SUMMARY OF "THE OPTIMUM STRATEGY IN BLACKJACK"

PAPER BY ROGER R. BALDWIN, WILBERT E. CANTEY, HERBERT MAISEL, AND JAMES P. MCDERMOTT. SUMMARY BY AARON FORD

This paper deals with the house game of blackjack, one of the top four casino games played in America. The house game consists of a permanent dealer, governed by common house rules concerning the deal, the draw, splitting pairs, and so on. There are two basic strategies to be considered, one when the player's hand has one unique value not exceeding 21, and the other when the player's hand is also under 21 but has two distinct values, meaning that the hand contains 1 or 2 aces.

The decision equation the derive is $E_{d,x} - E_{s,x} = -2P(T < x) - P(T = x) - 2P(T > 21)P(J > 21) + 2P(T < J \leq 21) + P(T = J \leq 21)$, where $E_{d,x}$ is the expectation of a player with a total of x who draws exactly one card, $E_{s,x}$ is the expectations of a player standing on a total of x, T is the final total obtained by the dealer, and J is the total obtained by the player after drawing one more card. As is expected, the greater the probability that the dealer goes over 21 and busts, the lower the player's minimum standing number should be. They then go ahead and calculate these above probabilities, under the assumptions that the probability of drawing any card in the deck is $1/52$ regardless of stage in the game.

Furthermore, they evaluate the conditional probabilities of soft hands, doubling down, and splitting pairs in great detail. After combining these results, they determine that the mathematical expectation of the player is equal to $E(W) = 1/13 \sum_{D \neq 10} E(W_D) + 4/13E(W_{10})$, where W_D is the amount won by the player when the dealer's shown card is D. The reason that 10 is a unique value is that one must calcuate $E(W_D)$ first if the dealer doesn't have a natural, and second that he does.

In the end, there are 3 parts to a player's strategy, when to draw and stand, when to double down, and when to split pairs. Let D be the card the dealer has showing, M(D) be the minimum standing numbers for unique hands, an M*(D) be the same for soft hands. The optimum strategy becomes

$$M(D) = \begin{cases} 13 & \text{if } D = 2, 3 \\ 12 & \text{if } D = 4, 5, 6 \\ 17 & \text{if } D \geq 7, D = (1, 11) \end{cases}$$

and

Date: December 4, 2009.

$$M*(D) = \begin{cases} 18 & \text{if } D \geq 8, D + (1, 11) \\ 19 & \text{if } D = 9, 10 \end{cases}$$

The surprising result at the time was the recommendation to stand on 12 if the dealer showed a 4, 5, or 6. This contradicted other papers at the time. After going through further tables of strategy that are too in-depth for this summary, their final surprising result is that the player's conditional expectation, given that the dealer's up card is D, is positive for 7 out of 10 values of D. The only advantage the dealer exhibits is the fact that if both the player and the dealer bust, the dealer wins. This disadvantage is the smallest of the popular house games surveyed.

E-mail address: ajf2@williams.edu