Math 341: Probability
First Lecture

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public_html/341/

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Introduction and Objectives

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Probability theory: model the real world, predict likelihood of events.

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Objectives

- Model problems and analyze model.
- Emphasize techniques / asking the right questions.
- Elegant solutions vs brute force (parameters in closed form versus numerical solutions).
- Looking at equations and getting a sense: $\log -5$ Method: $\frac{p \pm pq}{p+q \pm 2pq}$.

Types of Problems

- Biology: will a species survive?
- Physics / Chemistry / Number Theory: Random Matrix Theory.
- Gambling: Double-plus-one.
- Economics: Stock market / economy.
- Finance: Monte Carlo integration.
- Cryptography: Markov Chain Monte Carlo.
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My (applied) experiences

- Marketing: parameters for linear programming (SilverScreener).
- Data integrity: detecting fraud with Benford's Law (IRS, Iranian elections).
- Sabermetrics: Pythagorean Won-Loss Theorem.

Course Mechanics

Material

- Move at fast pace, responsible for preparing for class.
- Excellent book: phenomenal problems, detailed explanations.
- Supplemental book: interesting tidbits (cards at break: not Mafia!).
- Will cover most of Chapters 1 through 4, some combinatorics, generating functions and the Central Limit Theorem (CLT), and topics TBD by class.
- Pre-reqs: mostly calculus and basic combinatorics / set theory, need some multivariable calculus for some computations, linear algebra helps interpret some results, complex analysis for proof of the CLT.

Administrative

Grading

- HW 20%, midterms 40%, final 40%.
- Will be 2 or 3 midterms, at least 1 or 2 take home, lowest dropped.
- Tests will always have at least one question ‘do 1 of 2’.
- Project option: 10%, scale back rest.

Office hours / feedback

- Regular TBD, weekly dinner (?), whenever I’m in my office (schedule online).
- Feedback: mathephs@gmail.com, password first 7 Fibonacci numbers (1235813).

Office hour explanation

Other

- Webpage: numerous handouts, additional comments each day (mix of review and optional advanced material).
- Clickers: see how well we can estimate probabilities, always anonymous.
- Probability Lifesaver: opportunity to help write a book, lots of worked examples.
- Scribe: optional: taking turns summarizing lecture / takeaways.
- Creating HW problems: mix of ones you can solve and ones you want to learn about.
- Gather and analyze some data set of interest.
- PREPARE FOR CLASS! Must do readings before each class.

Clicker Problems

Birthday Problem I

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Birthday Problem I

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How large must N be for there to be at least a 50% probability that two of the N people share a birthday?

- (A) 11 people
- (B) 22 people
- (C) 33 people
- (D) 44 people
- (E) 90 people
- (F) 180 people
- (G) 365 people
- (H) 500 people.

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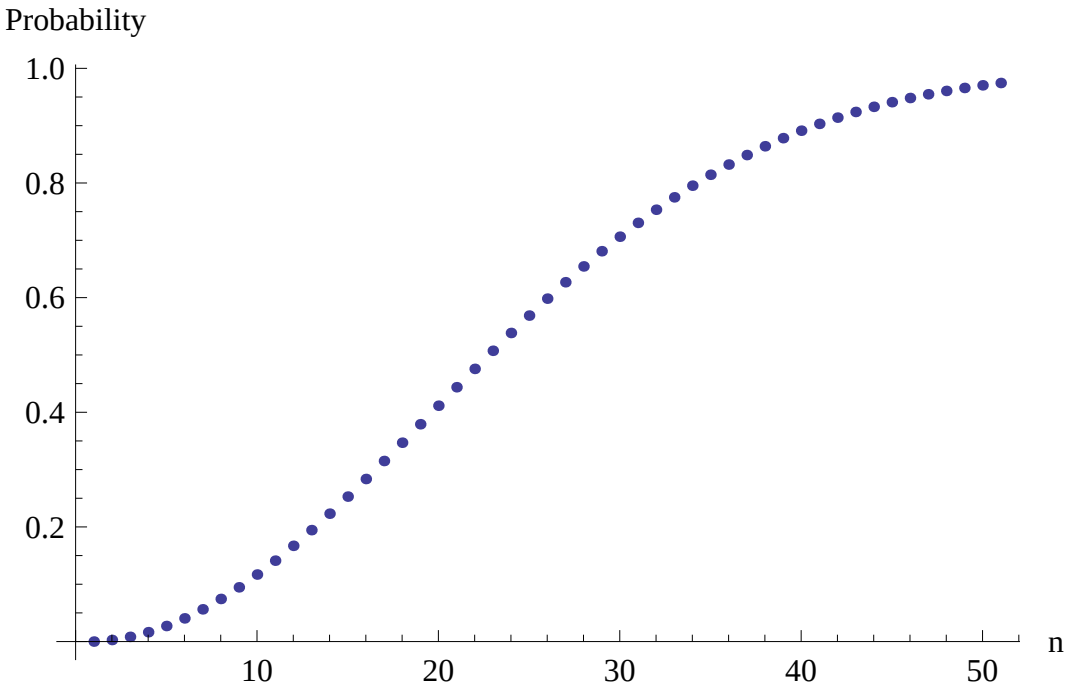

Birthday Problem II

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- (A) 110 people
- (B) 220 people
- (C) 330 people
- (D) 440 people
- (E) 1,000 people
- (F) 5,000 people
- (G) 10,000 people
- (H) 20,000 people
- (I) more than 30,000 people.

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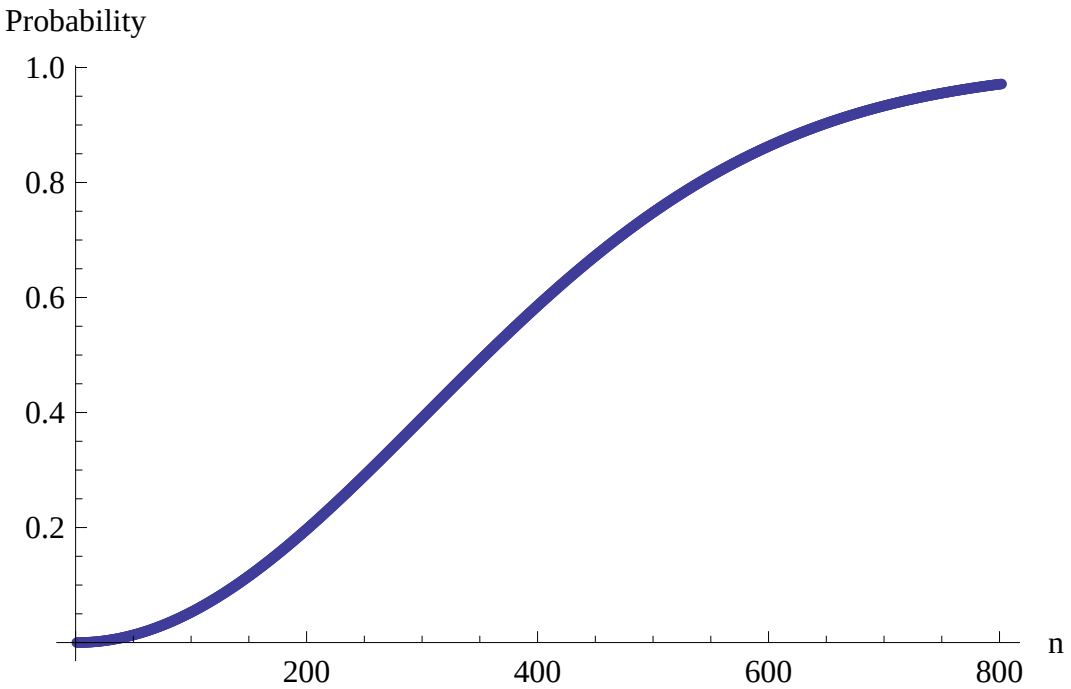

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- (A) 1 / 10
- (B) 1 / 100
- (C) 1 / 1,000
- (D) 1 / 10,000
- (E) 1 / 100,000
- (F) 1 / 1,000,000 (one in a million)
- (G) 1 / 1,000,000,000 (one in a billion).

Voting: Democratic Primaries (continued)

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Prof. Kim's calculation ... was based on the assumption that Syracuse voters were likely to vote in equal proportions to the state as a whole, which went for Ms. Clinton, its junior senator, 57%-40%. .... Prof. Kim said she had little time to make the calculation, so she made the questionable assumption ... for simplicity.