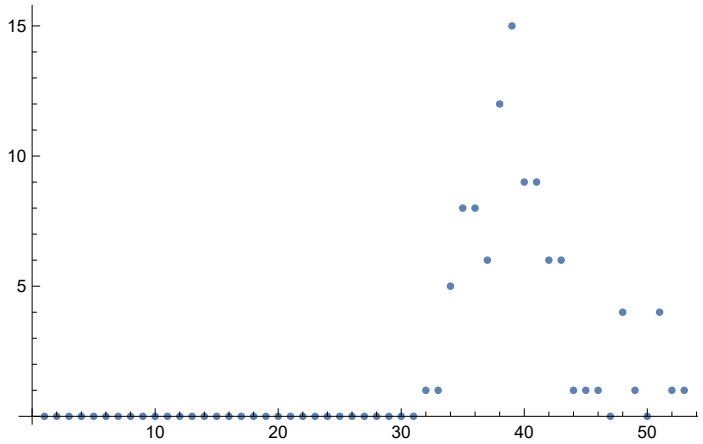
{20, 0.}, {21, 0.}, {22, 0.}, {23, 0.}, {24, 0.}, {25, 0.}, {26, 0.}, {27, 0.}, {28, 0.},
{29, 0.}, {30, 0.}, {31, 0.}, {32, 0.}, {33, 0.}, {34, 10.}, {35, 0.}, {36, 10.}, {37, 0.},
{38, 20.}, {39, 10.}, {40, 0.}, {41, 0.}, {42, 20.}, {43, 0.}, {44, 10.}, {45, 10.}, {46, 10.}}

Mean = 40.4 and StDev = 3.8.

Out[•]=

{51.2969, Null}

In[•]:= **Timing[dealstillseeall[10000, 100]]**



{{1, 0.}, {2, 0.}, {3, 0.}, {4, 0.}, {5, 0.}, {6, 0.}, {7, 0.}, {8, 0.}, {9, 0.},
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{19, 0.}, {20, 0.}, {21, 0.}, {22, 0.}, {23, 0.}, {24, 0.}, {25, 0.}, {26, 0.}, {27, 0.},
{28, 0.}, {29, 0.}, {30, 0.}, {31, 0.}, {32, 1.}, {33, 1.}, {34, 5.}, {35, 8.}, {36, 8.},
{37, 6.}, {38, 12.}, {39, 15.}, {40, 9.}, {41, 9.}, {42, 6.}, {43, 6.}, {44, 1.},
{45, 1.}, {46, 1.}, {47, 0.}, {48, 4.}, {49, 1.}, {50, 0.}, {51, 4.}, {52, 1.}, {53, 1.}}

Mean = 39.9 and StDev = 4.5111.

Out[•]=

{219.438, Null}

In[•]:=
```
n
```

Out[•]=

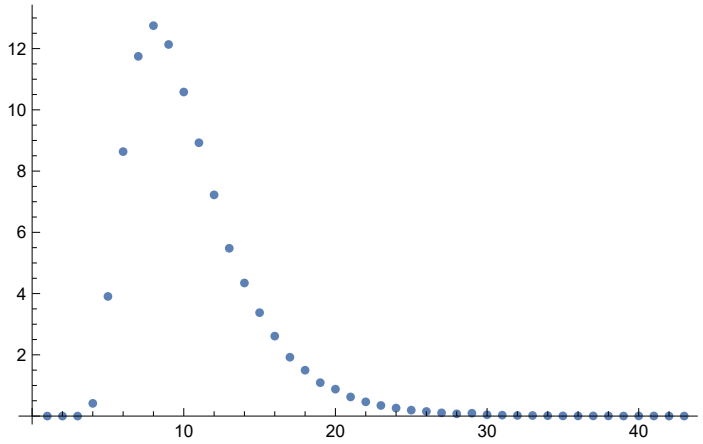
2

In[•]:=
```
52.0 HarmonicNumber[52]
```

Out[•]=

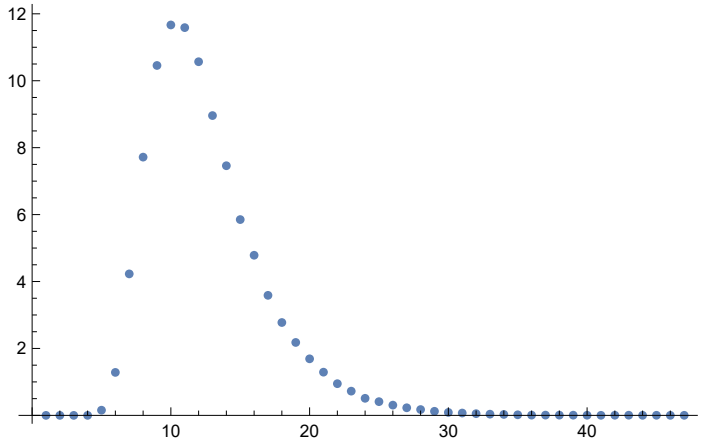
235.978

In[•]:=
```
For[r = 1, r ≤ 10, r++, Timing[dealstillseeall[2^r, 100000]]]
```




```
{{1, 0.}, {2, 0.}, {3, 0.}, {4, 0.415}, {5, 3.907}, {6, 8.636}, {7, 11.747}, {8, 12.749},
 {9, 12.134}, {10, 10.582}, {11, 8.925}, {12, 7.223}, {13, 5.479}, {14, 4.346}, {15, 3.377},
 {16, 2.611}, {17, 1.922}, {18, 1.498}, {19, 1.091}, {20, 0.879}, {21, 0.625}, {22, 0.465},
 {23, 0.346}, {24, 0.262}, {25, 0.194}, {26, 0.151}, {27, 0.105}, {28, 0.074}, {29, 0.093},
 {30, 0.043}, {31, 0.031}, {32, 0.02}, {33, 0.019}, {34, 0.012}, {35, 0.006}, {36, 0.005},
 {37, 0.006}, {38, 0.006}, {39, 0.002}, {40, 0.005}, {41, 0.004}, {42, 0.001}, {43, 0.001}}
```

```
Mean = 10.3883 and StDev = 4.01798.
```



```
{{1, 0.}, {2, 0.}, {3, 0.}, {4, 0.}, {5, 0.155}, {6, 1.284}, {7, 4.228}, {8, 7.717},
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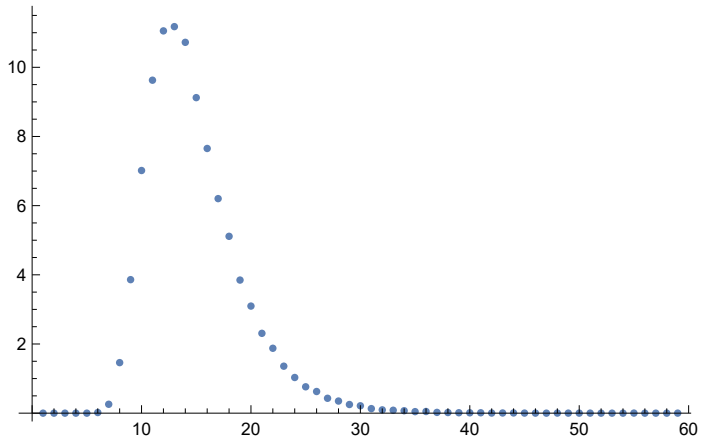
 {15, 5.851}, {16, 4.785}, {17, 3.587}, {18, 2.775}, {19, 2.179}, {20, 1.69}, {21, 1.291},
 {22, 0.947}, {23, 0.724}, {24, 0.51}, {25, 0.41}, {26, 0.306}, {27, 0.227}, {28, 0.174},
 {29, 0.122}, {30, 0.088}, {31, 0.07}, {32, 0.051}, {33, 0.036}, {34, 0.028}, {35, 0.017},
 {36, 0.007}, {37, 0.008}, {38, 0.007}, {39, 0.006}, {40, 0.004}, {41, 0.002},
 {42, 0.004}, {43, 0.005}, {44, 0.002}, {45, 0.003}, {46, 0.001}, {47, 0.002}}
```

```
Mean = 12.5701 and StDev = 4.18176.
```




```
{{1, 0.}, {2, 0.}, {3, 0.}, {4, 0.}, {5, 0.}, {6, 0.018}, {7, 0.257}, {8, 1.461}, {9, 3.86},
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 {17, 6.203}, {18, 5.113}, {19, 3.848}, {20, 3.096}, {21, 2.308}, {22, 1.876}, {23, 1.358},
 {24, 1.031}, {25, 0.762}, {26, 0.626}, {27, 0.43}, {28, 0.35}, {29, 0.25}, {30, 0.213},
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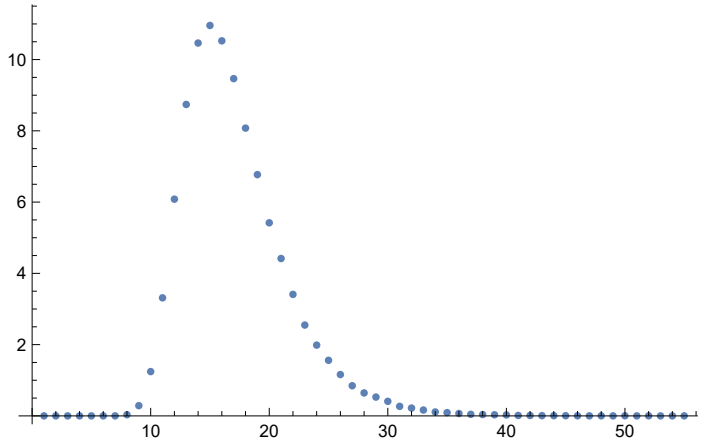
 {52, 0.}, {53, 0.}, {54, 0.}, {55, 0.002}, {56, 0.}, {57, 0.}, {58, 0.}, {59, 0.001}}
```

```
Mean = 14.7982 and StDev = 4.31215.
```



```
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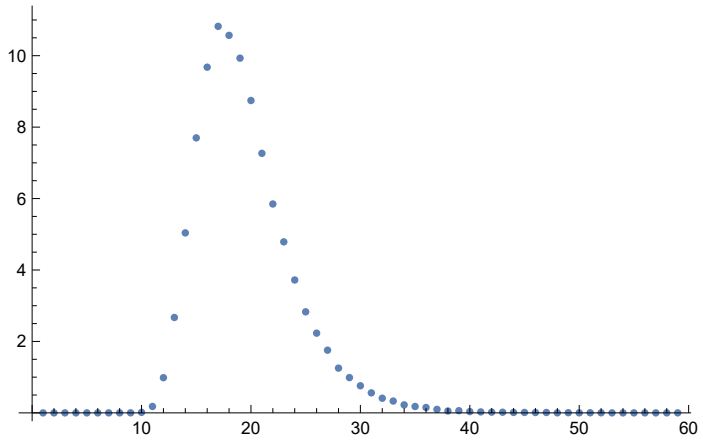
 {16, 10.526}, {17, 9.465}, {18, 8.076}, {19, 6.771}, {20, 5.419}, {21, 4.417},
 {22, 3.41}, {23, 2.549}, {24, 1.986}, {25, 1.56}, {26, 1.16}, {27, 0.846}, {28, 0.644},
 {29, 0.528}, {30, 0.409}, {31, 0.268}, {32, 0.22}, {33, 0.165}, {34, 0.11}, {35, 0.091},
 {36, 0.067}, {37, 0.044}, {38, 0.038}, {39, 0.031}, {40, 0.027}, {41, 0.013},
 {42, 0.012}, {43, 0.006}, {44, 0.006}, {45, 0.005}, {46, 0.003}, {47, 0.001}, {48, 0.002},
 {49, 0.}, {50, 0.003}, {51, 0.}, {52, 0.002}, {53, 0.}, {54, 0.}, {55, 0.001}}
```

```
Mean = 17.1044 and StDev = 4.36368.
```

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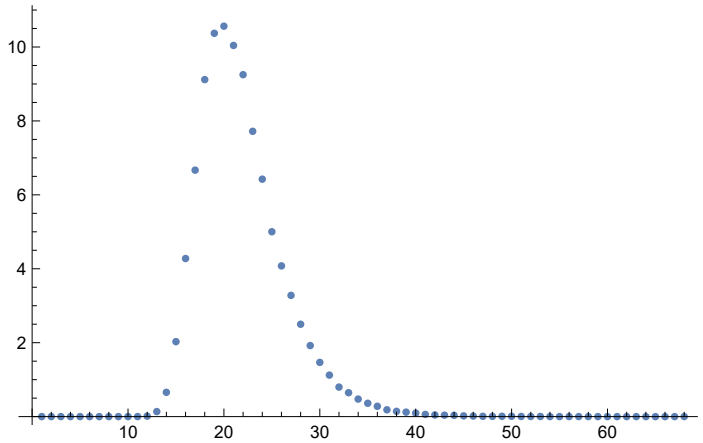
 {24, 3.72}, {25, 2.829}, {26, 2.232}, {27, 1.755}, {28, 1.253}, {29, 0.988}, {30, 0.759},
 {31, 0.56}, {32, 0.41}, {33, 0.331}, {34, 0.224}, {35, 0.179}, {36, 0.15}, {37, 0.098},
 {38, 0.053}, {39, 0.061}, {40, 0.036}, {41, 0.027}, {42, 0.02}, {43, 0.018}, {44, 0.012},
 {45, 0.012}, {46, 0.01}, {47, 0.011}, {48, 0.002}, {49, 0.005}, {50, 0.001}, {51, 0.003},
 {52, 0.002}, {53, 0.}, {54, 0.}, {55, 0.}, {56, 0.001}, {57, 0.}, {58, 0.001}, {59, 0.001}}
```

```
Mean = 19.4637 and StDev = 4.39266.
```



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 {25, 5.002}, {26, 4.078}, {27, 3.28}, {28, 2.499}, {29, 1.922}, {30, 1.467}, {31, 1.123},
 {32, 0.799}, {33, 0.647}, {34, 0.474}, {35, 0.361}, {36, 0.279}, {37, 0.184}, {38, 0.143},
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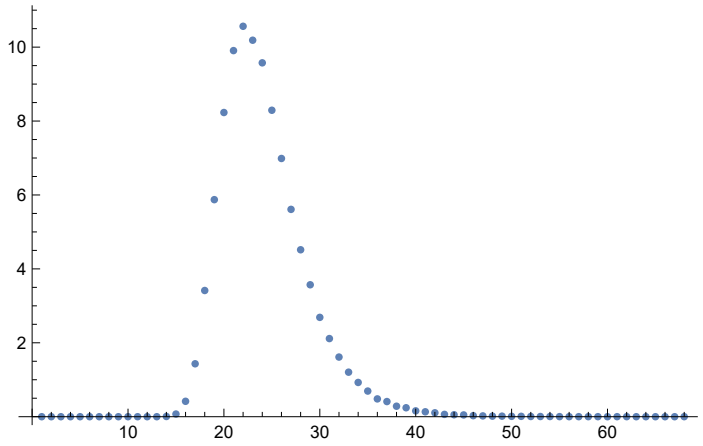
 {46, 0.015}, {47, 0.008}, {48, 0.006}, {49, 0.009}, {50, 0.009}, {51, 0.004}, {52, 0.003},
 {53, 0.003}, {54, 0.001}, {55, 0.}, {56, 0.001}, {57, 0.001}, {58, 0.}, {59, 0.}, {60, 0.},
 {61, 0.}, {62, 0.}, {63, 0.}, {64, 0.}, {65, 0.}, {66, 0.}, {67, 0.}, {68, 0.001}}
```

```
Mean = 21.8392 and StDev = 4.44233.
```

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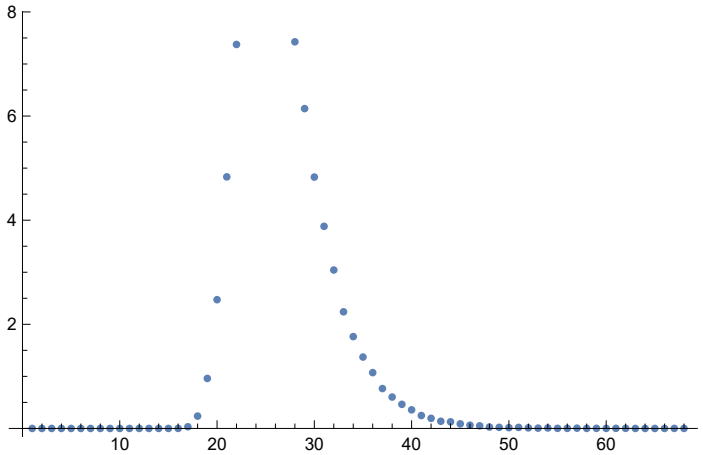
 {40, 0.158}, {41, 0.133}, {42, 0.106}, {43, 0.063}, {44, 0.053}, {45, 0.048}, {46, 0.028},
 {47, 0.022}, {48, 0.015}, {49, 0.015}, {50, 0.01}, {51, 0.01}, {52, 0.006}, {53, 0.005},
 {54, 0.005}, {55, 0.002}, {56, 0.001}, {57, 0.001}, {58, 0.001}, {59, 0.001}, {60, 0.},
 {61, 0.001}, {62, 0.001}, {63, 0.}, {64, 0.}, {65, 0.}, {66, 0.}, {67, 0.}, {68, 0.001}}
```

```
Mean = 24.2057 and StDev = 4.44666.
```



```
{{1, 0.}, {2, 0.}, {3, 0.}, {4, 0.}, {5, 0.}, {6, 0.}, {7, 0.}, {8, 0.}, {9, 0.}, {10, 0.},
 {11, 0.}, {12, 0.}, {13, 0.}, {14, 0.}, {15, 0.}, {16, 0.001}, {17, 0.032}, {18, 0.237},
 {19, 0.96}, {20, 2.473}, {21, 4.832}, {22, 7.375}, {23, 9.238}, {24, 10.336}, {25, 10.684},
 {26, 10.049}, {27, 8.79}, {28, 7.426}, {29, 6.143}, {30, 4.828}, {31, 3.882}, {32, 3.043},
 {33, 2.24}, {34, 1.764}, {35, 1.371}, {36, 1.072}, {37, 0.766}, {38, 0.603}, {39, 0.463},
 {40, 0.357}, {41, 0.248}, {42, 0.194}, {43, 0.138}, {44, 0.127}, {45, 0.088}, {46, 0.062},
 {47, 0.05}, {48, 0.027}, {49, 0.021}, {50, 0.015}, {51, 0.02}, {52, 0.011}, {53, 0.006},
 {54, 0.009}, {55, 0.002}, {56, 0.002}, {57, 0.005}, {58, 0.001}, {59, 0.002}, {60, 0.001},
 {61, 0.002}, {62, 0.002}, {63, 0.}, {64, 0.}, {65, 0.}, {66, 0.}, {67, 0.001}, {68, 0.001}}

Mean = 26.6078 and StDev = 4.45831.
```

Out[•]=

```
$Aborted
```

THEORY

Below is an approach to solve the problem through recurrences / iteratively finding. Some notation first:

s = number of suits

c = number of cards

p = number of players (often p = s)

h = number of cards in a hand (often h = c)

d = number of cards in the deck: d = h p = s c

The number of ways to choose a hand (order doesn't matter) is (d choose h)

Let x_m be the wait when missing m cards (so have d-m cards seen)

The number of ways to choose a hand and get k of the m missing cards (so have d-m already) is

(m choose k) (d - m choose h - k)

Note that (m choose -i) = 0 for any positive i, as is (m choose m+i)

In[•]:=
```
recurrencetofindwaittime[suits_, cards_, players_, hands_] := Module[{},
  s = suits; c = cards; p = players; h = hands; d = s * c;
  (* x[m] is average weight time when missing m;
  we find recursively, start knowing x[0] = 0 and x[1] = 1/p,
  both from the formula and there are p hands! *)
  (* it is convenient to set y[i] =
   x[i] for i < m if trying to find x[m] and y[m] = 0 *)
  (* want y[m] = 0 from bringing things over to solve for x[m] *)
  For[i = 0, i ≤ d + 1, i++,
   {
    x[i] = 0; y[i] = 0; (* initialize quantities to zero *)
   }];
  x[1] = p; y[1] = x[1];
  numhands = Binomial[d, h]; (* number of ways to choose h cards from deck of d *)
  For[m = 2, m ≤ d, m++,
   {
    x[m] = Sum[ Binomial[m, k] Binomial[d - m, h - k] (y[m - k] + 1) / numhands,
       {k, 0, Min[h, m]}] / (1 - (Binomial[d - m, h] / numhands));
    (* this is the recursive formula, do only for k at most min(h,m) *)
    y[m] = x[m]; (* update y[m] from 0 to x[m] *)
    Print["Wait time x[", m, "] = ", 1.0 x[m], "."];
   }]; (* end of n loop *)
  Print["x[", m - 1, "] = ", x[m - 1]];
 ]
```

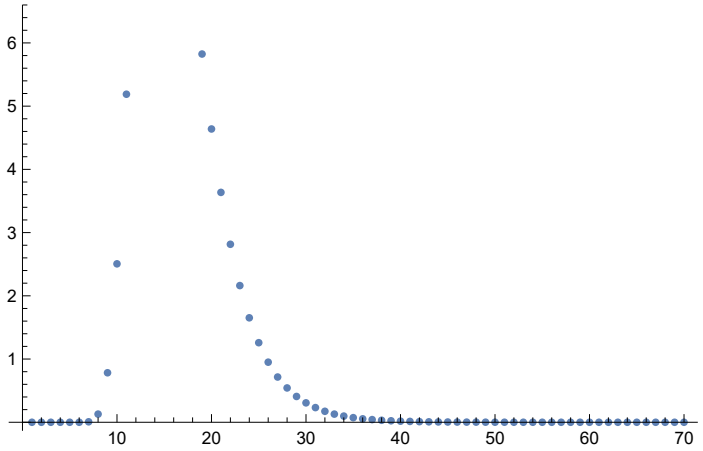
In[•]:=
```
recurrencetofindwaittime[4, 13, 4, 13]
```

```
Wait time x[2] = 5.73333.
Wait time x[3] = 6.90512.
Wait time x[4] = 7.78406.
Wait time x[5] = 8.48704.
Wait time x[6] = 9.07285.
Wait time x[7] = 9.57498.
Wait time x[8] = 10.0144.
Wait time x[9] = 10.4049.
Wait time x[10] = 10.7564.
Wait time x[11] = 11.0759.
Wait time x[12] = 11.3688.
Wait time x[13] = 11.6392.
Wait time x[14] = 11.8903.
Wait time x[15] = 12.1246.
Wait time x[16] = 12.3443.
Wait time x[17] = 12.5511.
```

```
Wait time x[18] = 12.7463.
Wait time x[19] = 12.9313.
Wait time x[20] = 13.1071.
Wait time x[21] = 13.2745.
Wait time x[22] = 13.4342.
Wait time x[23] = 13.587.
Wait time x[24] = 13.7335.
Wait time x[25] = 13.8741.
Wait time x[26] = 14.0093.
Wait time x[27] = 14.1395.
Wait time x[28] = 14.265.
Wait time x[29] = 14.3862.
Wait time x[30] = 14.5034.
Wait time x[31] = 14.6168.
Wait time x[32] = 14.7266.
Wait time x[33] = 14.8331.
Wait time x[34] = 14.9365.
Wait time x[35] = 15.0369.
Wait time x[36] = 15.1346.
Wait time x[37] = 15.2295.
Wait time x[38] = 15.322.
Wait time x[39] = 15.4122.
Wait time x[40] = 15.5.
Wait time x[41] = 15.5858.
Wait time x[42] = 15.6695.
Wait time x[43] = 15.7512.
Wait time x[44] = 15.8311.
Wait time x[45] = 15.9092.
Wait time x[46] = 15.9856.
Wait time x[47] = 16.0604.
Wait time x[48] = 16.1336.
Wait time x[49] = 16.2054.
Wait time x[50] = 16.2757.
Wait time x[51] = 16.3446.
Wait time x[52] = 16.4122.
```

```
x[52] =
 (545 928 847 452 258 491 492 389 829 320 450 626 160 841 197 170 151 499 543 576 764 239 417 023 255 451 861 085 -
    626 905 883 156 684 405 195 107 023 780 479 317 372 271 274 509 508 567 967 387 626 740 665 018 015 009 371 549 -
    436 149 407 597 300 658 313 676 235 368 117 130 572 713 596 038 893 838 341 598 601 195 428 485 082 034 721 825 -
    135 468 987 /
   33 263 651 815 411 301 455 093 132 853 409 898 491 173 712 201 961 171 863 009 501 862 172 850 150 290 107 365 -
    581 703 955 244 599 875 291 300 187 598 652 081 762 922 197 224 670 489 260 819 393 711 347 009 189 515 918 -
    586 716 637 050 967 309 053 192 156 474 313 569 986 447 499 658 535 026 680 206 545 307 025 594 664 981 127 -
    461 443 909 937 520)
```

In[•]:=
```
Timing[dealstillseeall[13, 10 000 000]]
```




```
{{1, 0.}, {2, 0.}, {3, 0.}, {4, 0.}, {5, 0.}, {6, 0.00005}, {7, 0.00745}, {8, 0.12907}, {9, 0.78296},
 {10, 2.50582}, {11, 5.18848}, {12, 8.01843}, {13, 10.0628}, {14, 10.9782}, {15, 10.8006},
 {16, 9.90676}, {17, 8.59254}, {18, 7.19095}, {19, 5.82429}, {20, 4.63875}, {21, 3.63591},
 {22, 2.81515}, {23, 2.16187}, {24, 1.65213}, {25, 1.25848}, {26, 0.95041}, {27, 0.71555},
 {28, 0.54286}, {29, 0.40812}, {30, 0.307}, {31, 0.23202}, {32, 0.17344}, {33, 0.12906},
 {34, 0.09771}, {35, 0.07305}, {36, 0.05439}, {37, 0.042}, {38, 0.0314}, {39, 0.02348},
 {40, 0.01734}, {41, 0.01351}, {42, 0.00964}, {43, 0.00705}, {44, 0.00524}, {45, 0.00406},
 {46, 0.00319}, {47, 0.00228}, {48, 0.00205}, {49, 0.00124}, {50, 0.00076}, {51, 0.00071},
 {52, 0.00045}, {53, 0.00031}, {54, 0.00024}, {55, 0.00023}, {56, 0.00011}, {57, 0.00012},
 {58, 0.00004}, {59, 0.00005}, {60, 0.00004}, {61, 0.00007}, {62, 0.00003}, {63, 0.00002},
 {64, 0.00001}, {65, 0.00001}, {66, 0.}, {67, 0.}, {68, 0.}, {69, 0.00001}, {70, 0.00001}}

Mean = 16.4131 and StDev = 4.35065.
```

Out[•]=
```
{7874.61, Null}
```