MATH/STAT 341: Probability: MWF 11 - 11:50am Horm 103C

Professor Steven Miller (sjm1@williams.edu), Bascom 106D (617-835-3982)

Office hours: TBD and probably if in my office (click here for my schedule)

TA Office Hours: Sunday, Tuesday and Thursday: TBD

COURSE DESCRIPTION: While probability began with a study of games, it has grown to become a discipline with numerous applications throughout mathematics and the sciences. Drawing on gaming examples for motivation, this course will present axiomatic and mathematical aspects of probability. Included will be discussions of random variables, expectation, independence, laws of large numbers, and the Central Limit Theorem. Many interesting and important applications will also be presented, including some from coding theory, number theory and nuclear physics. **NOTE: this course will move at a very fast pace. We will cover a lot of material and applications, and you are required to read the book before the lecture. Whenever possible we will prove all results and theorems.**

Format: lecture. Evaluation will be based primarily on homework, classwork, writing and exams.

Prerequisites: Multivariable Calculus and Linear Algebra, or permission of instructor. *No enrollment limit (expected: 40).*

CONTACTING ME: You can reach me in Bascom 106D (if I'm there it's probably office hours), email sjm1@williams.edu, or ***anonymously through ephsmath@gmail.com (password 1793williams; it used to be the first eight Fibonacci numbers but annoyingly someone hacked the account and changed it, and Google wouldn't let me restore it; with new security features may not be usable….).***

OBJECTIVES: There are two main goals to this course: to explore probability theory and see the connections between various problems, and to learn problem solving skills. We will constantly emphasize the techniques we use to solve problems, as these techniques are applicable to a wide range of problems in the sciences. For a fuller statement as to the objectives of this course, please click here. This includes some fascinating videos with some thought provoking comments about what you should get out of your education.

TEXTBOOK/SYLLABUS: The textbook is The Probability Lifesaver. This book is designed to supplement any standard class, or be used as a stand-alone book. Tentatively we will cover most of the first 20 chapters, chapter 23, and additional topics as time permits; due to time constraints we may not do some of the topics (such as Chapter 6). **There will also be programming assignments (you may use any language).** There will be a weekly lunch series (if you do not have a meal plan lunch will be provided) to discuss additional topics if there is student interest; though in parallel to that course, these meetings are optional and have ***no impact*** on the course grade. You are expected to have read the corresponding material ***before*** class; if you have any questions you should email me beforehand. I strongly urge you to pick up an additional book for the class (almost any book is fine, feel free to grab one from the library). It is a valuable skill to learn how to read and discern the relevant information from a variety of sources. Here are a few particularly good books: (1) Probability and Random Processes by Geoffrey R. Grimmett and David R. Stirzaker (third edition). Excellent text, used it a few years ago, lots of great additional topics. (2) Probability by Lawrence M. Leemis. Not as high level as Grimmett and Stirzaker and not as many advanced applications, but extremely well written, friendly, and has lots of comments on how to attack the problems in R. (3) A nice read is Impossible?: Surprising Solutions to Counterintuitive Conundrums by Julian Havil (this is a *suggested* book for the class for those who want to see some additional fun problems; this book is **not** required for the course, and anything we use from the book will be explained completely in class).

GRADING POLICY: Homework: 15%. Preparing for class: 5%. Midterm: 40% (if there are two exams only best counts). Final exam: 40%. You may also do a project for 10% of your grade (which reduces all other categories proportionally). Exams are black tie optional. Homework is to be handed in on time, stapled and legible. *Late, messy or unstapled homework will not be graded.* I encourage you to work in small groups, but everyone must submit their own homework assignment. **Extra credit problems should not be included in the general homework, but handed in separately. Very little partial credit is given on these problems.**

COURSE DISCLAIMER: I may occasionally say things such as `Probability is one of the most useful courses you can take' or 'If you know probability, stats and a programming language then you'll always be able to find employment'. I really should write `you should always be able to find employment', as nothing is certain. Thus, please consider yourself warned and while you may savor the thought of suing me and/or Williams College, be advised against this! I'm saying this because of the recent lawsuit of a graduate who was upset that she didn't have a job, and sued her school!

- Article from the NYPost on the lawsuit.
- Another article, which notes that she misspells 'tuition'.
- You can view her motion here.
- **Week 2: September 9-13**
  - Read: Chapter 2 and Chapter 3 of my textbook
  - Videos: Monday: http://youtu.be/dmI9d-w-bM4 Wed: http://youtu.be/Mg7xZqolBKE Fri: http://youtu.be/0Rp8KgWmLi0
  - HW: Due September 13, 2019: #1: Section 1.2: Modify the basketball game so that there are 2015 players, numbered 1, 2, ..., 2015. Player i always gets a basket with probability $1/2^i$. What is the probability the first player wins? #2: Section 1.2: Is the answer for Example 1.2.1 consistent with what you would expect in the limit as c tends to minus infinity? #3: Section 1.2: Compute the first 42 terms of 1/998999 and comment on what you find; you may use a computer. #4: Section 2.2.1: Find sets A and B such that |A| = |B|, A is a subset of the real line and B is a subset of the plane (i.e., R^2) but is not a subset of any line. #5: Section 2.2.1: Write at most a paragraph on the continuum hypothesis (you may use Wikipedia or any source to look it up). #6: Section 2.2.2: Give an example of an open set, a closed set, and a set that is neither open nor closed (you may not use the examples in the book); say a few words justifying your answer. #7: Section 2.3: Give another proof that the probability of the empty set is zero. #8: Find the probability of rolling exactly k sixes when we roll five fair die for k = 0, 1, ..., 5. Compare the work needed here to the complement approach in the book. #9: If f and g are differentiable functions, prove the derivative of f(x)g(x) is f'(x)g(x) + f(x)g'(x). Emphasize where you add zero.
- **Week 1: September 6, 2019**
  - Read: Chapter 1 of the textbook and the graduation speech here: http://www.graduationwisdom.com/speeches/0018-uslan.htm
  - Homework: To be **emailed** to me by 10pm on Sunday, September 8: Email me a short note on what you want to get out of this course, and what lesson you learned from the graduation speech. Full credit, 20/20, if you answer both questions on time. If you've already said what you want from a previous course with me, please briefly repeat. Thanks!
  - click here for first day slides click here for first day's handout first day's video: http://youtu.be/pVQL4ivA08w (right now from an earlier year)

Objectives for Math/Stat 341: Probability

The following is entirely optional, but describes some of my thoughts about the course, ranging from why it is structured the way it is to what I want you to get out of this course in particular and your education in general. When I was in high school, I remember the Boston Globe ran an article where they asked numerous 'famous' people in the state: what 10 books should **every** high school student read? The answers were, for the most part, disappointing. The English professor had ten literary selections, ranging from some Shakespeare and Milton to stuff I can no longer remember. Most of the others had lists greatly biased towards the small part of the world they studied; very few had balanced lists that would help prepare the general student for the world (Governor Weld was one of the few who did).

I **strongly** urge you to view the following clip on YouTube: Did You Know (2009 version). Some of the more interesting statistics / quotes:

- Preparing students for jobs that don't exist using technologies that haven't been invented to solve problems we don't know are problems yet.
- We are living in exponential times:
    - 31 billion searches on google each month now; it was 2.7 billion in 2006.
    - The first text message was in 1992; more are sent today than the population of the planet.
    - Number of internet devices: 1,000 in 1984, 1,000,000 in 1992, 1,000,000,000 in 2008;
    - Estimated that 4 exabytes (4×10^{19} bytes) of unique information will be generated this year, which is more than has been generated in the past 5000 years!
    - Amount of new technical information doubles every two years.

Another great clip to listen to is the TED lecture of Malcolm Gladwell on spaghetti sauce. The main point of this lecture is

- Important to ask the right question; what you think is the right question frequently isn't. I won't do the lecture justice by summarizing, so here's a tantalizing tidbit: **this surprising question led to Prego making $600 million in 10 years on extra chunky spaghetti sauce.**
- There are lots of great clips on TED; another one of my favorites is Dan Pink on Motivation. Some very interesting observations here on how to create an optimal environment for creativity to flourish.

What does all this have to do with our class? The point is that it is hard to predict what will be useful to most of you in your career(**s**) after Williams (save for the few who go off to grad school to study number theory, where I can do a pretty good job predicting). It is imperative that you learn to be problem solvers. The content of a course matters; you need to learn the language, the basic facts, the key theorems, et cetera. But, at least as important, you need to learn how to use these on 'new' problems.

One of my biggest epiphanies as an educator was when I prepared my lecture notes for Math 209 (differential equations). This was the first class I'd ever taught as a college professor which I'd actually taken as an undergrad. I was looking through my old course notes, trying to decide what to include, when I noticed that we did the Bessel equation and function when I was a student. I was shocked; I had no memory of having done this, but I use the Bessel function in about a quarter of my number theory papers. What's the takeaway? You're going to forget much of the material you learn. That's to be expected. Hopefully you'll be able to re-learn it as needed / you'll know where to go to read more about it / you'll have some familiarity that such results exist. I had to relearn the Bessel function in grad school (and did, it's not that bad). What is more permanent is the techniques and methods. These are the things you'll use again and again. They can range from learning how to multiply by 1 or add 0 (two of the most difficult things to do well in mathematics!) to how to count something two ways to how to model the key features of a problem.

One of the goals of this course is to help you become problem solvers. The problems are from probability, but the methods and techniques, the mindset, should hopefully be applicable to a variety of problems. If you are taking an intro calculus class and Section 3.2 is on the Chain Rule, it's a safe bet that you should use the chain rule to solve problems from that section. The real world (or advanced academia) is not like that -- you frequently do not know what the 'right' way is to attack a problem (especially open problems that have stumped people for years). This is one of the reasons why I want you to create (and if possible solve!) your own homework problems. The act of creation is a huge part of research. **Most** math papers are **mostly** trivial (or, as I often say, trained monkey work). What does this mean? It means that over 95% of most papers is just straightforward manipulation of previous results (the further you go in math, the more things become straightforward). The most important parts of papers are usually the following two items: (1) asking the right question; (2) coming up with a novel way to attack a problem. Often once the question is asked and the method chosen, the paper writes itself; however, it is very hard at times to ask the right question, or to see a good way to attack it. (As an example, when I taught at Princeton years ago I wrote a handout for my students on how to prove the Fibonacci numbers satisfy Benford's law of digit bias. I decided to try and publish it, and did some research. I found a paper from the 1970s that was almost identical to mine -- basically same theorems and lemmas in the same order! This isn't too surprising, as once the question was asked, this truly was the most 'natural' way to go.)

There are remarkable connections between what seem at first very disparate branches of learning; if you are one of the first people to see such a connection, you have the potential of making a real breakthrough. I **strongly urge** you to tell me what you're interested in. I'll see if I can work it into the course (either in the lectures proper or in the additional comments). The more you explore, the more likely it is you can make one of these great connections. I've been fortunate enough to make some connections between nuclear physics and baseball, and between number theory and tax fraud. Because of this I've had a private tour of Petco Park (where the San Diego Padres play) and given a talk at the Boston headquarters of the IRS (with district attorneys, auditors, and secret service agents in the audience) and been interviewed by the Wall Street Journal about fraud in the recent Iranian elections.

Finally, here is a great website with 10 excellent commencement speeches. It's worth the time reading these; I particularly liked the one by Uslan (on how it's not enough to just have a good idea, but how to get noticed).