Math 341: Probability
Sixth Lecture (9/29/09)

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Clicker Questions

Exponential numbers

Question

Let x and y be any two 341 digit real numbers. What is $e^x e^y$?

Exponential numbers

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- (a) e^{x^y}
- (b) e^{xy}
- (c) e^{x+y}
- (d) e^{2xy}
- (e) It is undefined.
- (f) None of the above.
- (g) I remember the answer to this from another class with Professor Miller.

Exponential numbers

Question

Let A and B be two 341×341 matrices with real entries. What is $e^A e^B$?

Exponential numbers

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Let A and B be two 341×341 matrices with real entries. What is $e^A e^B$?

- (a) e^{A^B}
- (b) e^{AB}
- (c) e^{A+B}
- (d) e^{2AB}
- (e) It is undefined.
- (f) None of the above.
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Baker, Campbell and Hausdorff formula

Baker, Campbell and Hausdorff formula

Let A and B be two $n \times n$ real matrices, and define the commutator of A and B by $[A, B] = AB - BA$. Then if

$$e^A = I + A + \frac{A^2}{2!} + \frac{A^3}{3!} + \cdots$$

then

$$e^A e^B = e^{A+B+\frac{1}{2}[A,B]+\frac{1}{12}([[A,B],B]+[A,[A,B]])+\cdots}.$$

For more information, see

- `http://www.hep.anl.gov/czachos/CBH.pdf`
- `http://en.wikipedia.org/wiki/Baker−Campbell−Hausdorff_formula`

Section 3.1
Probability Mass and Density Functions

Definition (discrete)

Probability Mass Function

The Probability Mass Function of a discrete random variable X is a function $f : \mathbb{R} \to [0, 1]$ given by $f(x) = \mathbb{P}(X = x)$.

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Lemma: Standard properties:

- $F(x) = \sum_{x_i \leq x} f(x_i)$, and $f(x) = F(x) - \lim_{y \to x^-} F(y)$.
- $\{x : f(x) \neq 0\}$ is at most countable.
- $\sum_i f(x_i) = 1$ where $\{x_1, x_2, \ldots\}$ is where f is non-zero.

Definition (continuous)

Probability Density Function

The Probability Density Function of a continuous random variable X is the f such that $F(x) = \int_{-\infty}^{x} f(u)du$.

Definition (continuous)

Probability Density Function

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Lemma: Standard Properties:

- $\int_{-\infty}^{\infty} f(x)dx = 1$.
- $\mathbb{P}(X = x) = 0$ for all $x \in \mathbb{R}$.
- $\mathbb{P}(a \leq X \leq b) = \int_{a}^{b} f(x)dx$.