Chapter 2: Sec 2.1: The Binomial Distribution

Will skip most of this chapter, do this sec as motivates

Recall Bernoulli: $\text{Prob}(X=1) = p$, $\text{Prob}(X=0) = 1-p = q$

Binomial: n indep Bernoulli trials

Can figure out probabilities of events

Ex: shoot n times, make p % of shots

(1) prob make (exactly) k baskets: $\binom{n}{k} p^k (1-p)^{n-k}$

(2) prob make at least k baskets: $\sum_{\ell=k}^{n} \binom{n}{\ell} p^\ell (1-p)^{n-\ell}$

↳ note no double counting

(3) prob make a basket: $1 - \binom{n}{0} p^0 (1-p)^n = 1-(1-p)^n$

if n is large, what is this?

Can't do $(1-p)^n = 1 - np + \frac{n(n+1)}{2} p^2 + \cdots$

$(1-p)^n = \exp(\log((1-p)^n)) = \exp(n \log(1-p))$

MVT: $1-(1-p)^n = n(1-c)^{n-1} p$ $\qquad f(x) = (1-x)^n$

↳ but what is c as a fn of p?

View $1-p$ as q, then answer is $1-q^n$

↳ This is already a great approx!

This is all doing from chapter, but have several Qs:

① What is expected/average number of heads if toss n times with prob p of head?

② What is scale of fluctuations?

Leads to concept of mean/variance/moments

HW: Pg 91: #2 (first part, not rel freq), #4–#7, #10

CHAPTER 3: RANDOM VARIABLES

SECTION 3.1: INTRODUCTION

Book is just doing for discrete; will do discrete and continuous simultaneously for unified approach.

__Discrete__: density p, have finite or countably infinite set $\{x_1, x_2, x_3, \dots\}$ where probabilities are non-negative, say $p(x_i) = p_i$, and $\sum p(x_i) =$

If want prob value b/w a and b it is
$$\sum_{a \le x_i \le b} p(x_i)$$
note endpoints matter. Will say have random variable $\underline{X}$ with density/distribution p. Let $F(x) = \sum_{x_i \le x} p(x)$ be the prob of X being at most x. Call this the cumulative distribution fn. To avoid confusion, often write f_X and F_X for the density and CDF.

__Continuous__: Now $f(x) \ge 0$, $\int_{-\infty}^{\infty} f(x)dx = 1$, prob X b/w a and b is $\int_a^b f(x)dx$ (endpoints don't matter), $CDF = \int_{-\infty}^{x} f(x)\,dx$.
By Fund Thm of Calculus, see density is the derivative of CDF, so if can find CDF can find density.

Section 3.1: Introduction

Example 1: Say X is uniform on $[0,2]$, let $Y = X^2$. Find density of Y.

Step 1: density of X is $\quad f_X(x) = \begin{cases} 1/2 & 0 \le x \le 2 \\ 0 & \text{otherwise} \end{cases}$

Step 2: Find CDF of X:

$$F_X(x) = \int_{-\infty}^{x} f_X(x) = \begin{cases} 0 & x \le 0 \\ x/2 & 0 \le x \le 2 \\ 1 & x \ge 1 \end{cases}$$

Step 3: Find CDF of Y:

$F_Y(y)$ Is prob Y at most y:

$$\begin{aligned} F_Y(y) &= \text{Prob}(Y \le y) \\ &= \text{Prob}(X^2 \le y) \\ &= \text{Prob}(-\sqrt{y} \le X \le \sqrt{y}) \\ &= \text{Prob}(X \le \sqrt{y}) = \begin{cases} 0 & y \le 0 \\ \sqrt{y}/2 & 0 \le y \le 4 \\ 1 & y \ge 4 \end{cases} \end{aligned}$$

Step 4: Find pdf (prob density fn) of ~~X~~ Y

$$f_Y(y) = \frac{d}{dy} F_Y(y) = \begin{cases} 0 & y \le 0 \\ 1/4\sqrt{y} & 0 \le y \le 4 \\ 0 & y \ge 4 \end{cases}$$

Step 5: Check!

↳ density is non-neg

↳ $\displaystyle \int_0^4 \frac{1}{4\sqrt{y}}\, dy = \int_0^4 \frac{1}{4} y^{-1/2}\, dy = \frac{y^{\frac{1}{2}}}{4 \cdot \frac{1}{2}} \Big|_0^4 = 1$

HW: If X is uniform on $[-1,1]$, find density of $Y = X^2$.

Section 3.1: Random Vars (cont)

Always have random variable $X : \Omega \to \mathbb{R}$.
Why make it real valued? What is $H+H+T$?
But can do $1+1+0$ or $1+1-1$.

Joint Distributions

Write $p(x,y)$ or maybe $P_{X,Y}(x,y)$ for the prob that $X = x$ and $Y = y$.
↳ can be cont or discrete

<u>Marginal Probability</u>: $\text{Prob}(X = x) = \sum_{y} P_{X,Y}(x,y)$

or $\int_{y=-\infty}^{\infty} P_{X,Y}(x,y)\,dy$

<u>Great Example</u>: 2 draws w/o replacement from $\{1,2,3\}$, all equally likely, X result 1st draw, Y of second.

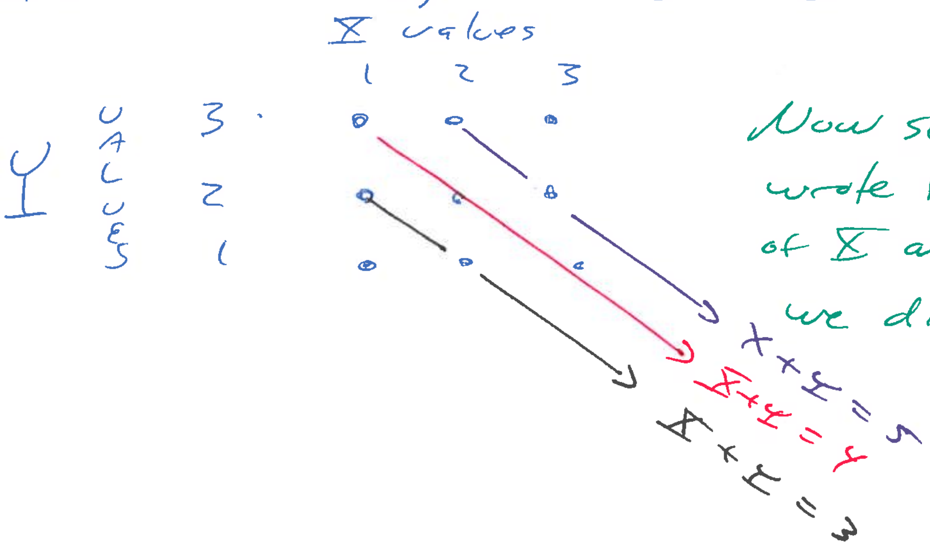
		values of X: 1	2	3	dist of Y (marginal)
values of Y	3	1/6	1/6	0	1/3
	2	1/6	0	1/6	1/3
	1	0	1/6	1/6	1/3
dist of X (marginal)		1/3	1/3	1/3	total sum

Have $3 \cdot 2 = 6$ pairs, each equally likely
Note X and Y have same distribution
but $X \neq Y$, in fact, $\text{Prob}(X = Y) = 0$

SECTION 3.1: INTRO: CONTINUED

Won't do too much more on change of variable;
I prefer the CDF approach.

Book has nice pictures on page 148 on sums of random variables, basically way to view it



Now see why wrote the values of X and Y as we did.

Aside: Continuous Case

Say X has density f_X, Y has density f_Y.
$Z = X + Y$. What is f_Z?

If discrete: $f_Z(z) = \sum_x f_X(x)\, f_Y(z-x)$

If continuous: $f_Z(z) = \int_{x=-\infty}^{\infty} f_X(x)\, f_Y(z-x)\, dx$

$$= (f_X * f_Y)(z)$$

Often nice answer by Calculus!

SECTION 3.1: INTRO: CONT

Skip the maximum/minimum on page 149

Multiplication Rule

$Prob(X = x, Y = y) = Prob(X = x)\, Prob(Y = y \mid X = x)$

Independence: $Prob(X = x, Y = y) = Prob(X = x)\, Prob(Y = y)$

More generally: $Prob(X \in A, Y \in B) = Prob(X \in A)\, Prob(Y \in B)$

Good example on pg 152 on whether or not two random vars are indep — depends on perspective.

Will not do rest of section 3.1 for now

HW: Pg 158: #3, #10, #16b (but look at #16a).