

PLOTS FROM RANDOM MATRIX THEORY

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1. PLOTS FROM CHAPTERS 15 – 17 OF AN INVITATION TO MODERN NUMBER THEORY

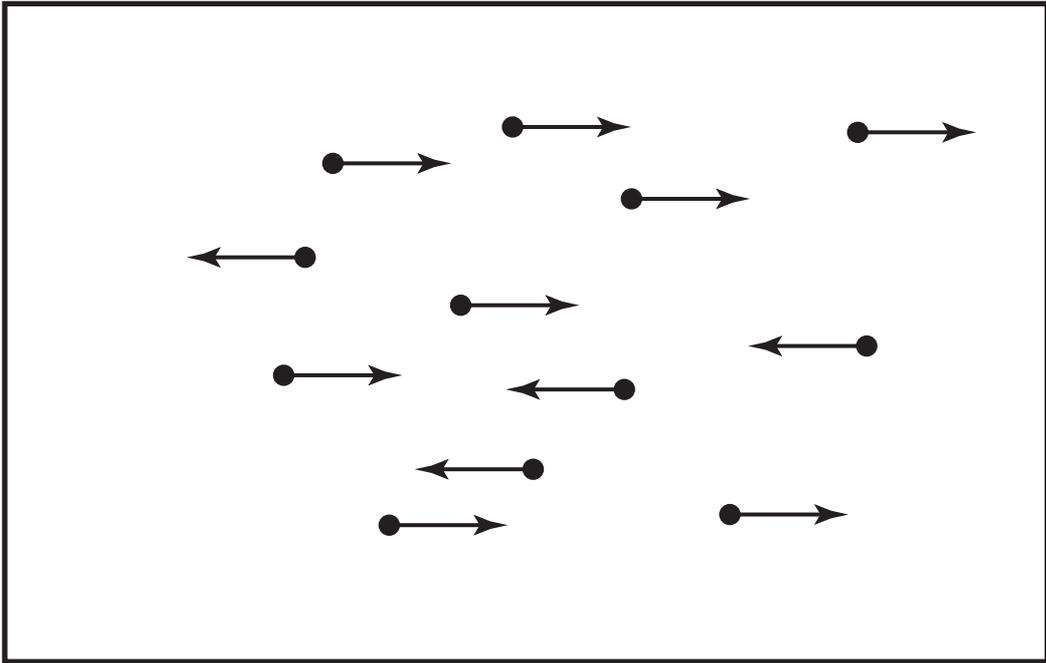


FIGURE 1. Molecules in a box

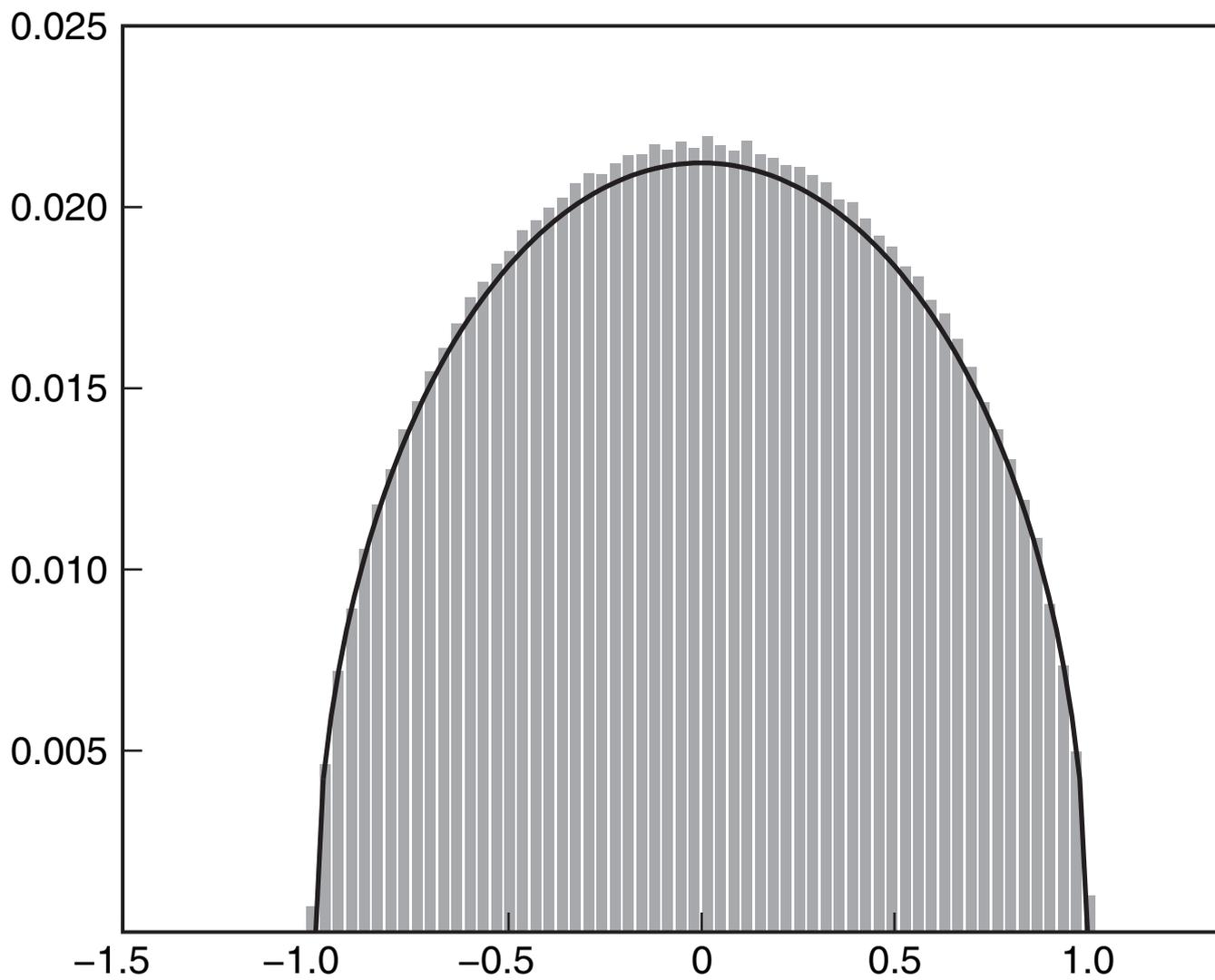


FIGURE 2. Distribution of eigenvalues: 500 Gaussian matrices (400×400)

First we look at the density of eigenvalues when p is the standard Gaussian, $p(x) = \frac{1}{\sqrt{2\pi}} e^{-x^2/2}$. In Figure 2 we calculate the density of eigenvalues for 500 such matrices (400×400), and note a great agreement with the semi-circle.

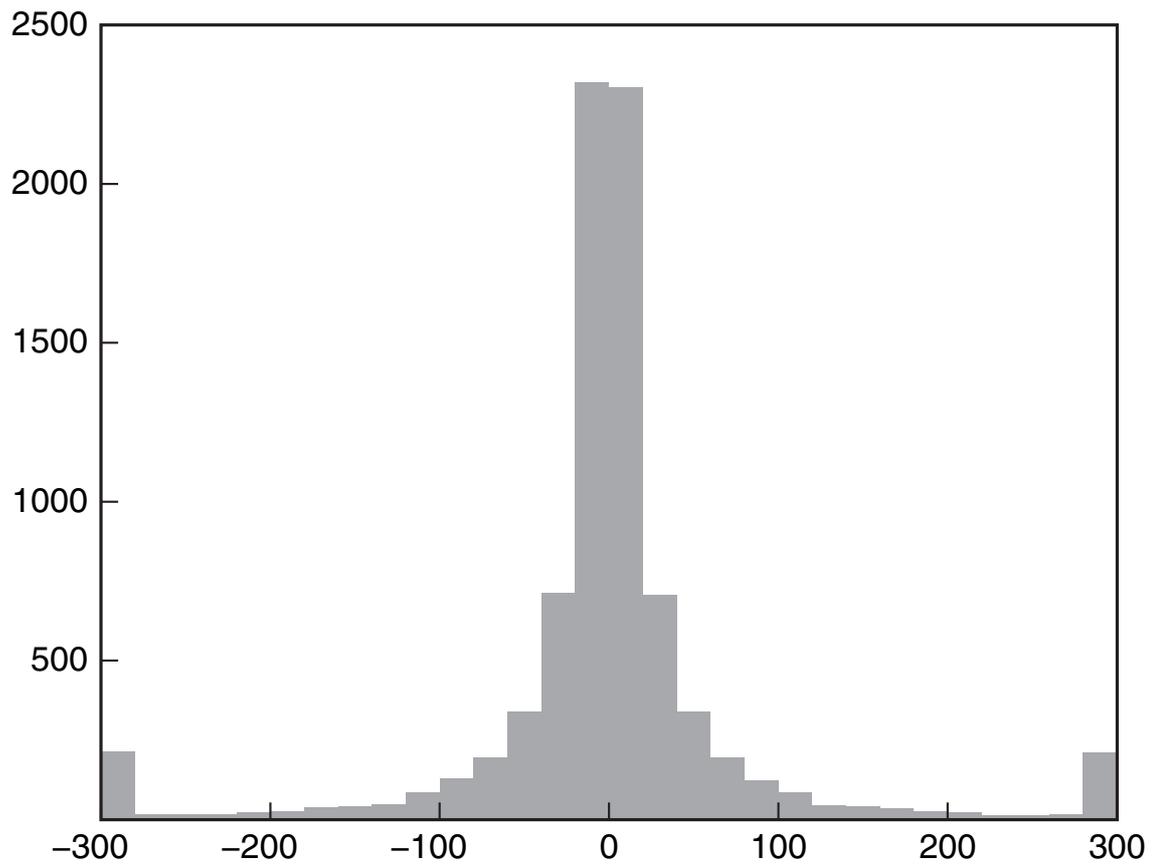


FIGURE 3. Distribution of eigenvalues: 5000 Cauchy matrices (300×300)

What about a density where the higher moments are infinite? Consider the Cauchy distribution,

$$p(x) = \frac{1}{\pi(1+x^2)}. \quad (1.1)$$

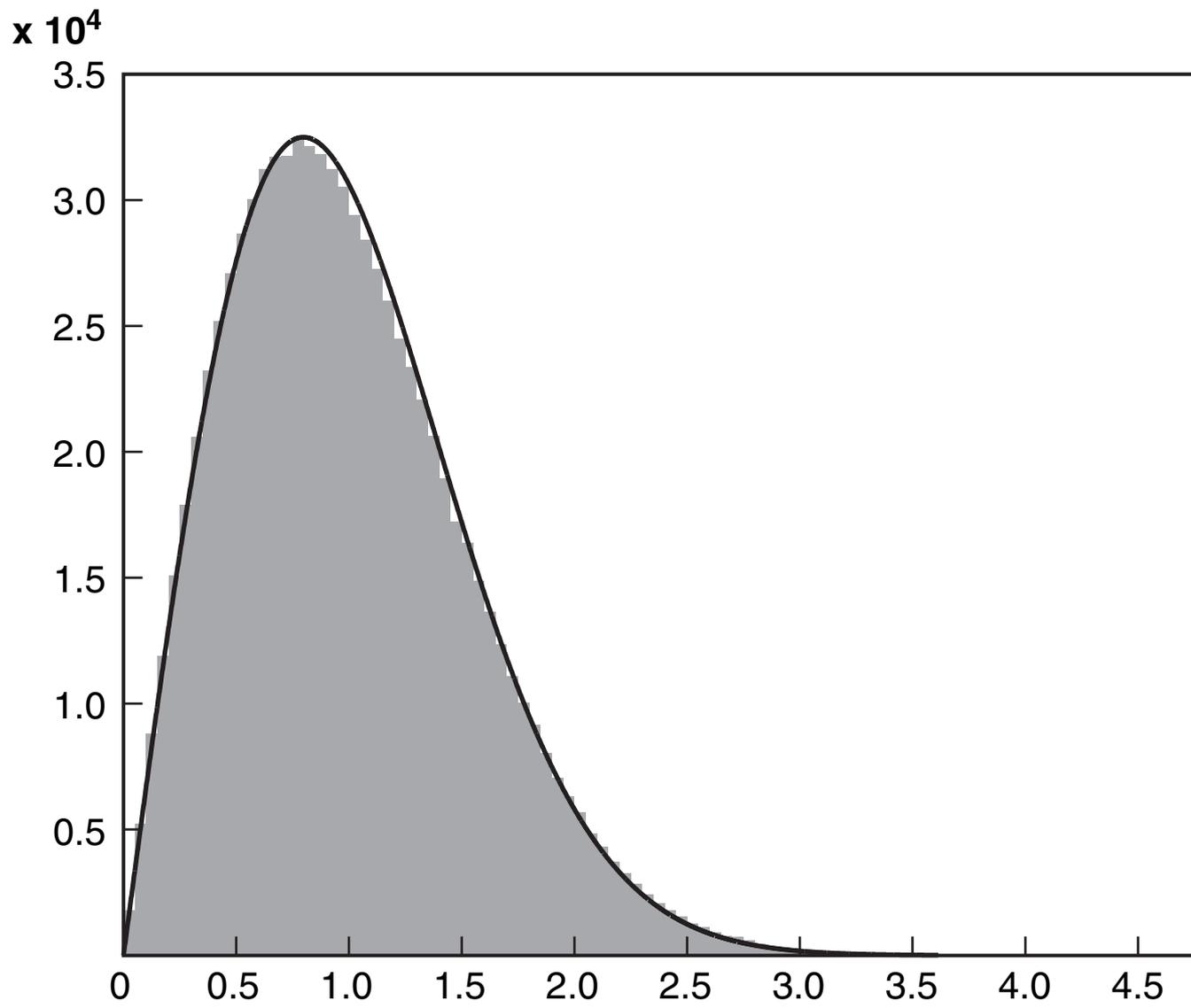


FIGURE 4. The local spacings of the central three-fifths of the eigenvalues of 5000 matrices (300×300) whose entries are drawn from the Uniform distribution on $[-1, 1]$

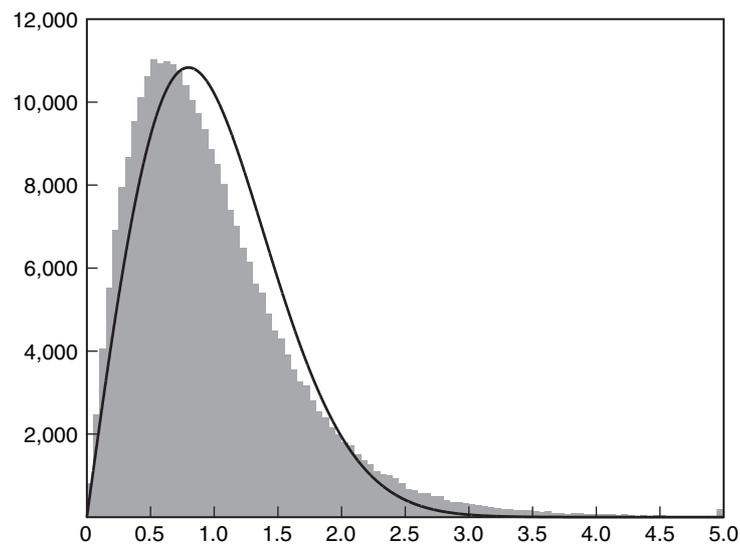


FIGURE 5. The local spacings of the central three-fifths of the eigenvalues of 5000 matrices (100×100) whose entries are drawn from the Cauchy distribution

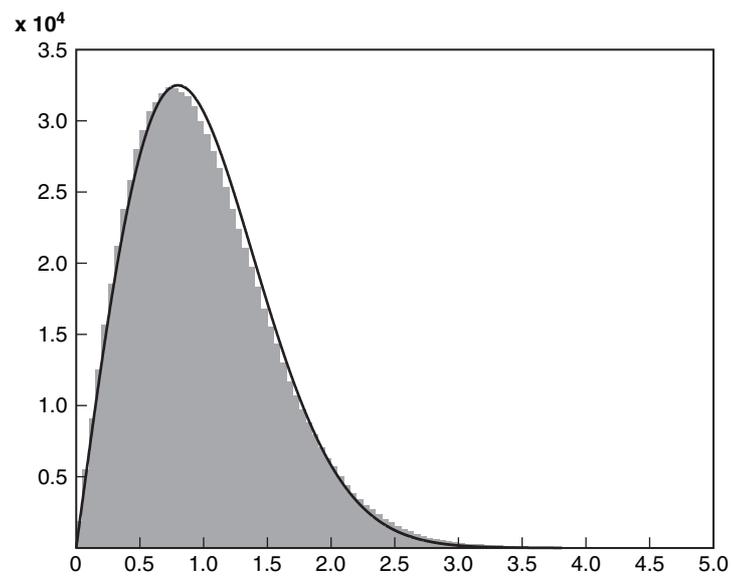


FIGURE 6. The local spacings of the central three-fifths of the eigenvalues of 5000 matrices (300×300) whose entries are drawn from the Cauchy distribution

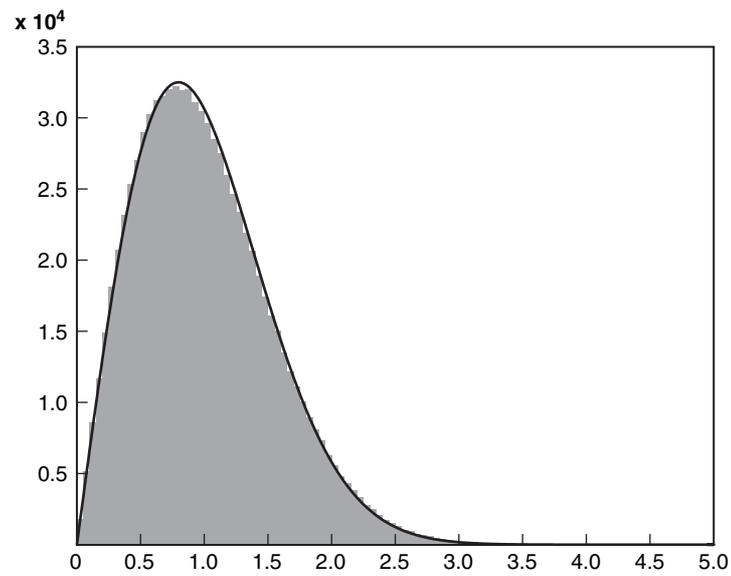


FIGURE 7. The local spacings of the central three-fifths of the eigenvalues of 5000 matrices (300×300) whose entries are drawn from the Poisson distribution ($\lambda = 5$)

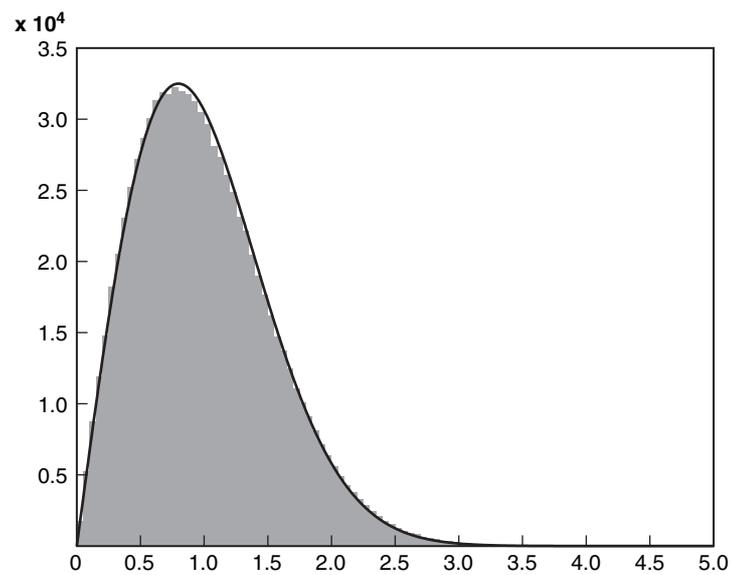


FIGURE 8. The local spacings of the central three-fifths of the eigenvalues of 5000 matrices (300×300) whose entries are drawn from the Poisson distribution ($\lambda = 20$)

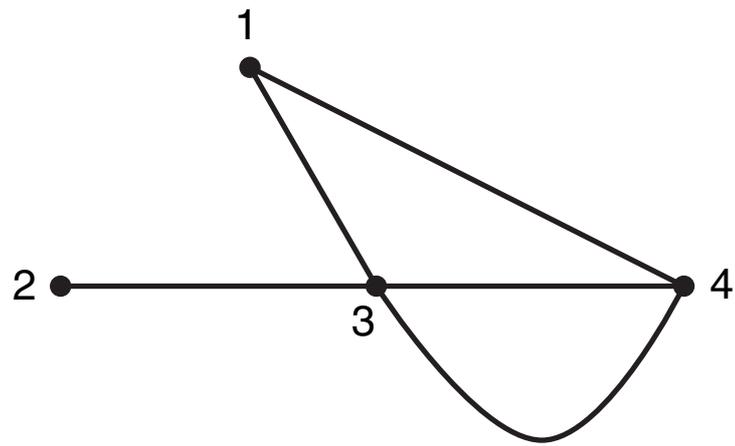


FIGURE 9. A typical graph

The degrees of vertices are 2, 1, 4 and 3, and vertices 3 and 4 are connected with two edges.

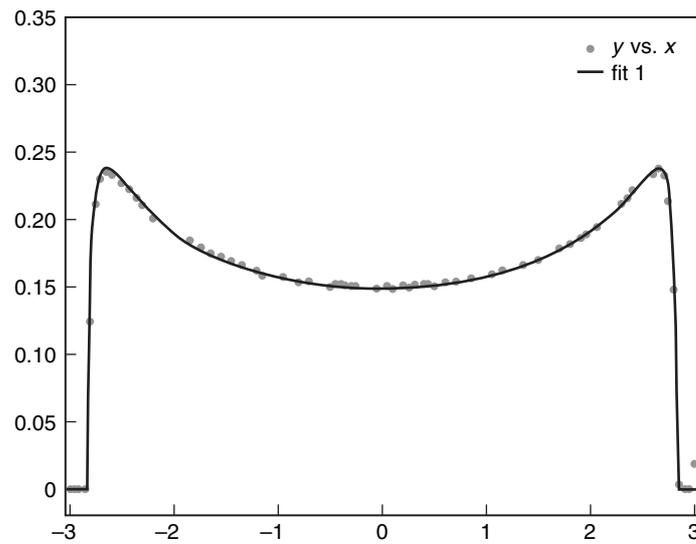


FIGURE 10. Comparison between theory (solid line) and experiment (dots) for 1000 eigenvalues of 3-regular graphs (120 bins in the histogram)

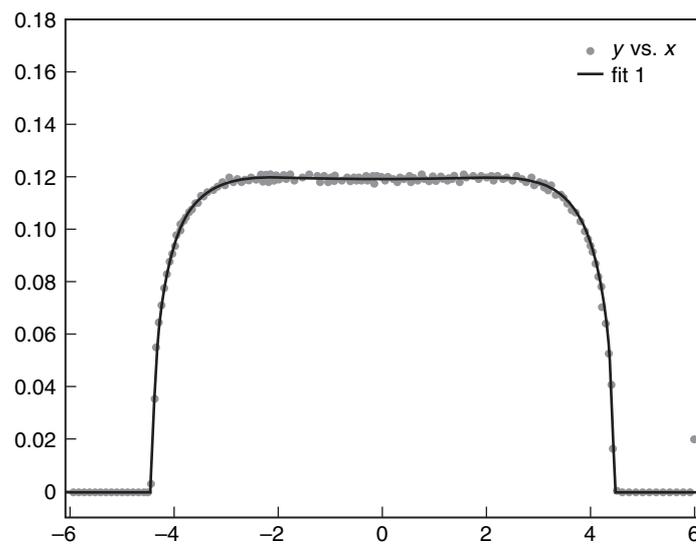


FIGURE 11. Comparison between theory (solid line) and experiment (dots) for 1000 eigenvalues of 6-regular graphs (240 bins in the histogram)

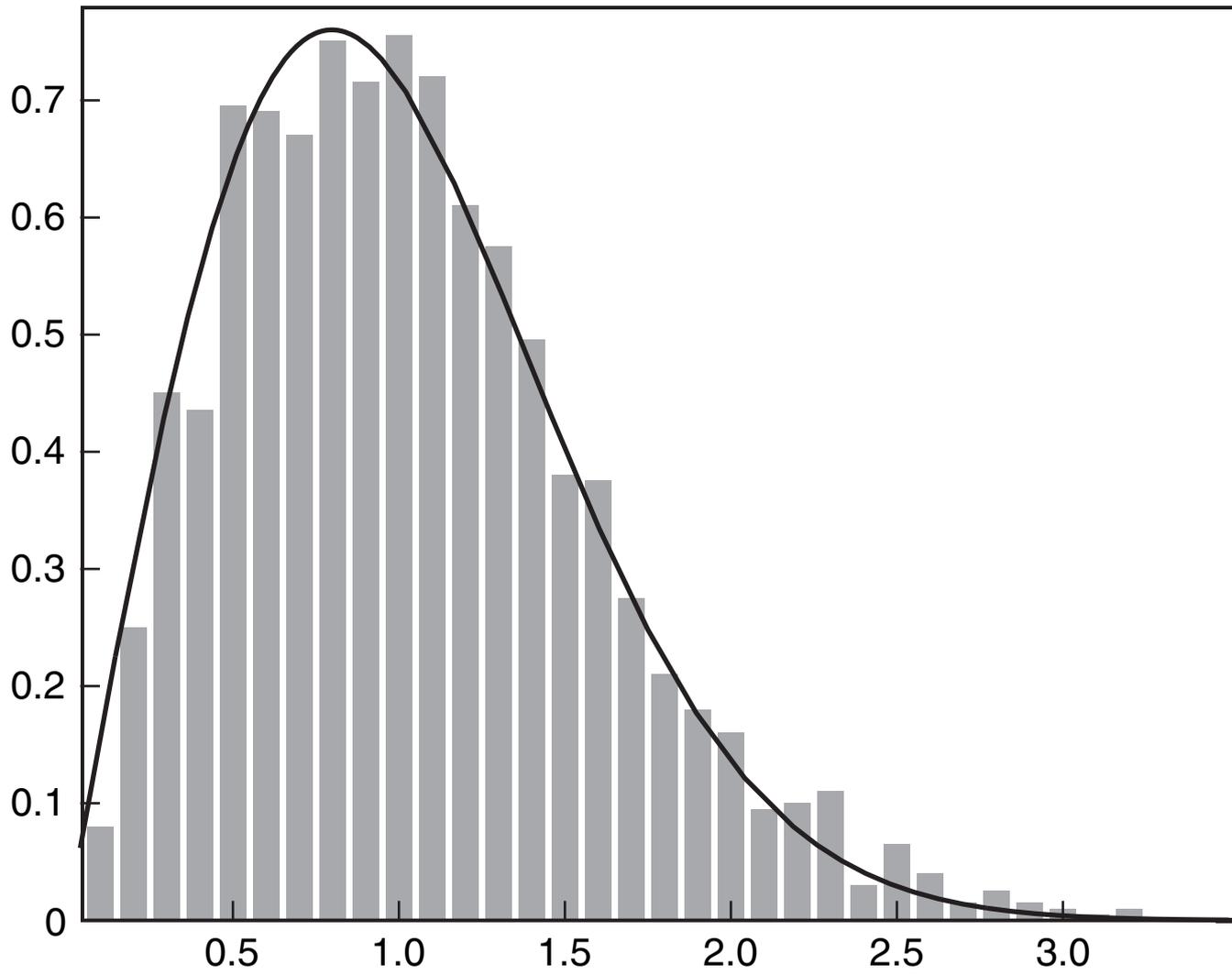


FIGURE 12. 3-regular, 2000 vertices (graph courtesy of [?])

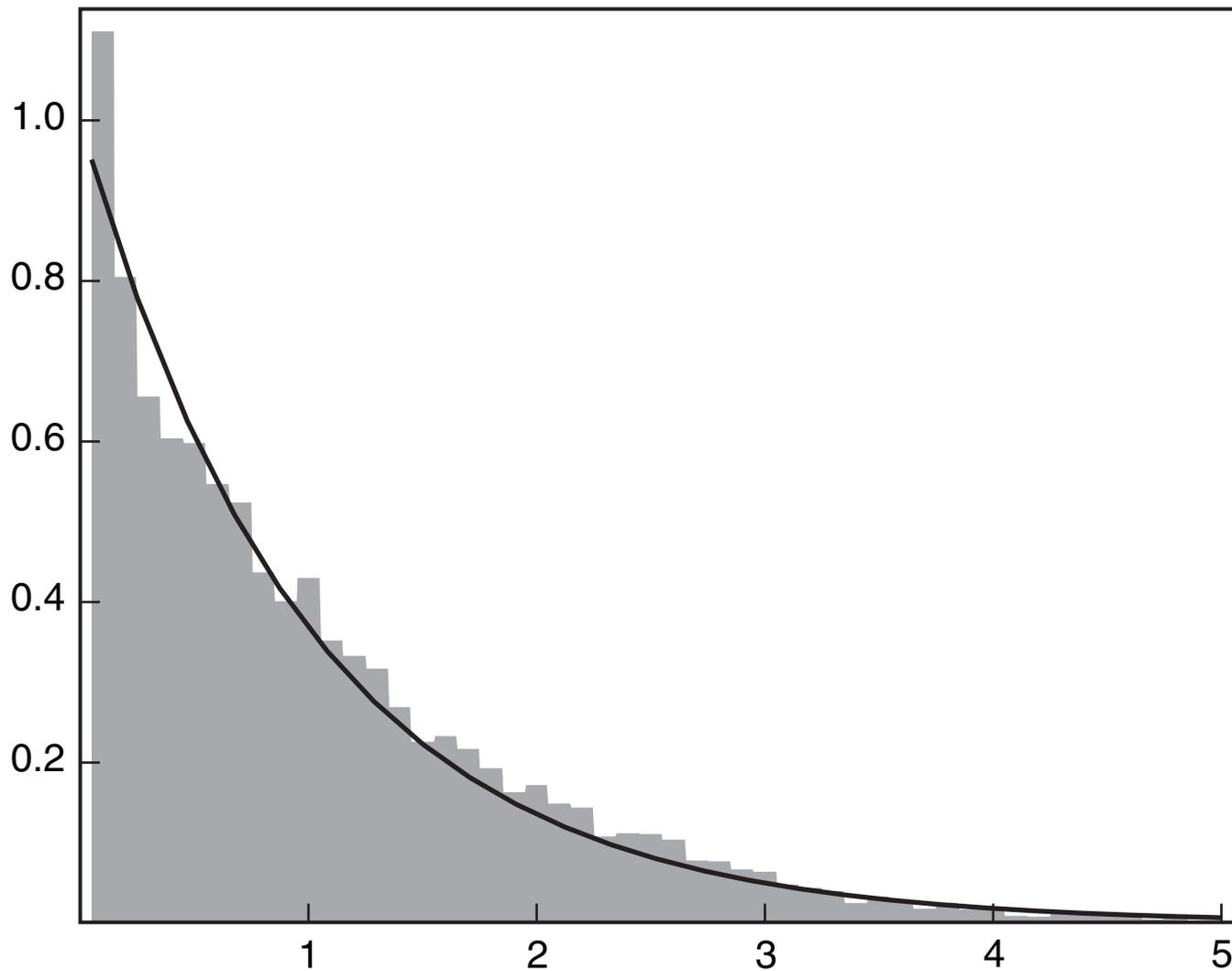


FIGURE 13. Spacings between normalized eigenvalues from 1000 Real Symmetric Toeplitz matrices (1000×1000)

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