Math 341: Probability
Tenth Lecture (10/15/09)

Steven J Miller
Williams College

Steven.J.Miller@williams.edu
http://www.williams.edu/go/math/sjmiller/public_html/341/

Bronfman Science Center
Williams College, October 15, 2009

Summary for the Day

Summary for the day

- Common Distributions
- Linearity of Expectation:
  - ◇ Mean of binomial random variable.
  - ◇ Fermat primes.
- Chebyshev's Theorem:
  - ◇ Application: Monte Carlo Integration.
- Questions from the class
- Dependence

Chebyshev's Inequality

Chebyshev's Inequality (Statement)

Chebyshev's Inequality

Let X be a random variable with finite mean μ and finite variance σ^2. Then

$$\text{Prob}(|X - \mu| \geq k\sigma) \leq \frac{1}{k^2}.$$

Sometimes called Chebyshev's Theorem.

Chebyshev's Inequality (Proof)

Proof: Letting f denote the density of X:

$$
\begin{aligned}
\text{Prob}(|X-\mu| \geq k\sigma) &= \int_{|x-\mu|/k\sigma \geq 1} f(x)dx \\
&\leq \int_{|x-\mu|/k\sigma \geq 1} \left(\frac{x-\mu}{k\sigma}\right)^2 f(x)dx \\
&\leq \frac{1}{k^2\sigma^2} \int_{-\infty}^{\infty} (x-\mu)^2 f(x)dx \\
&= \frac{\sigma^2}{k^2\sigma^2} = \frac{1}{k^2}.
\end{aligned}
$$

Clicker Questions

The n-dimensional sphere

Volume of the n-dimensional sphere

Consider the n-dimensional sphere of radius $1/2$ centered at the origin, which lives inside the n-dimensional unit cube. Let ρ_n be the ratio of the sphere's volume to that of the cube (i.e., the sphere's volume). How large must n be before $\rho_n < .01$ (i.e., before the n-dimensional sphere occupies less than 1% of the volume of the n-dimensional cube)?

The n-dimensional sphere

Volume of the n-dimensional sphere

Consider the n-dimensional sphere of radius $1/2$ centered at the origin, which lives inside the n-dimensional unit cube. Let ρ_n be the ratio of the sphere's volume to that of the cube (i.e., the sphere's volume). How large must n be before $\rho_n < .01$ (i.e., before the n-dimensional sphere occupies less than 1% of the volume of the n-dimensional cube)?

- (a) 2 (e) 10
- (b) 4 (f) 20
- (c) 6 (g) It is always greater than 1%.
- (d) 8 (h) Beats & is beaten by 1% infinitely often