MATH 372: COMPLEX ANALYSIS: MWF 9 – 9:50am Stetson Court 109

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http://web.williams.edu/Mathematics/sjmiller/public_html/372Fa17/

TA Hours: Tuesdays and Thursdays 7-8pm, HW due Fridays

COURSE DESCRIPTION: The calculus of complex-valued functions turns out to have unexpected simplicity and power. As an example of simplicity, every complex-differentiable function is automatically infinitely differentiable and equals its Taylor series expansion! As examples of power, the residue calculus permits the computation of difficult integrals, and conformal mapping reduces physical problems on very general domains to problems on the round disc. The easiest proof of the Fundamental Theorem of Algebra, not to mention the first proof of the Prime Number Theorem, used complex analysis. **NOTE: this course will move at a good pace and will provide the complex analysis background graduate programs in mathematics desire. We will cover a lot of material and applications. Whenever possible we will prove all results and theorems, either in class or as part of the homework or reading.**

Format: lecture. Evaluation will be based primarily on homework, classwork, and exams.

Prerequisites: Mathematics 350 --**OR**-- permission of instructor. *No enrollment limit.*

CONTACTING ME: I'm in Bronfman 202 (if I'm there it's office hours). Email sjm1@williams.edu, or ***anonymously through ephsmath@gmail.com (password williams1793). Cell is 617-835-3982.***

OBJECTIVES: There are two main goals to this course: to explore complex analysis and see the connections between various subjects, and to learn problem solving skills. We will constantly emphasize the techniques we use to solve problems and prove theorems, as these are applicable to a wide range of problems in the sciences. For a fuller statement on the objectives, click the online link. This includes some fascinating videos with some thought provoking comments about what you should get out of your education.

GRADING / HW: Prepared for Class: 5%, Homework 15%, Midterms 40%, Final 40%. Homework is to be handed in on time, stapled and legible. *Late, messy or unstapled homework will not be graded.* I encourage you to work in small groups, but everyone must submit their own hw assignment. **Extra credit problems should not be included in the general homework, but handed in separately. Very little partial credit is given on these problems. There will be opportunities to do a project and present to the class. If people are interested in a weekly lunch to discuss advanced material, let me know.**

TEXTBOOK: Complex Analysis by Stein and Shakarchi (Princeton University Press, ISBN13: 978-0-691-11385-2). The introduction and chapter two are online.

SYLLABUS: My lecture notes are online. There will be the option of student choice of topics (either to be covered in the main lectures or in small groups, probably some physics such as the Heisenberg Uncertainty Principle, and possible a rigorous proof of the Central Limit Theorem). We will cover.

- Chapter 1: Preliminaries to Complex Analysis (includes review of Real Analysis and problems you should be able to do to succeed in the course).
- Chapter 2: Cauchy's theorem and its applications
- Chapter 3: Meromorphic functions and the logarithm
- Chapter 5: Entire functions (for now plan is to just do Sections 3 and 4)
- Chapter 8: Conformal mappings (for now the plan is not to do Section 4)

- **Week 1: September 8, 2017:** Read: Chapter 1 and my online notes, review your real analysis. Useful resources are the online real analysis book. Homework: To be **emailed** to me by noon on Sunday, September 9: Email me a short note on what you want to get out of this course, and what lesson you learned from the graduation speech. Full credit, 20/20, if you answer both questions on time.

Objectives for Math 372: Complex Analysis: *The following is entirely optional, but describes some of my thoughts about the course, ranging from why it is structured the way it is to what I want you to get out of this course in particular and your education in general. As with all material in the course, I'm happy to discuss anything here in greater detail; I hope this will help start a dialogue about your undergraduate education (as well as my continuing education!).* ***Feel free to contact me by email about anything, either directly or anonymously through ephsmath@gmail.com (password 1793williams).***

When I was in high school, I remember the Boston Globe ran an article where they asked numerous 'famous' people in the state: what 10 books should **every** high school student read? The answers were, for the most part, disappointing. The English professor had ten literary selections, ranging from some Shakespeare and Milton to stuff I can no longer remember. Most of the others had lists greatly biased towards the small part of the world they studied; very few had balanced lists that would help prepare the general student for the world (Governor Weld was one of the few who did).

Complex analysis is one of the courses graduate programs in mathematics (and some other fields) love to see. It is not required, as some people do not have the opportunity to take it as undergraduates, but it is strongly recommended to take it if at all possible.The results are used in a variety of fields, ranging from my personal favorite number theory to dynamics to physics to engineering.... We will explore the basics of the subject and several applications, including the Riemann zeta function and the Gamma function, as well as other examples to be determined by class interest.

I **strongly** urge you to view the following clip on YouTube: Did You Know (2009 version). Some of the more interesting stats / quotes from this video:

- We're preparing students for jobs that don't exist yet using technologies not yet invented to solve problems we don't know are problems yet.
- We are living in exponential times:
  - 31 billion searches on google each month now; it was 2.7 billion in 2006.
  - The first text message was in 1992; more are sent today than the population of the planet.
  - Number of internet devices: 1,000 in 1984, 1,000,000 in 1992, 1,000,000,000 in 2008;
  - Estimated that 4 exabytes (4 x 10^{19} bytes) of unique information generated this year, more than generated in the past 5000 years!
  - Amount of new technical information doubles every two years (hey juniors: a lot of what you learned, if you're in a technical school, as a freshman is outdated!)
- Click here for another video in the series; contains a lot of the same info, some interesting facts for educations, but no cool (or uncool) soundtrack.

  Another great clip to listen to is the TED lecture of Malcolm Gladwell on spaghetti sauce. The main point of this lecture is
- It is important to ask the right question; what you think is the right question frequently isn't. I won't do the lecture justice by summarizing it, so I'll give the following tantalizing tidbit: this surprising question led to Prego making $600 million in 10 years on extra chunky spaghetti sauce.
- There are lots of great clips on TED; another one of my favorites is Dan Pink on Motivation. Some very interesting observations here on how to create an optimal environment for creativity to flourish.

What does this have to do with our class? It's hard to predict what'll be useful in your career(**s**) after Williams (save for the few who go off to grad school to study number theory, where I can do a pretty good job). It's imperative that you become problem solvers. The content of a course matters; you need to learn the language, the basic facts, the key theorems, etc. But, at least as important, you need to learn how to use these on 'new' problems.

One of my biggest epiphanies as an educator was when I prepared my lecture notes for Math 209 (differential equations). This was the first class I'd ever taught as a college professor which I'd actually taken as an undergrad. I was looking through my old course notes, trying to decide what to include, when I noticed that we did the Bessel equation and function when I was a student. I was shocked; I had no memory of having done this, but I use the Bessel function in about a quarter of my number theory papers (in fact, I'm using it crucially in a paper with a SMALL student right now). What's the takeaway? You're going to forget much of the material you learn. That's to be expected. Hopefully you'll be able to re-learn it as needed / you'll know where to go to read more about it / you'll have some familiarity that such results exist. I had to relearn the Bessel function in grad school (and did, it's not that bad). What is more permanent is the techniques and methods. These are the things you'll use again and again. They can range from learning how to multiply by 1 or add 0 (two of the most difficult things to do well in mathematics!) to how to count something two ways to how to model the key features of a problem.

One of the goals of this course is to help you become problem solvers. The problems will come from complex analysis, but the methods and techniques, the mindset, should hopefully be applicable to a variety of problems. If you are taking an intro calculus class andSection 3.2 is on the Chain Rule, it's a safe bet that you should use the chain rule to solve problems from that section. The real world (or advanced academia) is not like that -- you frequently do not know what the 'right' way is to attack a problem (especially open problems that have stumped people for years). This is one of the reasons why I want you to create (and if possible solve!) your own homework problems. The act of creation is a huge part of research. **Most** math papers are **mostly** trivial (or, as I often say, trained monkey work). What does this mean? It means that over 95% of most papers is just straightforward manipulation of previous results (the further you go in math, the more things become straightforward). The most important parts of papers are usually the following two items: (1) asking the right question; (2) coming up with a novel way to attack a problem. Often once the question is asked and the method chosen, the paper writes itself; however, it is very hard at times to ask the right question, or to see a good way to attack it. (As an example, when I taught at Princeton years ago I wrote a handout for my students on how to prove the Fibonacci numbers satisfy Benford's law of digit bias. I decided to try and publish it, and did some research. I found a paper from the 1970s that was almost identical to mine -- basically same theorems and lemmas in the same order! This isn't too surprising, as once the question was asked, this truly was the most 'natural' way to go.)

Finally, there are remarkable connections between what seem at first very disparate branches of learning; if you are one of the first people to see such a connection, you have the potential of making a real breakthrough. I **strongly urge** you to tell me what you're interested in. I'll see if I can work it into the course (either in the lectures proper or in the additional comments). The more you explore, the more likely it is you can make one of these great connections. I've been fortunate enough to make some connections between nuclear physics and baseball, and between number theory and tax fraud. Because of this I've had a private tour of Petco Park (where the San Diego Padres play) and given a talk at the Boston headquarters of the IRS (with district attorneys, auditors, and secret service agents in the audience) and been interviewed by the Wall Street Journal about fraud in the recent Iranian elections.