MATH 389: Advanced Analysis: MWF 9 – 9:50am: Bronfman B34

Professor Steven Miller (sjm1@williams.edu), 202 Bronfman Science Center (413-597-3293)
My office hours: TBD and whenever I'm in my office (click here for my schedule)

COURSE DESCRIPTION: This is a **post-core** 300 level course. It is designed to give you an excellent feel for what grad school is like, as well as maximize my ability to write a strong letter for you to get into a top program. in addition to learning topics in advanced analysis (with a special emphasis on number theory), many of the assignments involve broader impact activities. These range from reviewing papers for MathSciNet, refereeing papers for research journals, working with me on a book (on a centennial list of problems commemorating the 100th anniversary of the math honor society), ... It is likely that there will be class presentations in addition to homeworks and exams.
Format: Evaluation will be based primarily on homework, presentations and write-ups, and scholarship.

CONTACTING ME: You can reach me in Bronfman 202 (if I'm there it's office hours), email sjm1@williams.edu, or ***anonymously through ephsmath@gmail.com (password 011235813, the first eight Fibonacci numbers).***

OBJECTIVES: The goal is prepare you for graduate school (not just in math), and to help you decide if that is a path you want to take. Also, of course, to learn some advanced analysis and applications!

TEXTBOOK/SYLLABUS: Miller and Takloo-Bighash's `An Invitation to Modern Number Theory', plus numerous supplemental material.

GRADING POLICY: Homework: 20%, Midterm 20%, Final / Research Project 20%, Class Presentation: 20%, Broader Impact Projects: 20%

COURSE DISCLAIMER: I may occasionally say things such as `Probability is one of the most useful courses you can take' or 'If you know probability, stats and a programming language then you'll always be able to find employment'. I should say `you should always be able to find employment', as nothing is certain. Thus, please consider yourself warned and while you may savor the thought of suing me and/or Williams College, be advised against this! I'm saying this because of the recent lawsuit of a graduate who was upset that she didn't have a job, and sued her school!

- **Week 1: Sept 1 to Sept 5, 2014**
  - Start exploring mathematical software (R, Mathematica). Learn LaTeX. I have handouts and videos: http://web.williams.edu/Mathematics/sjmiller/public_html/math/handouts/latex.htm
  - Homework: To be emailed to me by 8pm on Sunday, September 7: Email me a short note (a paragraph suffices) on what you want to get out of this course, and what lesson you learned from the graduation speech by Uslan (http://www.graduationwisdom.com/speeches/0018-uslan.htm). Full credit, 20/20, so long as you answer both questions on time.
  - Homework: To be done by start of class on Monday, September 8: Read the handout on Benford's law (Chapter 1 of the book I'm editing). If you do not know probability, read Chapter 8 of the class textbook (Invitation to Modern Number Theory); note this is how grad school operates: if you don't know something, you need to pick it up quickly. You should make sure you get up to page 206 at least, and the rest can wait till Wednesday. Also read Sections 11.1 and 11.2 of the class textbook (though feel free to keep reading Chapter 11, as that will be our first unit).

Objectives for Math 389: Advanced Analysis

The following is entirely optional, but describes some of my thoughts about the course, ranging from why it is structured the way it is to what I want you to get out of this course in particular and your education in general. When I was in high school, I remember the Boston Globe ran an article where they asked numerous 'famous' people in the state: what 10 books should **every** high school student read? The answers were, for the most part, disappointing. The English professor had ten literary selections, ranging from some Shakespeare and Milton to stuff I can no longer remember. Most of the others had lists greatly biased towards the small part of the world they studied; very few had balanced lists that would help prepare the general student for the world (Governor Weld was one of the few who did).

I **strongly** urge you to view the following clip on YouTube: Did You Know (2009 version). Some of the more interesting statistics / quotes:

- Preparing students for jobs that don't exist using technologies that haven't been invented to solve problems we don't know are problems yet.
- We are living in exponential times:
  - 31 billion searches on google each month now; it was 2.7 billion in 2006.
  - The first text message was in 1992; more are sent today than the population of the planet.
  - Number of internet devices: 1,000 in 1984, 1,000,000 in 1992, 1,000,000,000 in 2008;
  - Estimated that 4 exabytes (4 x 10^{19} bytes) of unique information will be generated this year, which is more than has been generated in the past 5000 years!
  - Amount of new technical information doubles every two years.

Another great clip to listen to is the TED lecture of Malcolm Gladwell on spaghetti sauce. The main point of this lecture is

- Important to ask the right question; what you think is the right question frequently isn't. I won't do the lecture justice by summarizing, so here's a tantalizing tidbit: this surprising question led to Prego making $600 million in 10 years on extra chunky spaghetti sauce.
- There are lots of great clips on TED; another one of my favorites is Dan Pink on Motivation. Some very interesting observations here on how to create an optimal environment for creativity to flourish.

What does all this have to do with our class? The point is that it is hard to predict what will be useful to most of you in your career(**s**) after Williams (save for the few who go off to grad school to study number theory, where I can do a pretty good job predicting). It is imperative that you learn to be problem solvers. The content of a course matters; you need to learn the language, the basic facts, the key theorems, et cetera. But, at least as important, you need to learn how to use these on 'new' problems.

One of my biggest epiphanies as an educator was when I prepared my lecture notes for Math 209 (differential equations). This was the first class I'd ever taught as a college professor which I'd actually taken as an undergrad. I was looking through my old course notes, trying to decide what to include, when I noticed that we did the Bessel equation and function when I was a student. I was shocked; I had no memory of having done this, but I use the Bessel function in about a quarter of my number theory papers. What's the takeaway? You're going to forget much of the material you learn. That's to be expected. Hopefully you'll be able to re-learn it as needed / you'll know where to go to read more about it / you'll have some familiarity that such results exist. I had to relearn the Bessel function in grad school (and did, it's not that bad). What is more permanent is the techniques and methods. These are the things you'll use again and again. They can range from learning how to multiply by 1 or add 0 (two of the most difficult things to do well in mathematics!) to how to count something two ways to how to model the key features of a problem.

One of the goals of this course is to help you become problem solvers. The problems will come analysis and linear algebra, but the methods and techniques, the mindset, should hopefully be applicable to a variety of problems. If you are taking an intro calculus class and Section 3.2 is on the Chain Rule, it's a safe bet that you should use the chain rule to solve problems from that section. The real world (or advanced academia) is not like that -- you frequently do not know what the 'right' way is to attack a problem (especially open problems that have stumped people for years). This is one of the reasons why I want you to create (and if possible solve!) your own homework problems. The act of creation is a huge part of research. **Most** math papers are **mostly** trivial (or, as I often say, trained monkey work). What does this mean? It means that over 95% of most papers is just straightforward manipulation of previous results (the further you go in math, the more things become straightforward). The most important parts of papers are usually the following two items: (1) asking the right question; (2) coming up with a novel way to attack a problem. Often once the question is asked and the method chosen, the paper writes itself; however, it is very hard at times to ask the right question, or to see a good way to attack it. (As an example, when I taught at Princeton years ago I wrote a handout for my students on how to prove the Fibonacci numbers satisfy Benford's law of digit bias. I decided to try and publish it, and did some research. I found a paper from the 1970s that was almost identical to mine -- basically same theorems and lemmas in the same order! This isn't too surprising, as once the question was asked, this truly was the most 'natural' way to go.)

There are remarkable connections between what seem at first very disparate branches of learning; if you are one of the first people to see such a connection, you have the potential of making a real breakthrough. I **strongly urge** you to tell me what you're interested in. I'll see if I can work it into the course (either in the lectures proper or in the additional comments). The more you explore, the more likely it is you can make one of these great connections. I've been fortunate enough to make some connections between nuclear physics and baseball, and between number theory and tax fraud. Because of this I've had a private tour of Petco Park (where the San Diego Padres play) and given a talk at the Boston headquarters of the IRS (with district attorneys, auditors, and secret service agents in the audience) and been interviewed by the Wall Street Journal about fraud in the recent Iranian elections.

Finally, here is a great website with 10 excellent commencement speeches. It's worth the time reading these; I particularly liked the one by Uslan (on how it's not enough to just have a good idea, but how to get noticed).