DIPLOMA PROBLEMS

Given n people, what is the probability that your diploma's number is within 1 of the correct value? So if you are number 5, get either 4, 5 or 6; if there are n people we say 1 and n are adjacent. Answer is 3 / n if n ≥ 3. Next: what is the expected number of people to get a diploma within 1 of theirs? Linearity of expectation:

Let X_i be 1 if person i gets a diploma within 1 of theirs, and 0 else.

If X is the number of people with a diploma within 3, then X = X_ 1 + ... + X_n.So

So E[X] = Sum E[X_i] and E[X_i] = 1 * 3 / n + 0 * (n - 3) / n = 3 / n

SO E[X] = Sum_ {i = 1 to n} 3 / n = 3

In[54]:=
```
threegrad[printcheck_, numiter_, n_] :=
 Module[{},
  count = 0;
  list = {};
  For[j = 1, j ≤ n,
   j++, list = AppendTo[list, j]];
  (* creates list of numbers *)
```

```
 For[i = 1, i ≤ numiter, i++,
  {
   newlist = RandomSample[list, n];
   If[i == 1 && printcheck == 1,
    Print[list, " ", newlist]];
   currentcorrect = 0; (* initialize to zero,
   no one is within 1 yet *)
   For[d = 1, d ≤ n, d++,
    If[Abs[newlist[[d]] - d] ≤ 1,
     currentcorrect = currentcorrect + 1]];
   (* d statement ends *)
   (* have to handle wrap-around *)
   If[newlist[[1]] == n,
    currentcorrect = currentcorrect + 1];
   If[newlist[[n]] == 1,
    currentcorrect = currentcorrect + 1];
   If[i == 1 && printcheck == 1,
    Print["Number of matches is ",
     currentcorrect]];
   count = count + currentcorrect;
  }]; (* end of i loop *)
 Print[
  "The average number observed that are within
    1 is ", 1.00 count / numiter];
] (* end of module *)
```

In[57]:=
```
Timing[threegrad[0, 100000, 10]]
```
```
The average number observed
   that are within 1 is 2.99995
```
Out[57]= {2.57402, Null}

Now consider a success if diploma is the same parity as yours; even – even or odd – odd; not even – odd or odd – even. Now what is the expected number of successes? Recursion: if n = 1 expected number of successes is 1

If n = 2 expected number of successes is 1 = 2 * 1 / 2 + 0 * 1 / 2

If n = 3: 123 = 132 = 213 = 231 = 312 = 321 (3 + 1 + 1 + 1 + 1 + 3) / 6 = 10 / 6 = 5 / 3

a (2 n) = 1 / 2 (1 + a (2 n – 1)) + 1 / 2 (0 + a (2 n – 2))

a (2 n + 1) = ?? ?

In[72]:=
```
paritygrad[printcheck_, numiter_, n_] :=
 Module[{},
  count = 0;
  list = {};
  For[j = 1, j ≤ n, j++,
```

```
    list = AppendTo[list, Mod[j, 2]]];
   (* creates list of numbers *)
   For[i = 1, i ≤ numiter, i++,
    {
     newlist = RandomSample[list, n];
     If[i == 1 && printcheck == 1,
      Print[list, " ", newlist]];
     currentcorrect = 0; (* initialize to zero,
     no one is within 1 yet *)
     For[d = 1, d ≤ n, d++,
      If[newlist[[d]] == Mod[d, 2],
       currentcorrect = currentcorrect + 1]];
     (* d statement ends *)
     If[i == 1 && printcheck == 1,
      Print["Number of matches is ",
       currentcorrect]];
     count = count + currentcorrect;
    }]; (* end of i loop *)
   Print[
    "The average number of same parity is ",
    1.00 count / numiter];
  ] (* end of module *)

In[74]:= Timing[paritygrad[1, 10000, 10]]
```

```
{1, 0, 1, 0, 1, 0, 1, 0, 1, 0} {1, 0, 1, 0, 0, 1, 1, 0, 0, 1}
Number of matches is 6
The average number of same parity is 4.9782
```

Out[74]= {0.218401, Null}