Math 383: Complex Analysis: Fall ‘21 (Williams)

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Homepage:
https://web.williams.edu/Mathematics/sjmiller/public_html/383Fa21/

Lecture 01: 9-10-21:
https://youtu.be/8mrJvqbCqB8

Defn of the deriv: $f'(x)$

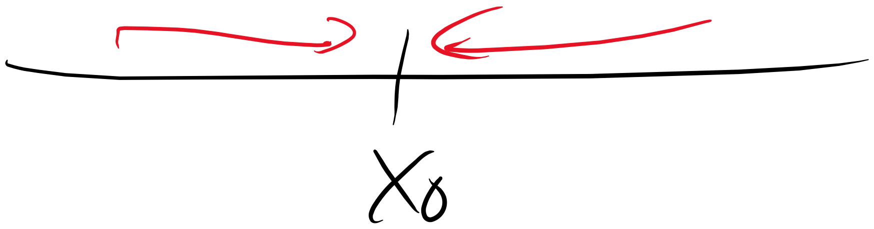
$$f'(x_0) = \lim_{h \to 0} \frac{f(x_0+h) - f(x)}{h}$$

$$= \lim_{x \to x_0} \frac{f(x) - f(x_0)}{x - x_0} \Rightarrow \lim_{x \to x_0} \frac{f(x) - f(x_0) - f'(x_0)(x - x_0)}{x - x_0} = 0$$

tangent line does well

$$\lim_{\vec{x} \to \vec{x}_0} \frac{|f(\vec{x}) - f(\vec{x}_0) - (\nabla f)(\vec{x}_0)(\vec{x} - \vec{x}_0)|}{\|\vec{x} - \vec{x}_0\|}$$

Paths



Real

Complex

Complex deriv:

$$f'(z_0) = \lim_{h\to 0} \frac{f(z_0+h) - f(z_0)}{h}$$

1) $f(z) = c \longrightarrow \lim_{h\to 0} \frac{c-c}{h} = \lim_{h\to 0} 0 = 0$

2) $f(z) = z \implies \lim_{h\to 0} \frac{(z_0+h) - z_0}{h} = \lim_{h\to 0} \frac{h}{h} = 1$

Complex conj: $z = x+iy$ then $\overline{z} = x - iy$, $i = \sqrt{-1}$

3) $f(z) = \overline{z} \longrightarrow \lim_{h\to 0} \frac{\overline{z_0+h} - \overline{z_0}}{h} = \lim_{h\to 0} \overline{h}/h = \begin{cases} 1 & h \in \mathbb{R} \\ -1 & h \in i\mathbb{R} \end{cases}$

Not diff!

$$z = x + iy \qquad \bar{z} = x - iy$$

$$\frac{z + \bar{z}}{2} = x \qquad \frac{z - \bar{z}}{2i} = y$$

$$(z, \bar{z}) \longleftrightarrow (x, y)$$

Property	real diff	Complex diff
f is ∞ diff	NO $x^3\sin(1/x)$	YES
fn equals Taylor Series	NO e^{-1/x^2}	YES
Can be bounded and not constant	YES	NO

$f, g, \ldots$ are diff fns