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>
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President Garfield and the Pythagorean Theorem

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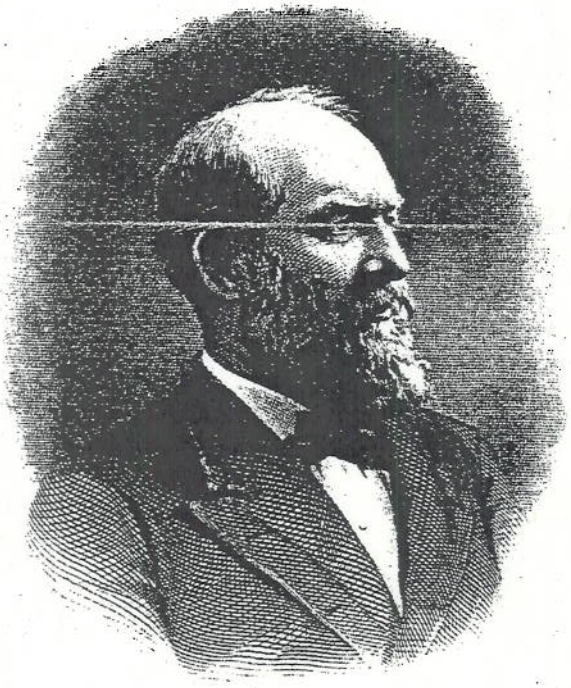
In 1876 a member of the House of Representatives produced a new proof of the Pythagorean Theorem. The proof itself is somewhat unusual in that it is based on a trapezoid, although inspection shows that it bears a resemblance to an argument found in the oldest known Chinese mathematics text. More interesting is the fact that Congressman James Abram Garfield, who later became the twentieth President of the United States, apparently had a rather mixed ability and background in mathematics, although he exhibited an unusually high level of overall intelligence.



James A. Garfield

Garfield was born in Ohio on 19 November 1831, the youngest of four children, and was barely two years old when his father succumbed to what is euphemistically recorded as a cold and sore throat, but as Jane Austen's Mrs. Bennet says, "People do not die of little trifling colds." It was almost certainly pneumonia, but the customs of the time did not allow for a grown man to be acknowledged to have been felled by an "unmanly" disease like pneumonia. Garfield's early schooling took place at the usual one-room schoolhouses near his rural home. His 13-year-old sister Mehitabel (named for her Puritan grandmother) carried her five-year-old brother through the snow on her back to attend classes. Eventually Eliza Garfield donated a portion of her land for a schoolhouse nearer their home. Garfield was also inspired by the visits of the local teacher when he boarded around with various neighborhood families. Here we have the beginning of the classic rags-to-riches story, fulfilling the heroic archetype of the day: the boy from the poor background who did grow up to be President of the United States. Garfield was, in fact, the last United States President to be born in a log cabin.

Garfield displayed rudimentary geometrical talent at an early age. His older brother Thomas sacrificed his own education in order to work for the family. Under his example and tutelage, James learned basic principles of carpentry; several surviving accounts relate that he had a good eye for angles, corners, and how pieces of wood would fit together. When he was about 15, he had a carpentry job building a woodshed for a blacksalter; the employer noticed that James could not only read and write, but was "death on figgers." Garfield accepted a job keeping the man's account books and used the time to devour his employer's library after hours.

Garfield subsequently went to Cleveland and was hired by a cousin as a "canal boy." One of his responsibilities was guiding a canal boat as it encountered another boat going in the opposite direction; this operation is tricky in that both boats have ropes attaching them to the mules or horses on shore, and care is required to ensure that the ropes do not tangle. Occasionally the ropes would get

snarled, and Garfield's job was to disentangle them without getting them caught on the bridges overhead; this task required some sense of spatial relations and perhaps, in modern terms, elementary knot theory.

In March 1849 Garfield entered Geauga Seminary in Ohio, having been encouraged to return to school by the cousin who employed him and who was impressed by his knowledge in all subjects. At Geauga he alternated between his studies and stints of teaching in the local school November–February and March–May. In his diary for May he recorded, "The *Algebra* has been nothing but *theory* for the past month. I like it however, very well." Three weeks later he wrote, "Upon this memorable (I guess) day, we have finished our Algebra, and commenced reviewing. I can look back to the time when it seemed a Herculean task to me but I [am] glad to know that *Perseverentia vincit omnia*." How much he learned may, however, be questioned since he noted two months later, "My Algebra was not examined because it was so small, and they had not time." In August, he wrote, "Agreed to take the Mental Arithmetic class reluctantly. Hope for success." These entries do not sound like the record of a particularly successful student of mathematics.

In August 1851 Garfield entered a college preparatory school, the Western Reserve Eclectic Institute (now Hiram College). During October he recorded in his diary, "I have today commenced the study of Geometry alone without class or teacher. I commenced this morning and in about two hours I got the definitions and 8 propositions and the scholiums and corollaries appended." That would suggest very rapid assimilation of material for two hours of work, but we have no further record of any interest or study of geometry in Garfield's diary, which makes his return to the Pythagorean Theorem some 25 years later all the more intriguing.

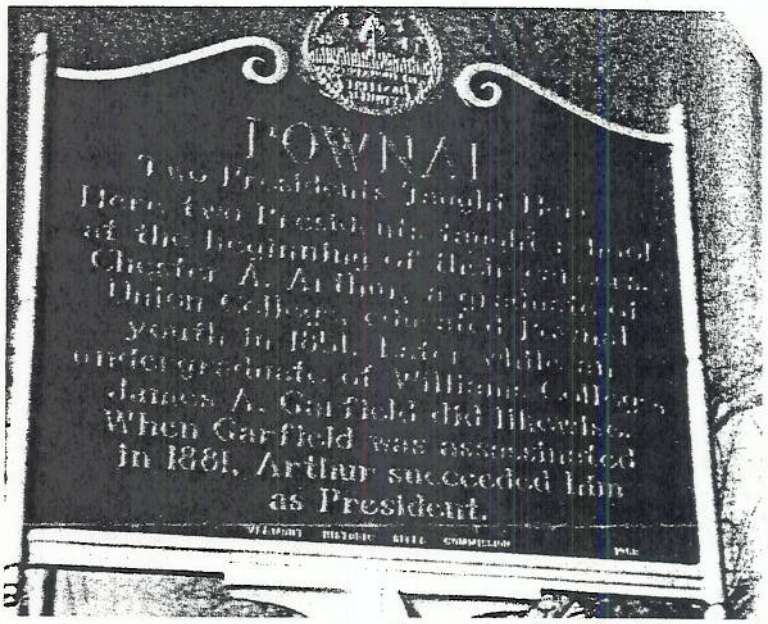


Photo courtesy of Victor E. Hill IV.

Garfield wrote to the presidents of Brown, Yale, and Williams Colleges, stating how much he knew and asking how long it would take him to finish his degree at each school. President Mark Hopkins of Williams replied, "If you come here, we shall be glad to do what we can for you," and more directly, "I can only say that if you come, we shall be glad to do for you what the circumstances will admit." Two other factors may have come into play. One is that the ranks of the junior class had just been significantly reduced by the mass expulsion of a bunch of its rowdier members following an incident apparently involving some spiked lemonade; having virtually no endowment or patronage at the time, the College could hardly afford to be overly selective. In addition, this was a time at which Hopkins took an interest in admitting "sincere yet rustic" men from the West to combat what he called "signs of effeteness" in his students. Garfield did decide to attend Williams, where he hoped to complete his studies in one year. On arrival at the beginning of the summer 1854, he was examined orally and was found highly proficient in Greek and Latin but unable to solve even simple geometry problems and was told to bring up his mathematics. Since Garfield was admitted as a junior, he would have had a deficiency in mathematics. The college catalog for those years shows a freshman requirement of algebra to the binomial theorem and Loomis' geometry, and a sophomore year of higher algebra and Loomis' analytical geometry and calculus, with no further mathematics requirement (or offerings) for upperclass students. Garfield, however, remained cheerful, as he recorded, "I recite…in Analytical Geometry and Conic Sections.... Privately I am bringing up the Trigonometries and Surveying and Navigation.... My mind seems unusually clear and vigorous in Mathematics, and I have considerable hope and faith in the future." Whatever he lacked in accomplishment, he made up for in confidence, an attribute he displayed throughout his life.

Williams marked the first time that Garfield experienced any sort of educational setback. In Ohio, which was really still frontier country, he had been teaching advanced courses, and his studies had always come easily to him. Indeed, at his first teaching appointment, he was accounted a wonder because he was the only instructor in two counties who could solve a problem requiring long division! Thus he arrived at the College with the expectation that he could study for nine weeks and enter as a senior. (It is important to realize that Garfield was 22 at the time, significantly older than most of the other students; his nickname among his peers was "Old Gar.") President Hopkins allowed him to attend the last few classes of the term

and Commencement; this experience, along with his oral entrance examination, enabled him to see that he was not as well prepared as he had believed.

Sadly, Garfield's diaries during his Williams years, 1854–56, are sparse, and information about what he learned about mathematics, presumably on his own, while a college student is entirely lacking. Like many students of limited means, he spent the long summer and winter vacations teaching in nearby communities. The winter months, in particular, were times when boys in their mid-teens (who could be spared then from farm work) were given to older male teachers such as Garfield. A historical marker in Pownal, Vermont (just over the Massachusetts state line), commemorates the teaching activity of both Garfield and his successor in the Presidency, Chester A. Arthur, in that community. Garfield was unusual in that, in addition to teaching, he filled in numerous times in the pulpits of churches of the Disciples of Christ and in conducting revival meetings, making him the only preacher to become a U.S. President.

In his second term at Williams, Garfield paid the bills by teaching two writing courses. The first grew out of a lecture engagement he had fulfilled in Pownal. The second took place in Poestenkill, New York, about 21 miles from Williamstown. He was due at college when the second course ended, but he was persuaded to stay an additional week to assist with a revival there. The local Disciples were so pleased that they gave him $20 and some new clothes to make up for his having missed the first three weeks of the term.

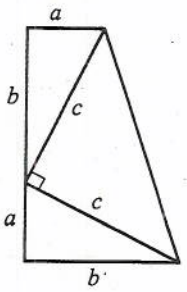
Garfield returned to teach at Hiram after graduating from Williams and married his former student Lucretia Rudolph on 11 November 1858. He enlisted in the Civil War and was made an officer, preparing himself by fashioning companies, troops, and soldiers out of wood and using them as physical examples to teach himself and his officers the principles of maneuvers and tactics—another example of his geometrical intuition at work. He maintained contacts with Williams alumni and, in fact, spent the day before his Presidential Inauguration attending a Williams reunion in Washington at which he also gave a speech—this caused him to stay up until 2:30 A.M. writing his inaugural address. (That inaugural was the first to be attended by the President's own mother.) It is ironic, in view of his longtime college connection, that Garfield was assassinated in 1881 en route to his 25th Williams College reunion.

Aside from the curiosity of the Pythagorean Theorem, Garfield did exercise mathematical skills in Congress. He served on the committee which oversaw the 1880 census, addressing the members of the American Social Science Association on the subject. Garfield also served (at his request) on the Ways and Means Committee, where he furthered his interest in the mathematics of finance. The same boy who had been "death on figgers" was entirely self-taught in these matters and is reported to have made something of a nuisance of himself on the committees with the tenacity of his mathematical interests.

Although Garfield was not a particularly strong student in college, he did have at least one remarkable ability, which he delighted to demonstrate: he could write in Latin with one hand and, simultaneously, in Greek with the other! This curiosity does reflect his extensive background in classical languages; in fact he and his wife routinely spoke Latin and Greek in their home. Another indication of Garfield's intellect is that he was admitted to the Bar while serving in Congress; he argued his first case ever before the Supreme Court—and won.

Garfield took his proof of the Pythagorean Theorem with him when he spoke at the Dartmouth College Chapel on 7 March 1876. He recorded in his diary, "After meeting [I] had a private conference of an hour with Professors

Garfield's Proof of the Pythagorean Theorem



Area of trapezoid

$\frac{1}{2}$ (sum of bases)(altitude)

$$= \tfrac{1}{2}(a+b)(a+b)$$
$$= \tfrac{1}{2}a^2 + ab + \tfrac{1}{2}b^2.$$

Sum of areas of three triangles

$$= \tfrac{1}{2}ab + \tfrac{1}{2}ab + \tfrac{1}{2}c^2$$
$$= ab + \tfrac{1}{2}c^2.$$

So $\frac{1}{2}a^2 + \frac{1}{2}b^2 = \frac{1}{2}c^2$ and $a^2 + b^2 = c^2$.

Quimby and Parker in reference to [ex-Senator James Wallis] Patterson's connection with the Credit Mobilier. I showed my solution of the *pons asinorum* to Professor Quimby, who said it was new and asked for a copy for publication in a Mathematical journal." (*Pons asinorum,* which is Latin for "bridge of fools," usually refers to Euclid's proof that the base angles of an isosceles triangle are equal.) Garfield's solution was published in the *New England Journal of Education* in 1876. The text begins:

> In a personal interview with Gen. James A. Garfield, Member of Congress from Ohio, we were shown the following demonstration of the pons asinorum, which he had hit upon in some mathematical amusements and discussions with other M.C.'s. We do not remember to have seen it before, and we think it something on which the members of both houses can unite without distinction of party.

Continued on p. 15.

Continued from p. 11.

The proof consists of two copies of the triangle, with their legs placed on a common straight line, as shown in the figure. The trapezoid is then completed by joining the remaining vertices of the triangles, computing the area of the trapezoid, and comparing it to the sum of the areas of the three triangles into which it is decomposed.

Adjoining the mirror reflection of this trapezoid, one obtains the square found in the ancient Chinese text, the *Arithmetic Classic of the Gnomon and the Circular Paths of Heaven*; however, there is no indication that Garfield or any of his colleagues in Congress had any knowledge of this or any similar historical proofs; indeed, what we have seen of Garfield's biography shows that his mathematical training, especially in geometry, was limited and largely self-taught. Thus he can be credited with some innate instinct and a worthy degree of geometrical curiosity, perhaps not often found in the halls of legislatures. ■

Acknowledgments

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For Further Reading

Read Allen Peskin's *Garfield: a Biography*, Kent State University Press, for more details of Garfield's life. *The Pythagorean Proposition* by Elisha S. Loomis, published by the National Council of Teachers of Mathematics, contains hundreds of proofs of the theorem: Garfield's is number 231. You can read about the *Arithmetic Classic of the Gnomon and the Circular Paths of Heaven* in David Burton's *The History of Mathematics: An Introduction.*