Math 341: Probability
Third Lecture (9/17/09)

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Clicker Questions

Poker hand

Question

A deck has 52 cards, with four aces, four kings, et cetera. How many ways are there to choose 5 cards from the 52 (without repetition) such that at least two cards are aces?

Let $x = \binom{4}{2}\binom{50}{3}$.

- (a) More than x.
- (b) Exactly x.
- (c) Fewer than x.

Poker hand

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- (a) More than x.
- (b) Exactly x.
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$\binom{4}{2}\binom{50}{3}/\binom{52}{5} \approx .0452.$

$\left(\binom{4}{2}\binom{48}{3} + \binom{4}{3}\binom{48}{2} + \binom{4}{4}\binom{48}{1}\right)/\binom{52}{5} \approx .0417.$

Question

Choose a number randomly from 1 through 9 inclusive, with each number equally likely.

For this question, press 1 for 1, 2 for 2, and so on.

Question

Choose a number from 1 through 9 inclusive; whomever is closest to one-half the class average is excused from one homework problem on the next assignment.

For this question, press 1 for 1, 2 for 2, and so on.

Section 1.5
Independence

Definition

Independence

A and B are independent if

$$\mathbb{P}(A \cap B) = \mathbb{P}(A)\mathbb{P}(B).$$

More generally, a family $\{A_i\}_{i \in I}$ is independent if

$$\mathbb{P}\left(\bigcap_{i \in J} A_i\right) = \prod_{i \in J} \mathbb{P}(A_i) \quad \text{for any } J \subset I.$$

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Question: If a set of positive integers are pairwise relatively prime, then they are relatively prime. Does a similar result hold for independence, namely if a collection of events are pairwise independent are they independent?

Roulette

Roulette

Consecutive colors

Imagine a simplified roulette game where red occurs 50% of the time and black occurs 50% of the time, and the spins are independent. What is the probability of getting at least 5 consecutive blacks when the wheel is spun 100 times?

Roulette

Consecutive colors

Imagine a simplified roulette game where red occurs 50% of the time and black occurs 50% of the time, and the spins are independent. What is the probability of getting at least 5 consecutive blacks when the wheel is spun 100 times?

- (a) less than 1%
- (b) about 5%
- (c) about 20%
- (d) about 50%
- (e) about 80%
- (f) about 95%
- (g) more than 99%

Roulette

Consecutive colors II

Imagine a simplified roulette game where red occurs 50% of the time and black occurs 50% of the time, and the spins are independent. About how many spins do we need to have about a 50% chance of observing at least 5 consecutive blacks?

Roulette

Consecutive colors II

Imagine a simplified roulette game where red occurs 50% of the time and black occurs 50% of the time, and the spins are independent. About how many spins do we need to have about a 50% chance of observing at least 5 consecutive blacks?

- (a) About 10
- (b) About 20
- (c) About 40
- (d) About 80
- (e) About 200
- (f) More than 500

Roulette

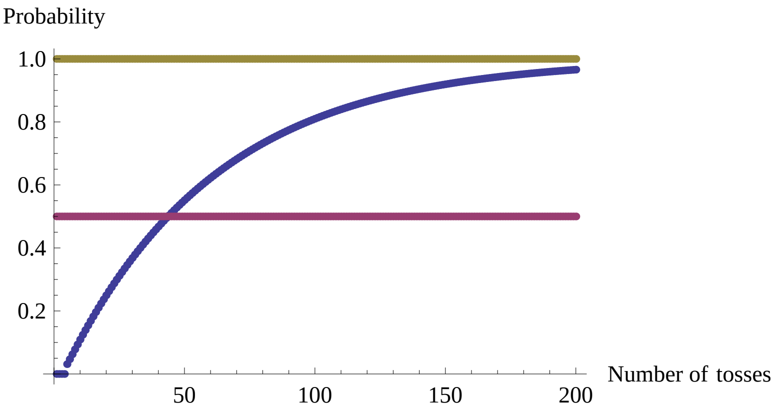

Figure: Plot of probability we have at least 5 consecutive black spins against the number of spins.