MATH 341: PROBABILITY: FALL 2009: HOMEWORK #9

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Due Thursday November 12 (though you may place in my mailbox anytime up till 10am on Friday 11/13):

(1) Let $X_1, \dots, X_n$ be iidrv random variables with the geometric distribution with parameter p, so $\text{Prob}(X_i = k) = (1-p)^{k-1}p$ for k a positive integer and 0 otherwise. Let $\overline{X} = (X_1 + \cdots + X_n)/n$. Find $\mathbb{E}[\overline{X}]$, $\text{Var}(\overline{X})$, and the moment generating function of $Y = (\overline{X} - \mathbb{E}[\overline{X}])/\text{StDev}(\overline{X})$.

(2) Calculate the Laplace transforms of the following densities (a) an exponential distribution with parameter λ; (b) uniform distribution on $[a, b]$ with $a \geq 0$.

(3) For each function compute the complex derivative at $z = 0$ or prove the function is not differentiable there: (a) $f(z) = z$; (b) $f(z) = z^2$; (c) $f(z) = \overline{z}$, where if $z = x + iy$ then $\overline{z} = x - iy$. Recall that the derivative is defined by
$$f'(z) \ = \ \lim_{h \to 0} \frac{f(z+h) - f(z)}{h},$$
where $h = h_1 + ih_2$ tends to $0 + 0i$ along any path but is *never* 0 in any calculation.

(4) Prove the product rule of differentiation, namely that if f and g are differentiable then the derivative of $f(x)g(x)$ is $f'(x)g(x) + f(x)g'(x)$. Using this, induction and the fact that the derivative of x is 1, compute the derivative of x^n for any positive integer n. Note that this proof *bypasses* having to use the binomial theorem to expand $(x+h)^n$!

(5) Calculate the limits as $(x, y) \to (0, 0)$, or prove the limit does not exist:
(a)
$$\lim_{(x,y)\to(0,0)} \frac{x^3 + 1701x^2y^2 + 24601y^4}{x^2 + y^2};$$
(b)
$$\lim_{(x,y)\to(0,0)} \left[\frac{x^8 + y^8}{x^2 + y^8} - \frac{x^{10} + y^{10}}{x^4 + y^{10}}\right].$$

(6) **Extra Credit** Prove or disprove: notation as in the first problem, the MGF of Y converges to the MGF of the standard normal as n tends to infinity.

Date: November 7, 2009.