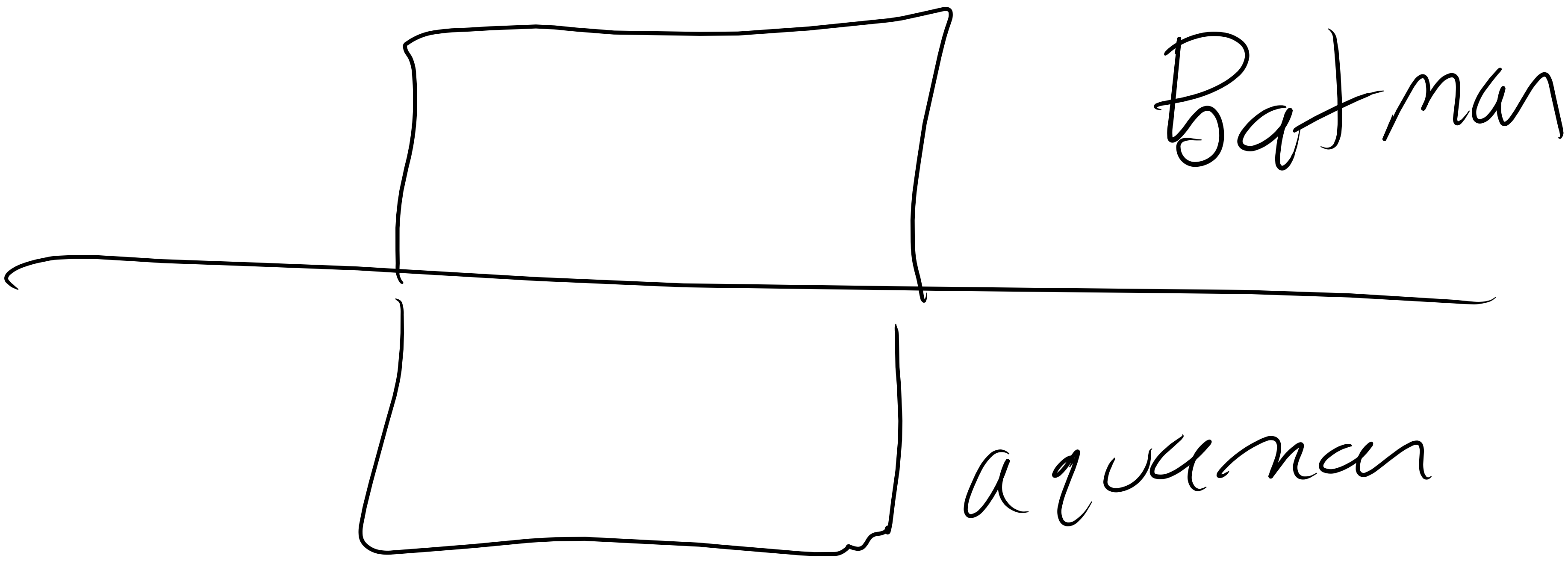
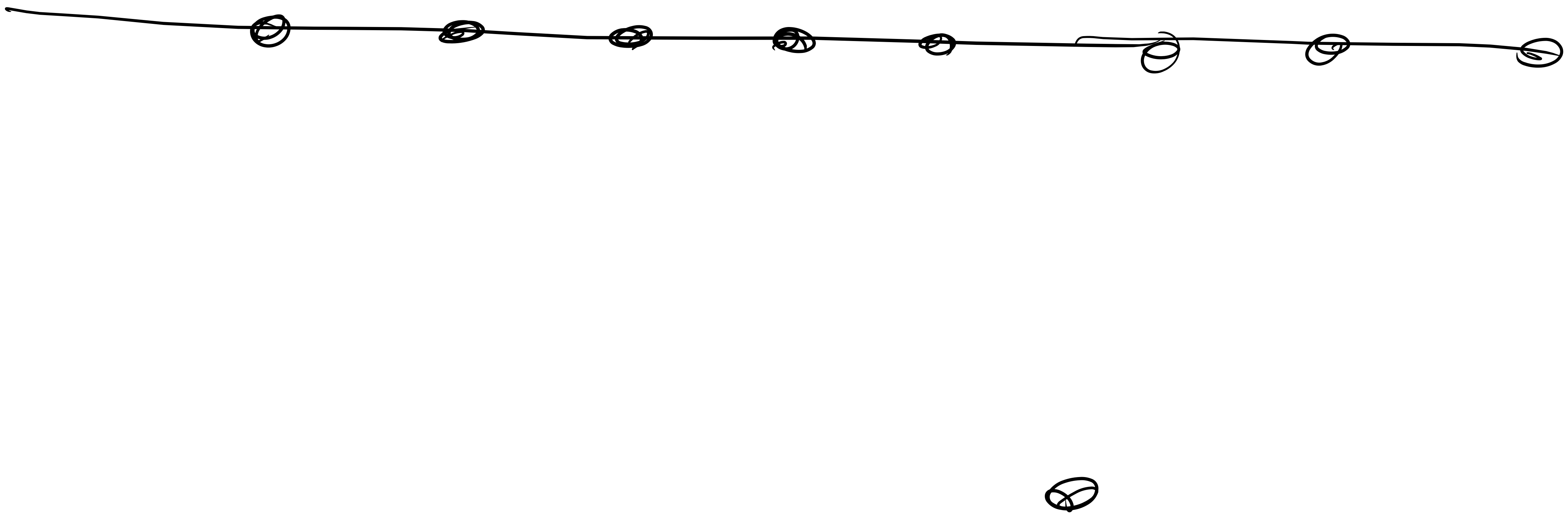
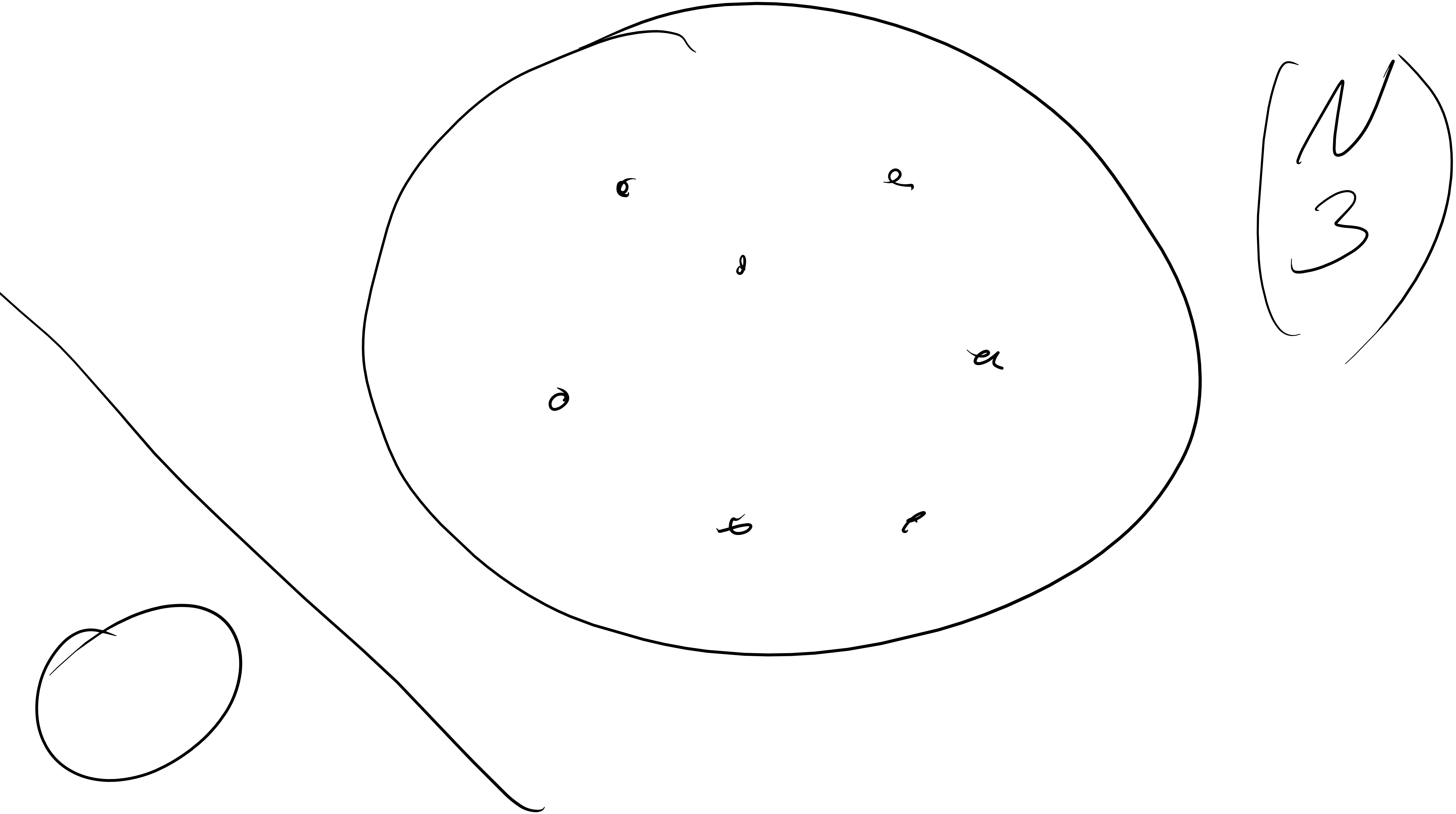
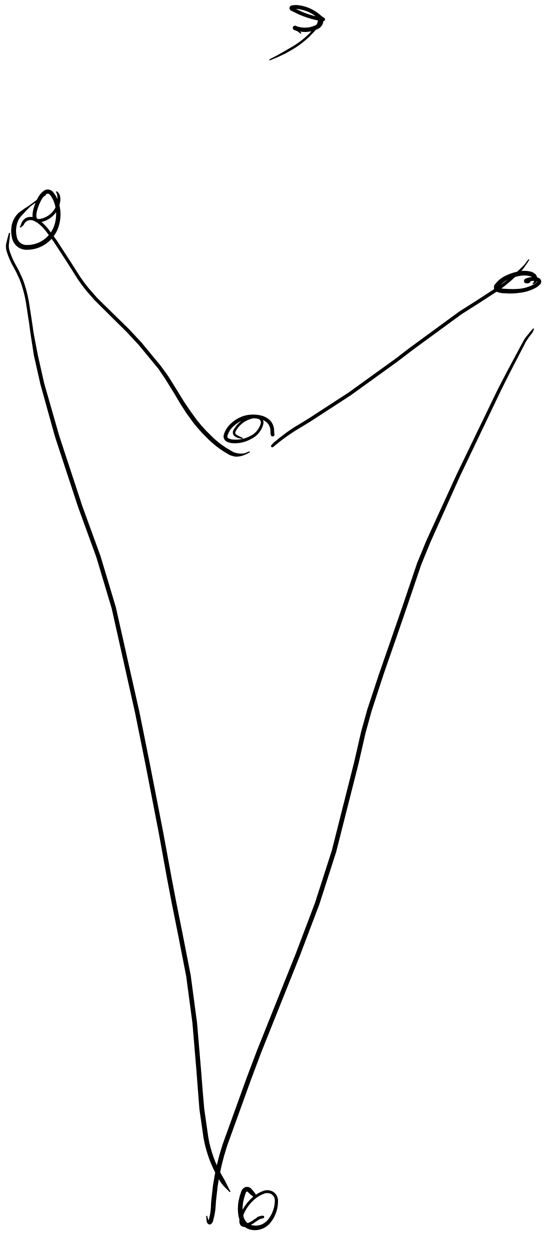
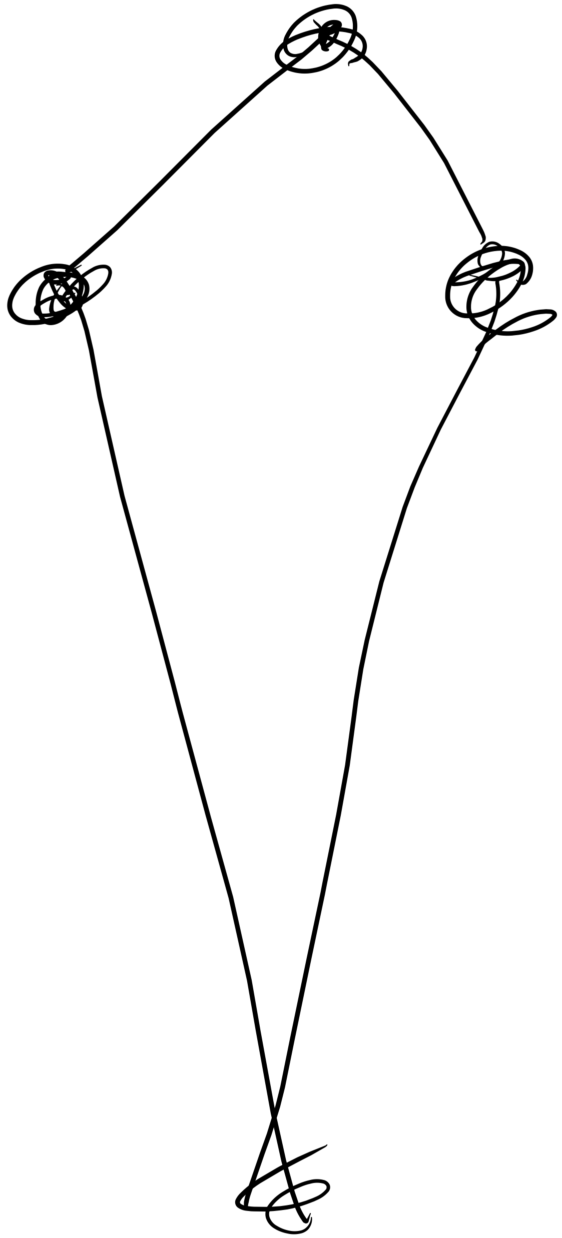


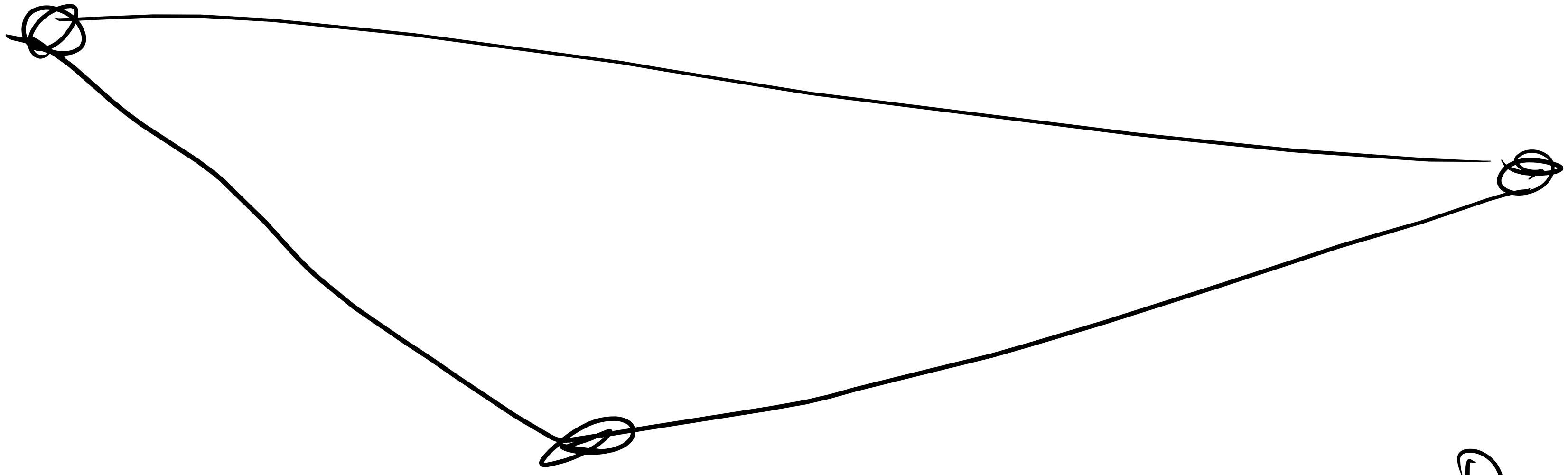
Batman
aquaman




N
3

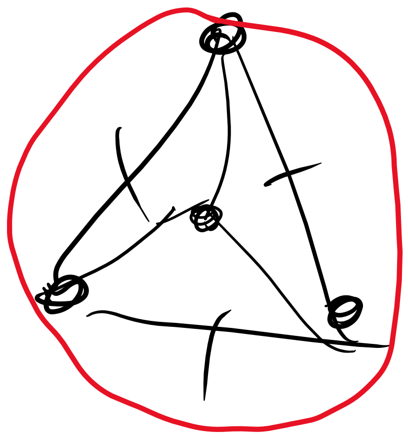


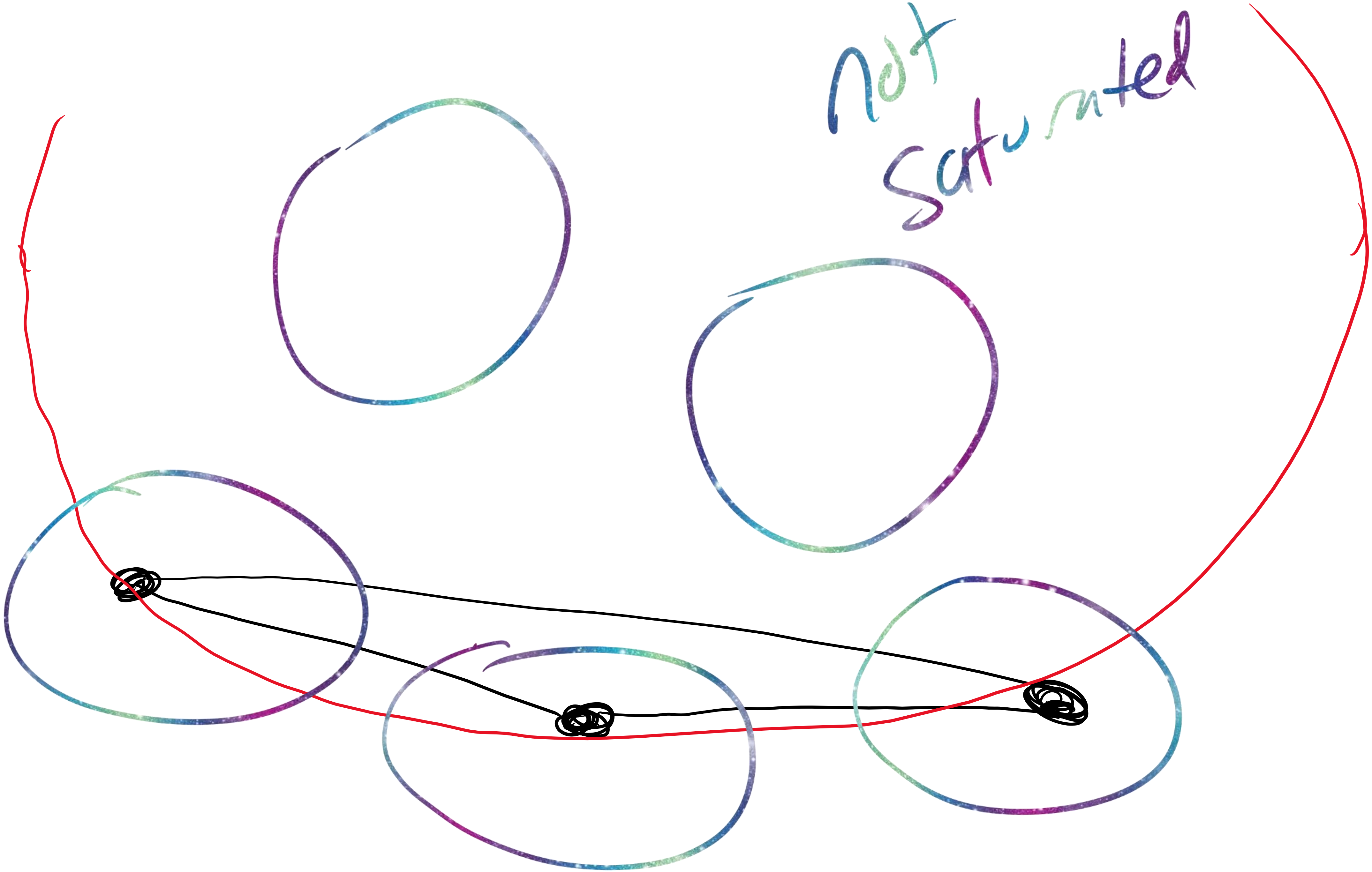


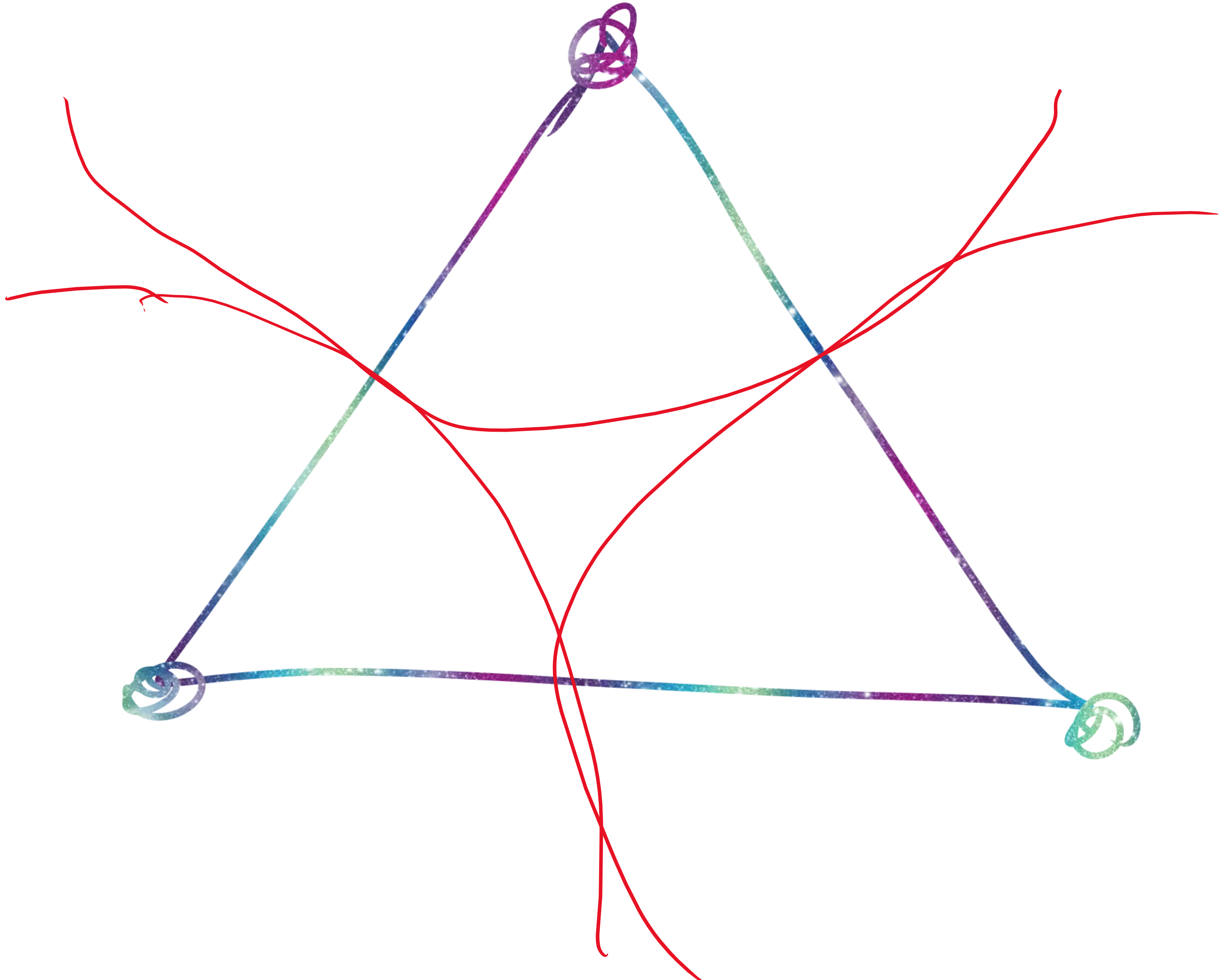


HW: show can always find a circum-circle for a triangle

Due Monday




not saturated



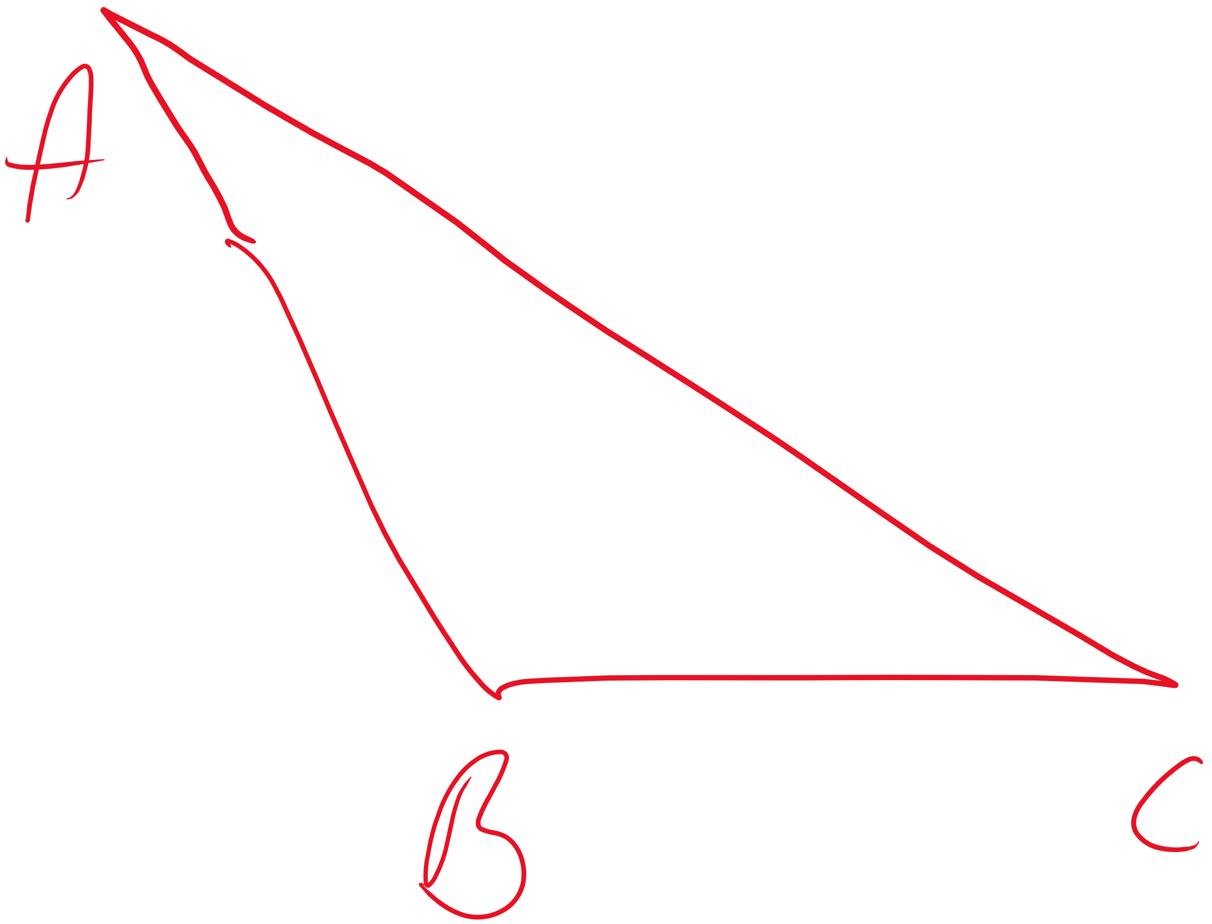
Lemma 2 *The density of a triangle ΔABC in a Delaunay triangulation for a saturated circle configuration $\mathcal{C}$ is less than or equal to $\pi/\sqrt{12}$. The equality holds only for the regular triangle with side-length 2.*

Proof: Let say that B is the largest internal angle of ΔABC. Then, by the above lemma,

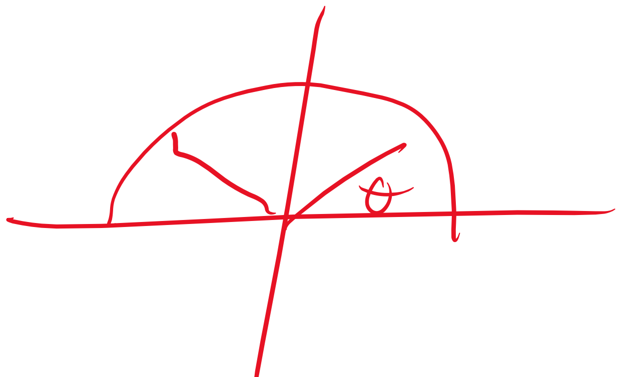
$$\text{the area of } \Delta ABC = \frac{1}{2}\overline{AB}\cdot\overline{BC}\cdot\sin B \geq \frac{1}{2}\cdot 2\cdot 2\cdot\frac{\sqrt{3}}{2} = \sqrt{3}.$$

Therefore, we have

$$\text{the density of } \Delta ABC = \frac{\pi/2}{\text{the area of } \Delta ABC} \leq \frac{\pi}{\sqrt{12}}.$$



$$\frac{\pi}{3} \le B < \frac{2\pi}{3}$$



It is obvious from the computation that the equality holds only when ΔABC is a regular triangle and side-length of ΔABC is 2. Q.E.D.

The density of the union of any finite Delaunay triangles in a saturated circle configuration is a weighted average of the densities of the Delaunay triangles. i.e.

$$\text{the density} = \frac{\sum_{\Delta_i:\text{Delaunay triangle}} (\text{the area of } \Delta_i) \times (\text{the density of } \Delta_i)}{\sum_{\Delta_i:\text{Delaunay triangle}} \text{the area of } \Delta_i}.$$

Since we have shown that the density of a Delaunay triangle is less than or equal to $\pi/\sqrt{12}$, the density of the union of any finite Delaunay triangles in a saturated circle configuration is also less than or equal to $\pi/\sqrt{12}$. Therefore, we obtain a simple proof of Thue theorem.

Is this a PROOF?

HW: State + Prove the C-S inequality
Cauchy-Schwarz
Due Monday
When is there equality?