Math 341: Probability
Fifth Lecture (9/24/09)

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Random Vectors

Joint Distribution

Joint Distribution of a Random Vector

The joint distribution function of a random vector $\vec{\mathbf{X}} = (X_1, \ldots, X_n)$ on a probability space $(\Omega, \mathcal{F}, \mathbb{P})$ is the function $F_{\vec{\mathbf{X}}} : \mathbb{R}^n \to [0, 1]$ given by $F_{\vec{\mathbf{X}}}(\vec{x}) = \mathbb{P}(\vec{\mathbf{X}} \leq \vec{x})$ for $\vec{x} \in \mathbb{R}^n$, where $\vec{x} \leq \vec{y}$ means each $x_i \leq y_i$, and $\{\vec{\mathbf{X}} \leq \vec{x}\} = \{\omega \in \Omega : \vec{\mathbf{X}}(\omega) \leq \vec{x}\}$.

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Satisfies the expected properties:

- $\lim_{x_1,\ldots,x_n \to -\infty} F_{\vec{\mathbf{X}}}(\vec{x}) = 0$, $\lim_{x_1,\ldots,x_n \to \infty} F_{\vec{\mathbf{X}}}(\vec{x}) = 1$.
- If $\vec{x} \leq \vec{x}'$ then $F_{\vec{\mathbf{X}}}(\vec{x}) \leq F_{\vec{\mathbf{X}}}(\vec{x}')$.
- $F_{\vec{\mathbf{X}}}$ continuous from above.

Jointly Discrete

Jointly Discrete

$X_1, \ldots, X_n$ random vectors on $(\Omega, \mathcal{F}, \mathbb{P})$ are jointly discrete if $\vec{\mathbf{X}} = (X_1, \ldots, X_n)$ takes values in a countable subset of $\mathbb{R}^n$ and has joint probability mass function $f : \mathbb{R}^n \to [0, 1]$ given by

$$f(x_1, \ldots, x_n) = \mathbb{P}(X_1 = x_1, \ldots, X_n - x_n).$$

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Jointly continuous defined analogously, with

$$F_{\vec{\mathbf{X}}}(\vec{x}) = \int_{u_1=-\infty}^{x_1} \cdots \int_{u_n=-\infty}^{x_n} f(u_1, \ldots, u_n) du_1 \cdots du_n$$

for some integrable function $f : \mathbb{R}^n \to [0, \infty)$.

Marginals

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Same set-up as above, the j^{th} marginal F_{X_j} is defined by

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Challenge problem: Find a set of marginals that can correspond to at least two different $F_{\vec{\mathbf{X}}}$'s, or show no set exists.