Math 331: Problem Solving

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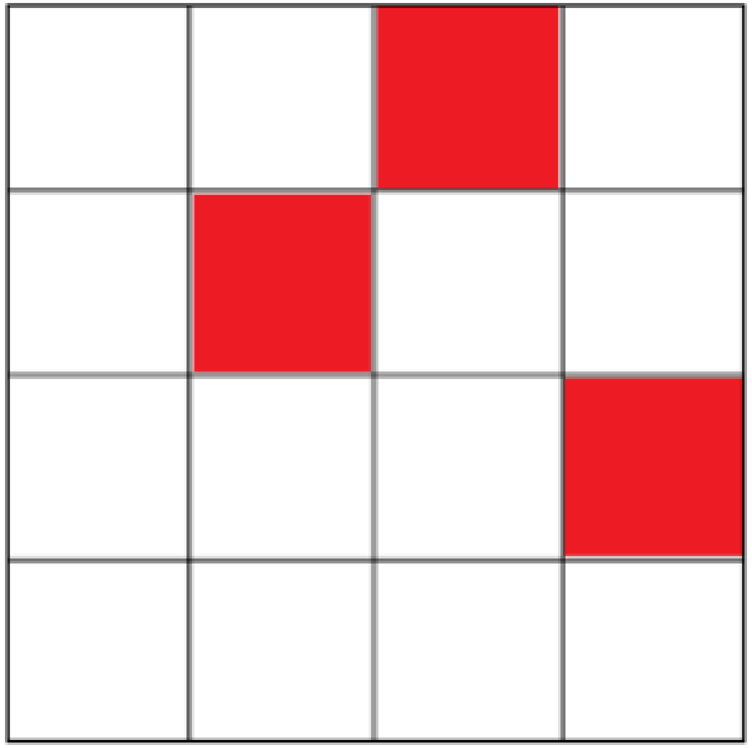
First Remote Participation Lecture
March 3, 2017

Zombie Infection: Rules

- If share walls with 2 or more infected, become infected.
- Once infected, always infected.



Initial Configuration

Zombie Infection: Rules

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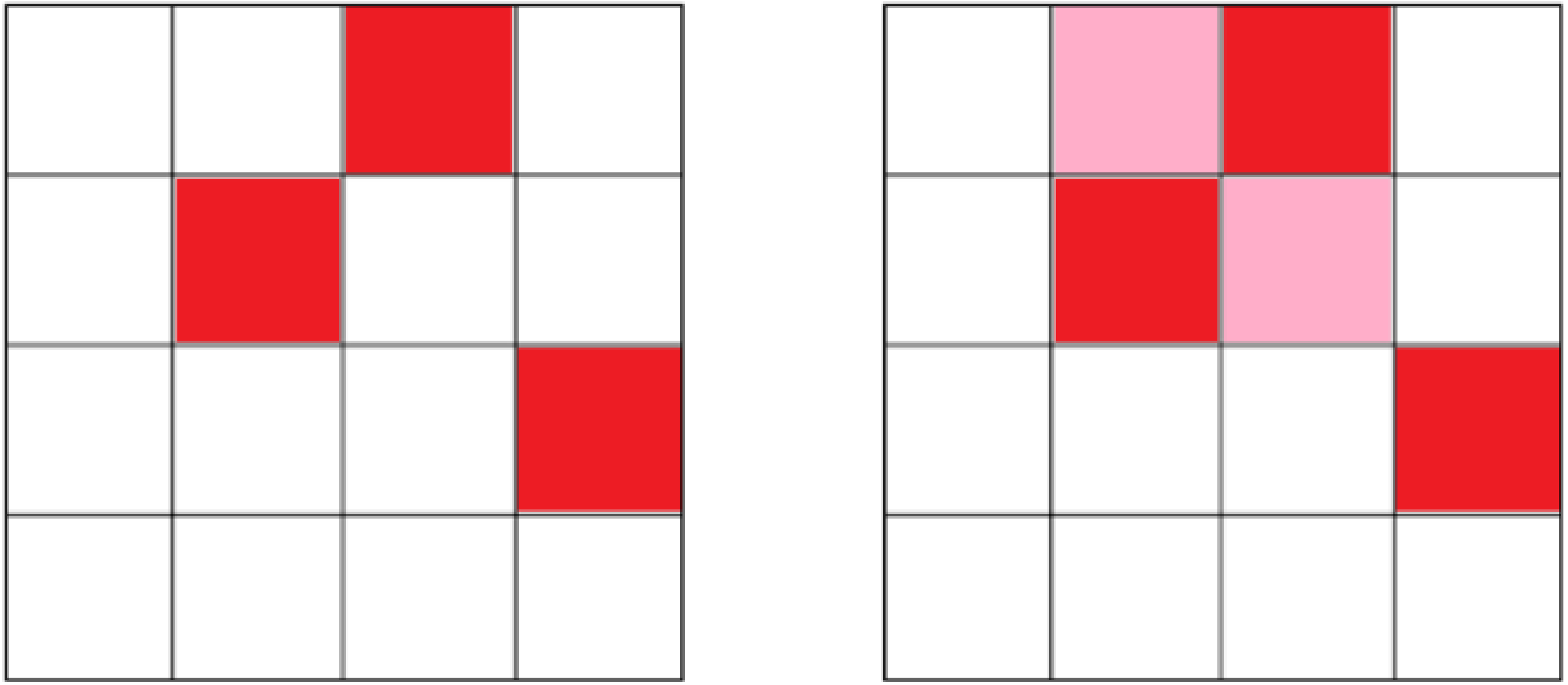

Initial Configuration *One moment later*

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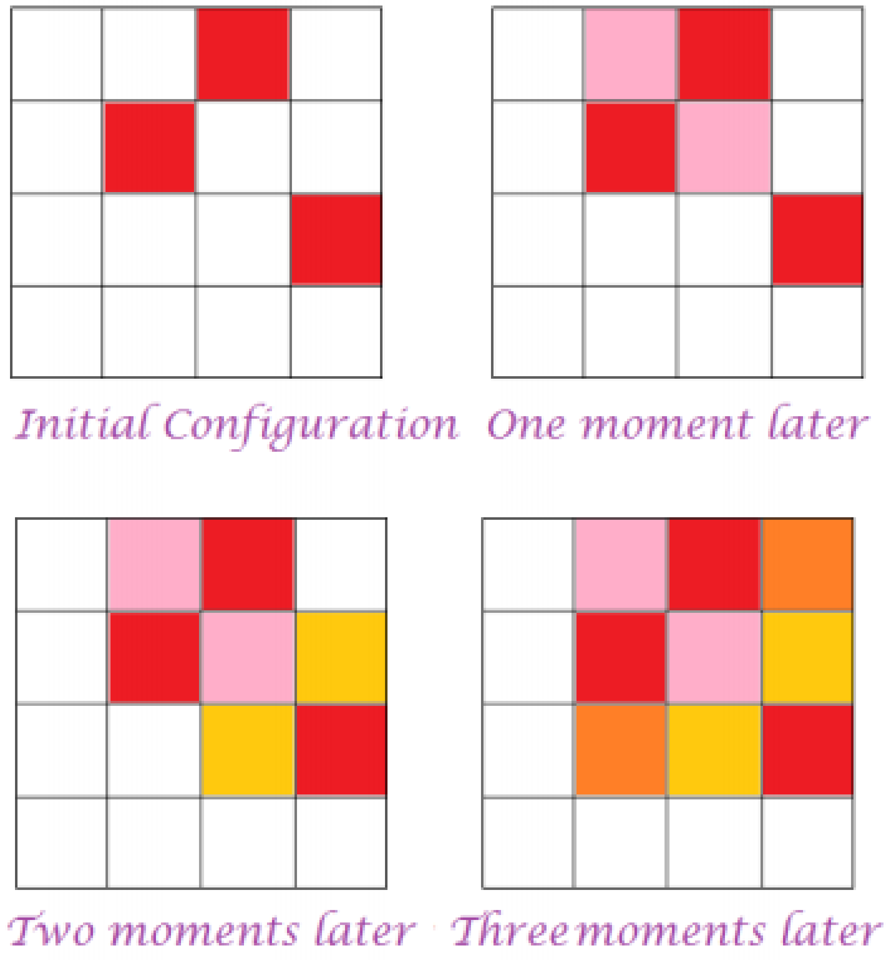
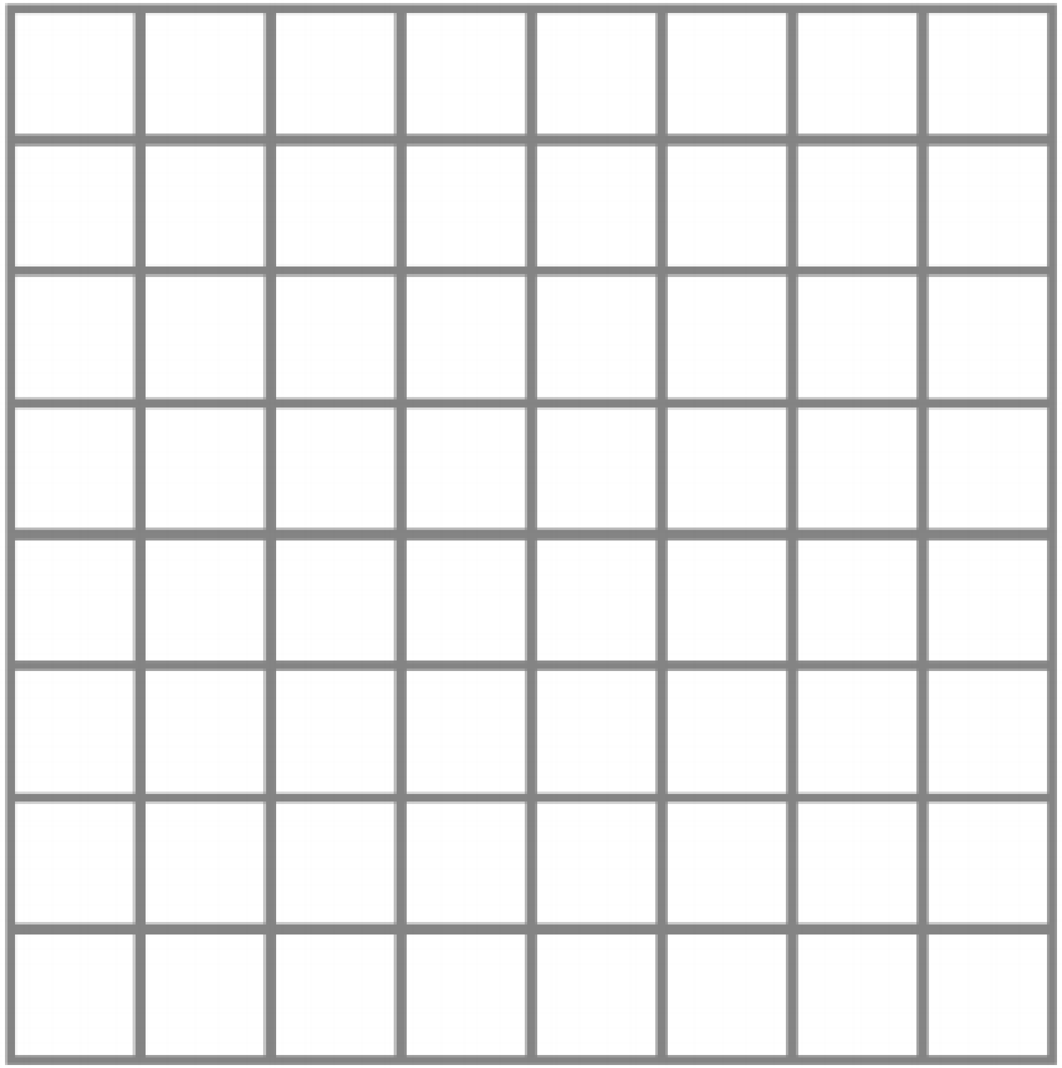


Zombie Infection: Conquering The World

Easiest initial state that ensures all eventually infected is...?



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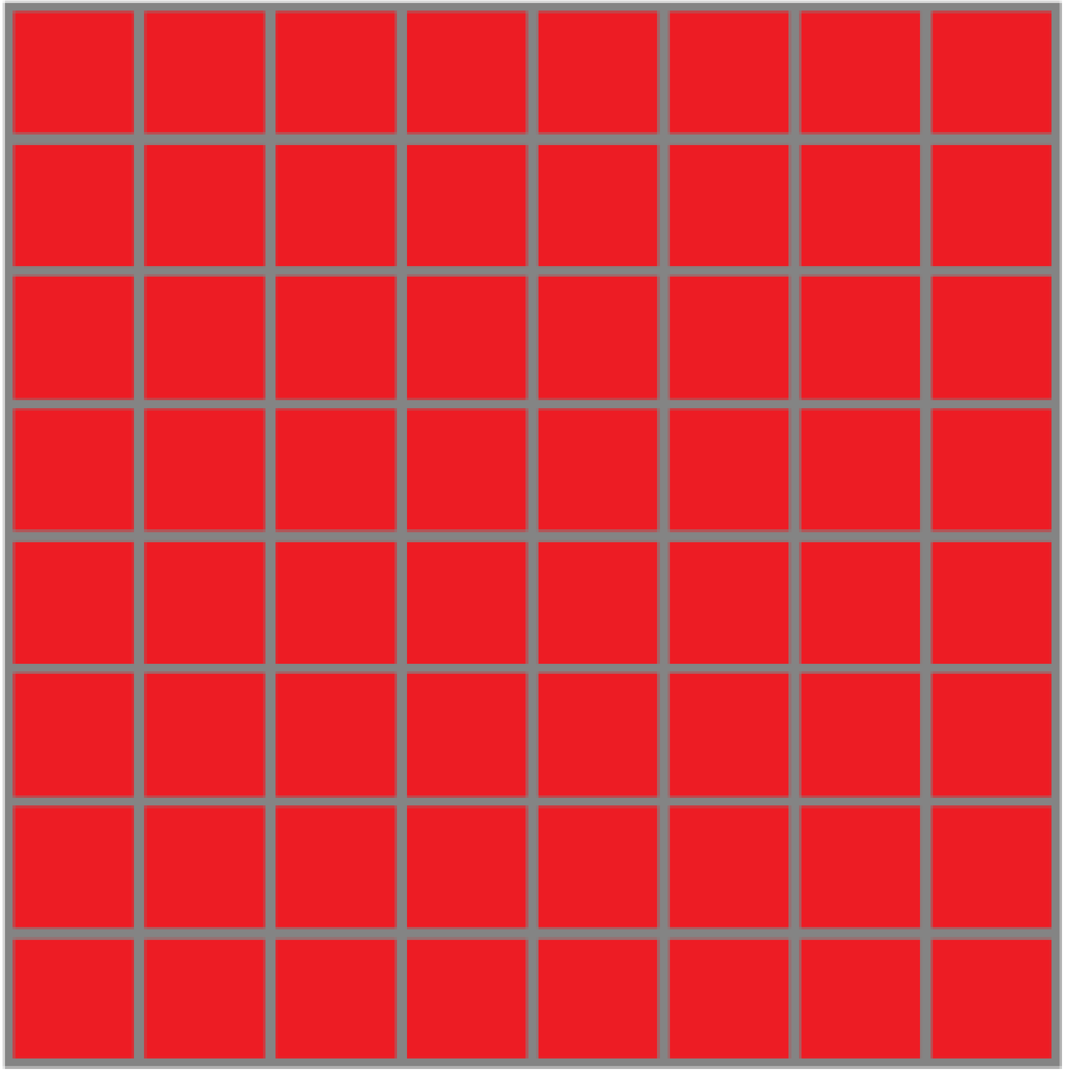
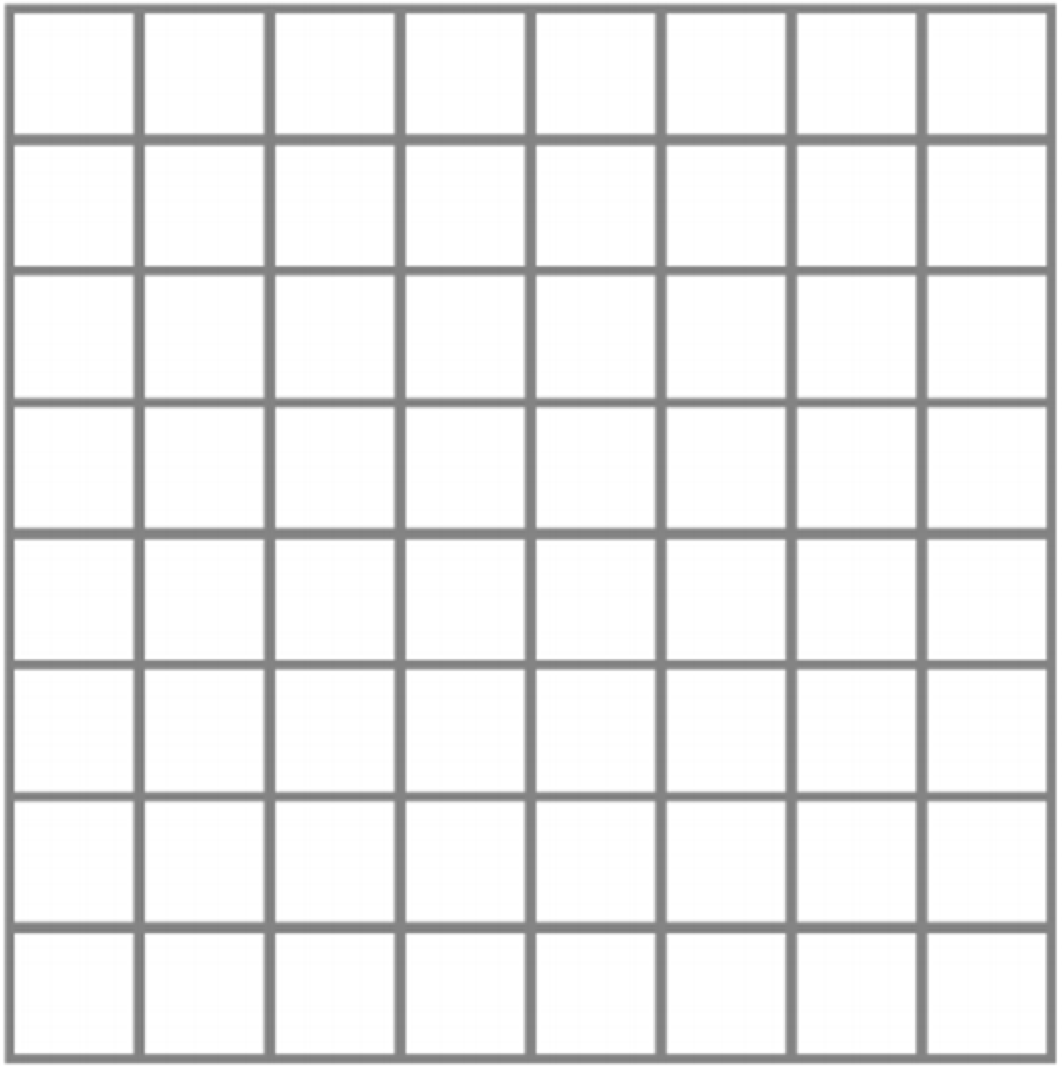


Zombie Infection: Conquering The World

Next simplest initial state ensuring all eventually infected...?



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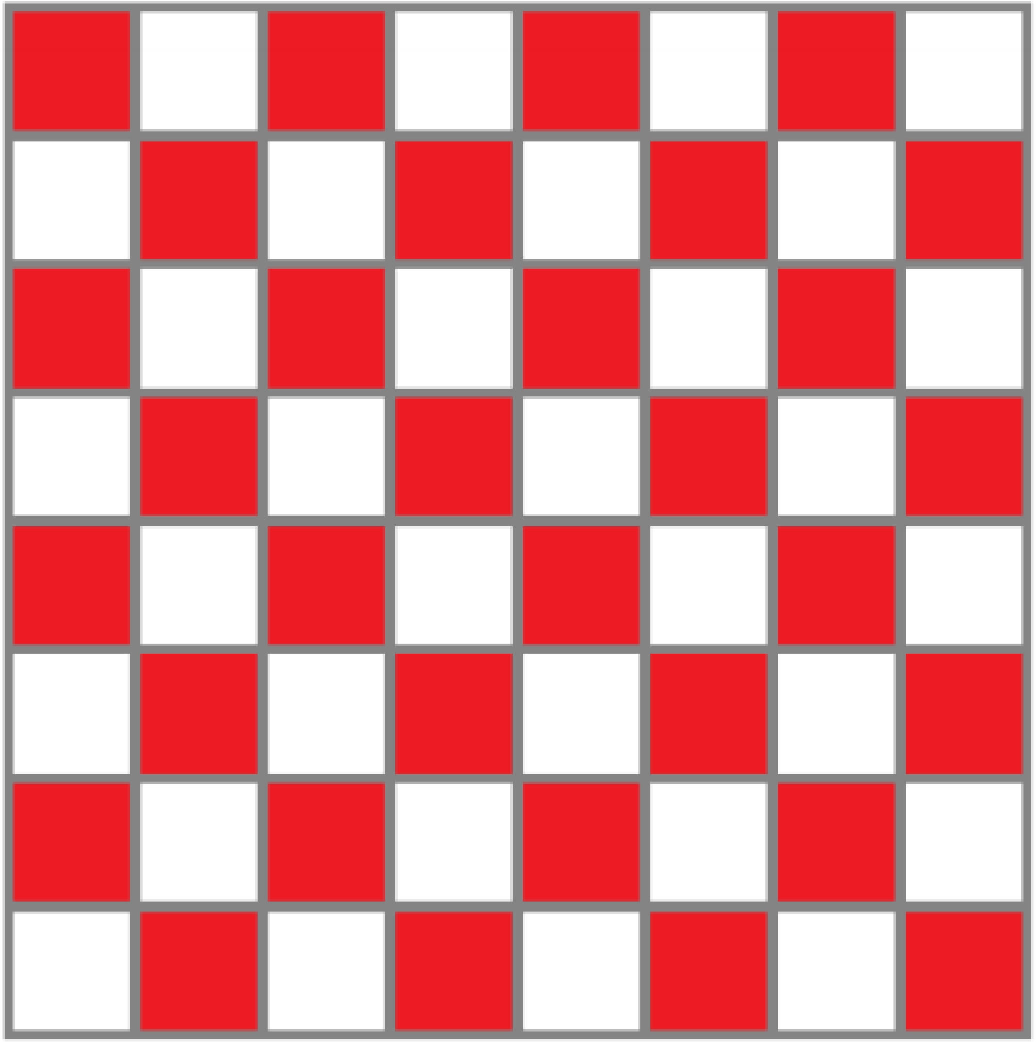
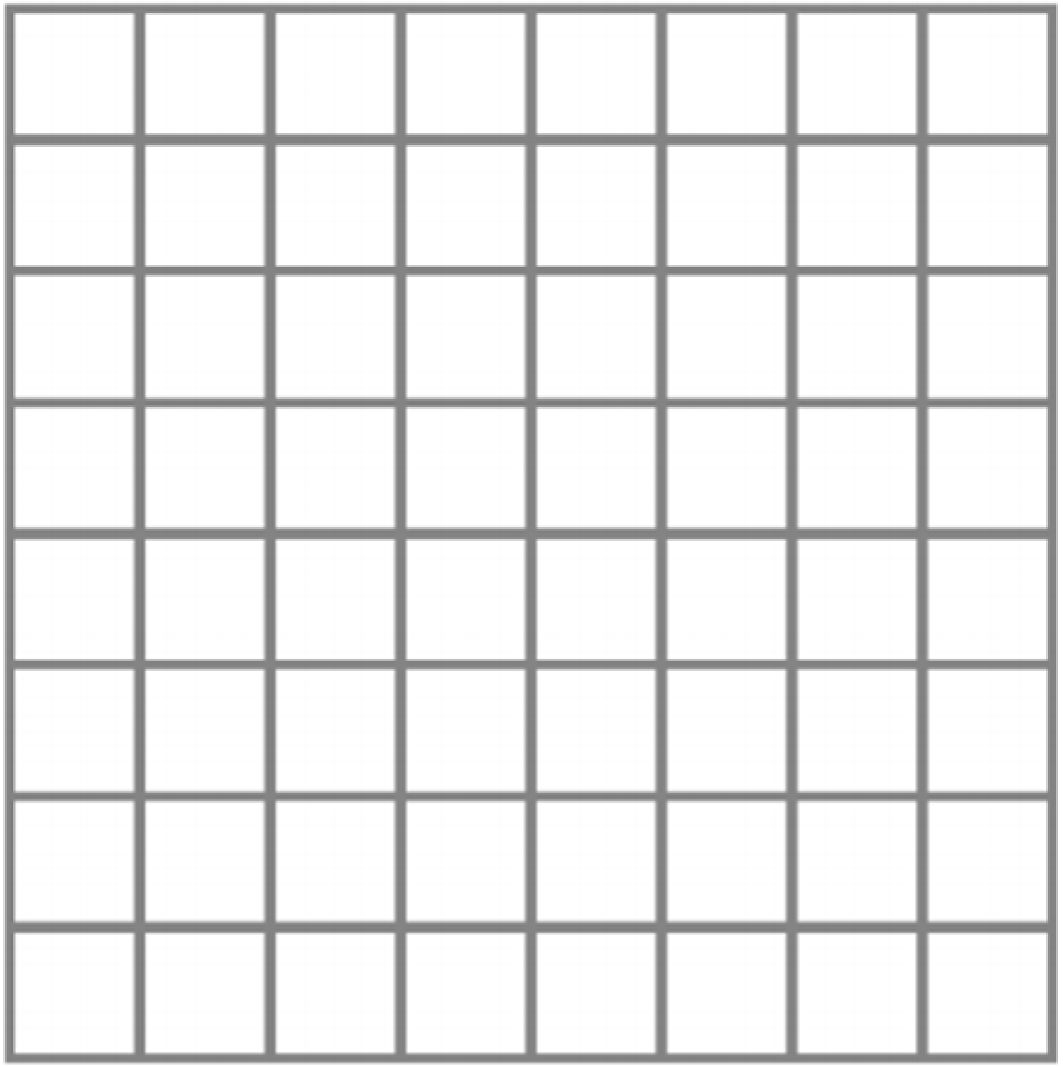


Zombie Infection: Conquering The World

Fewest number of initial infections needed to get all...?



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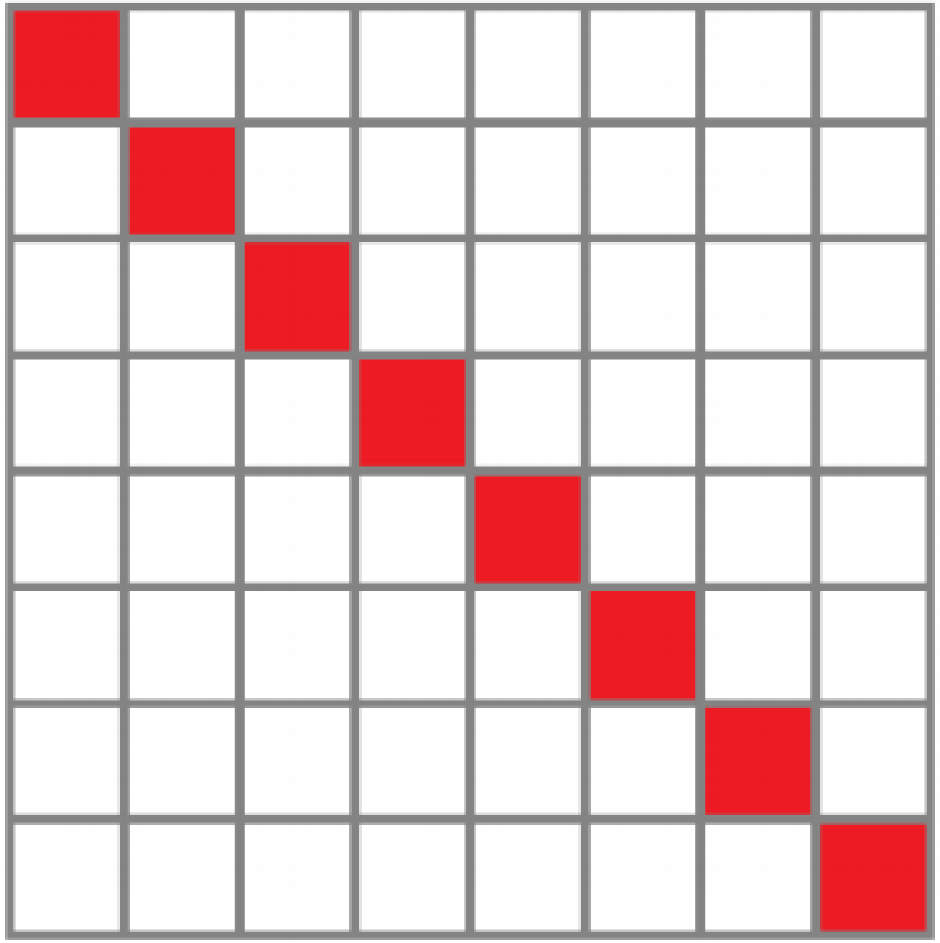

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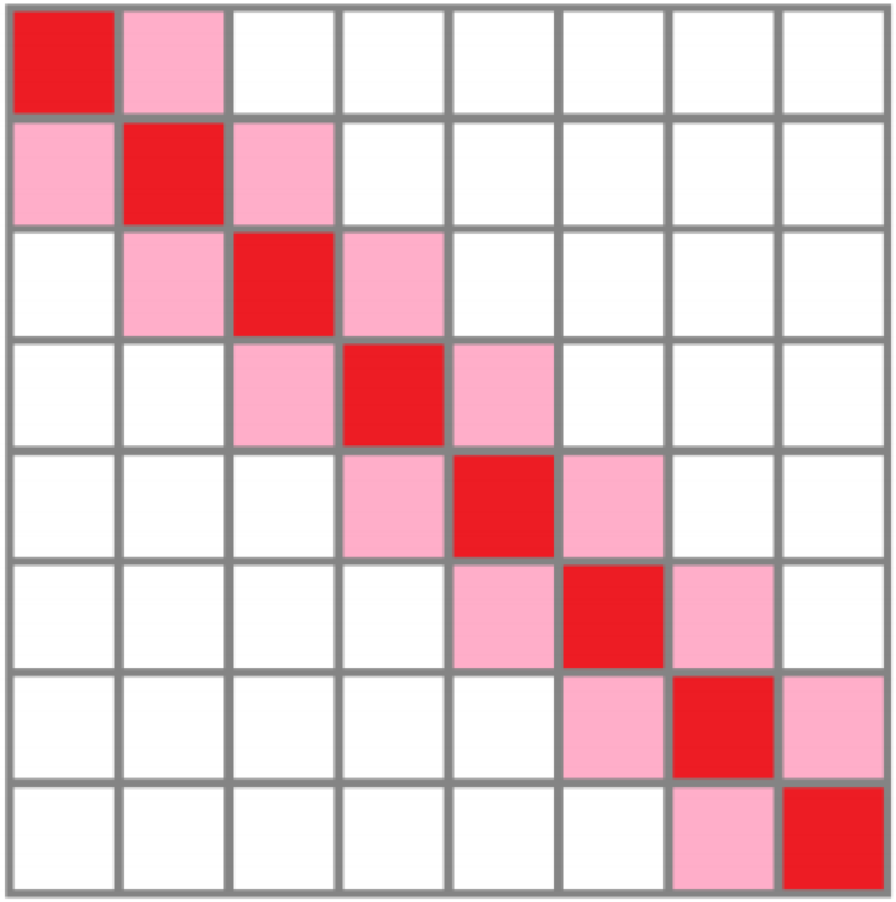

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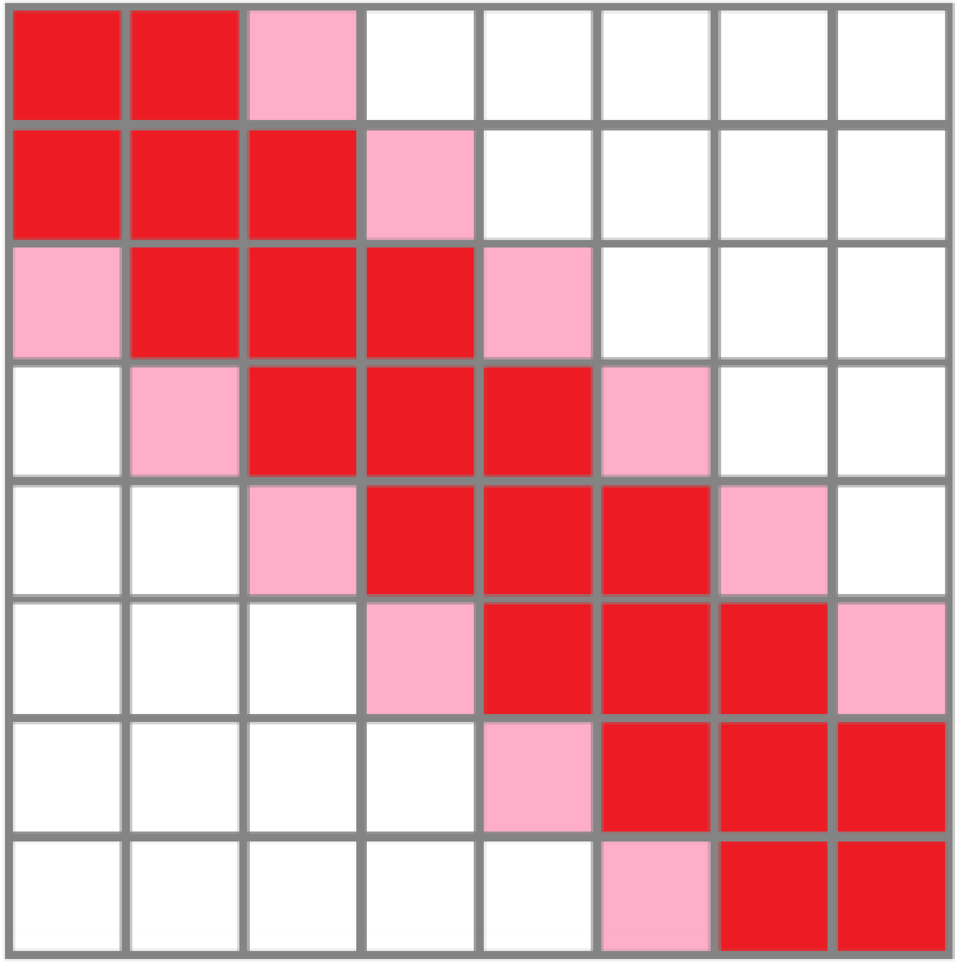

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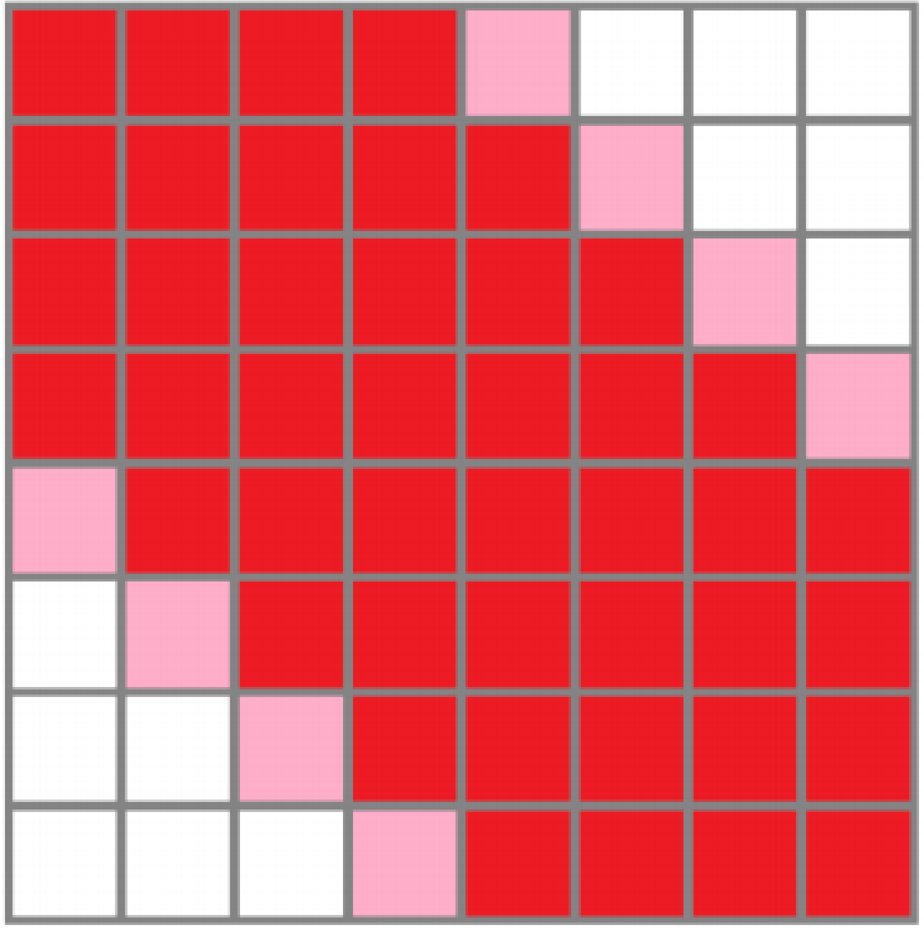

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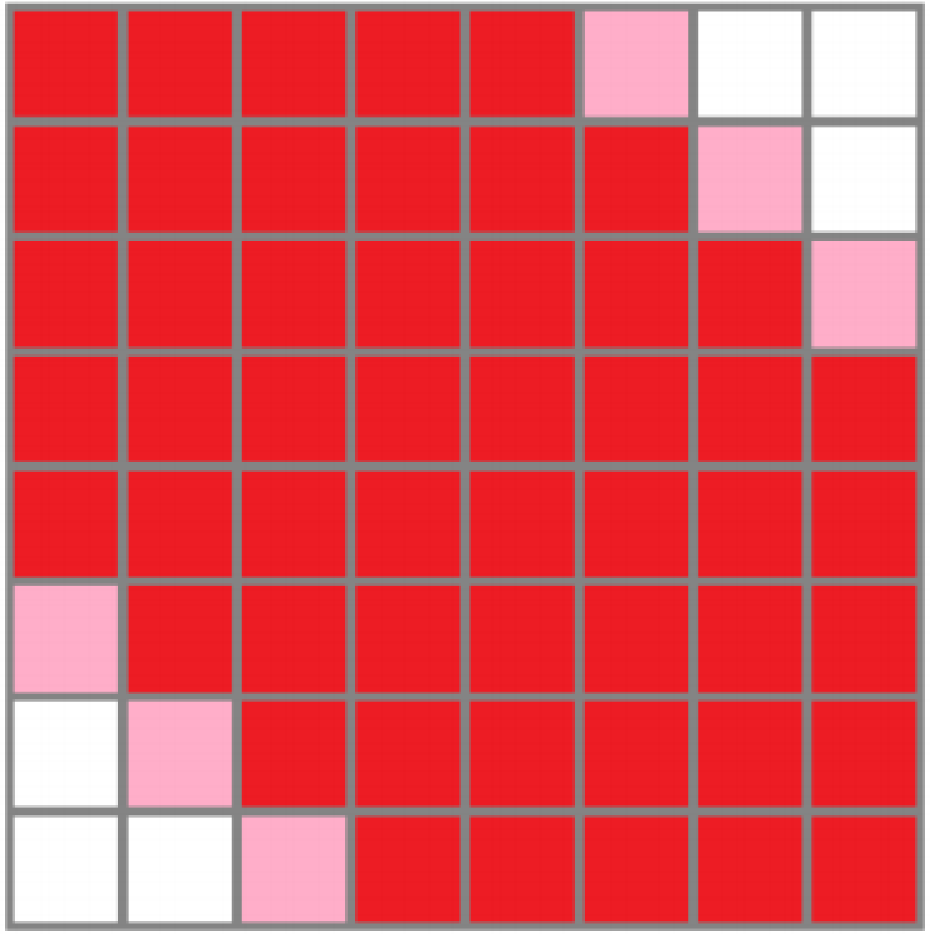

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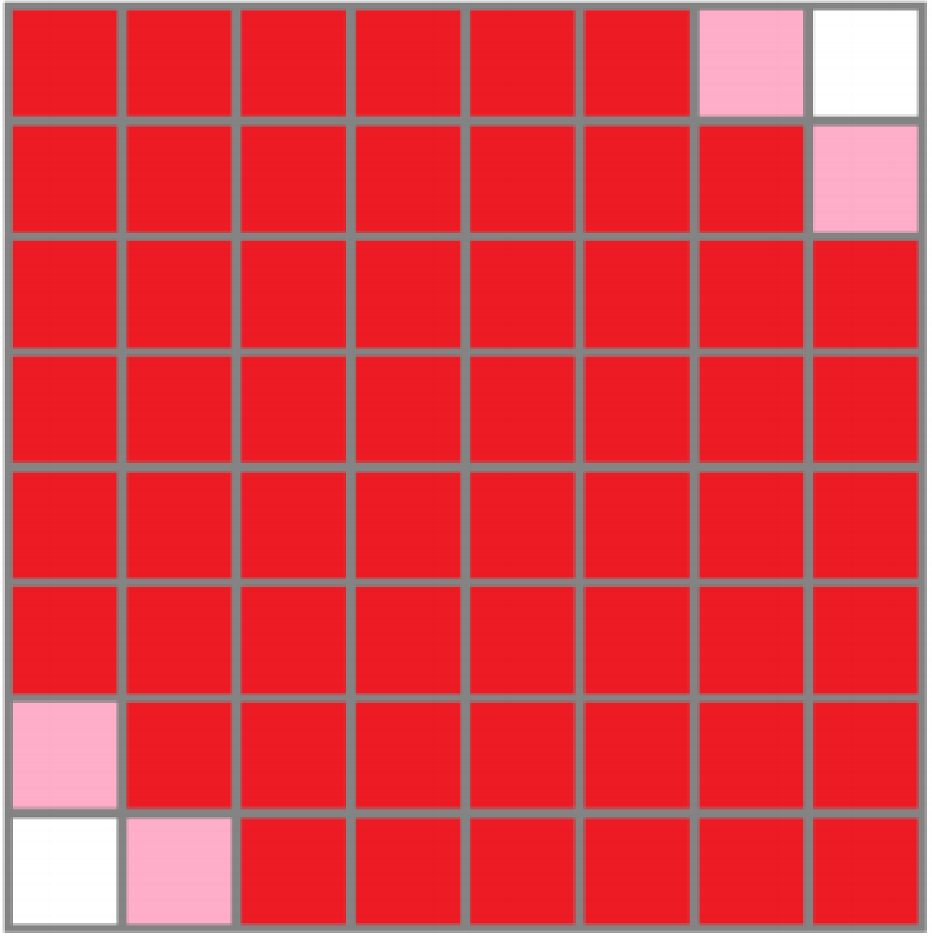

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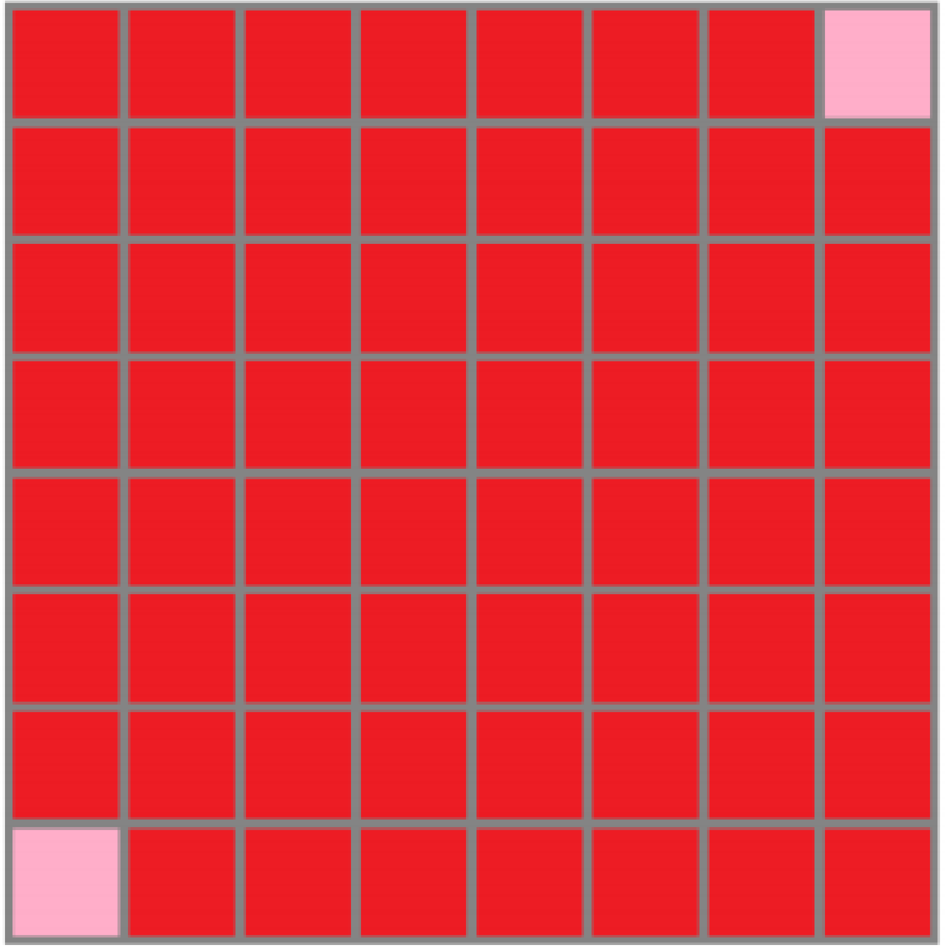

Zombie Infection: Can $n - 1$ infect all?

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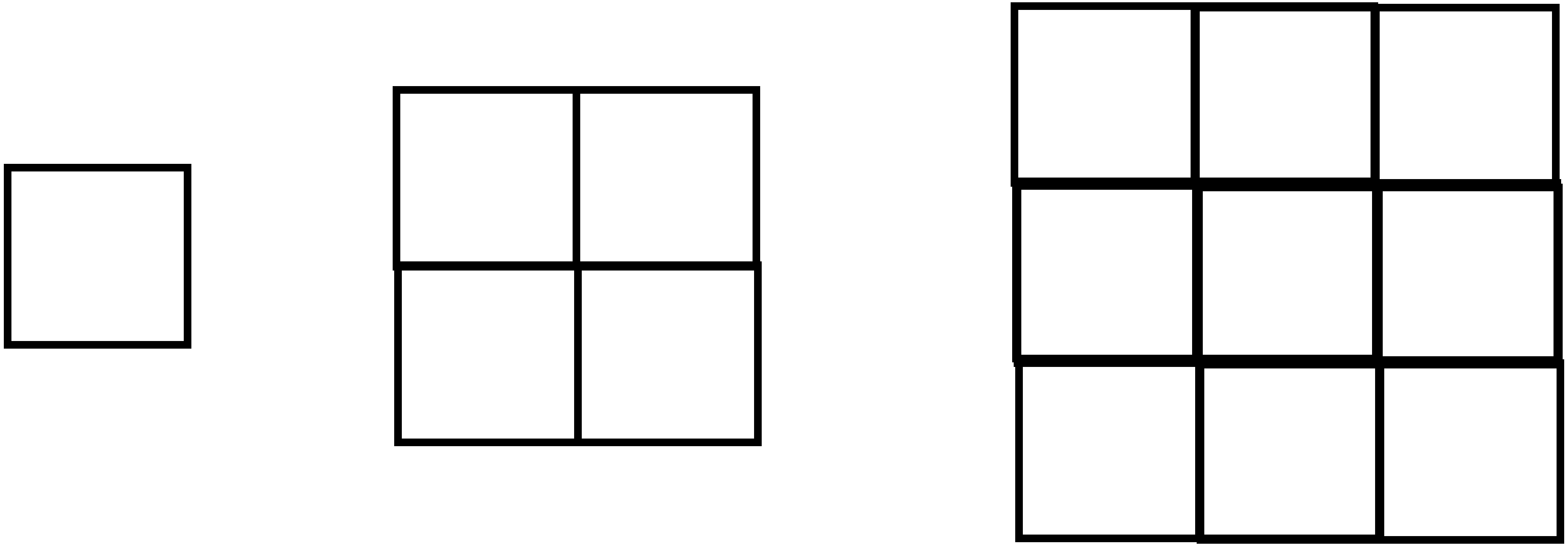

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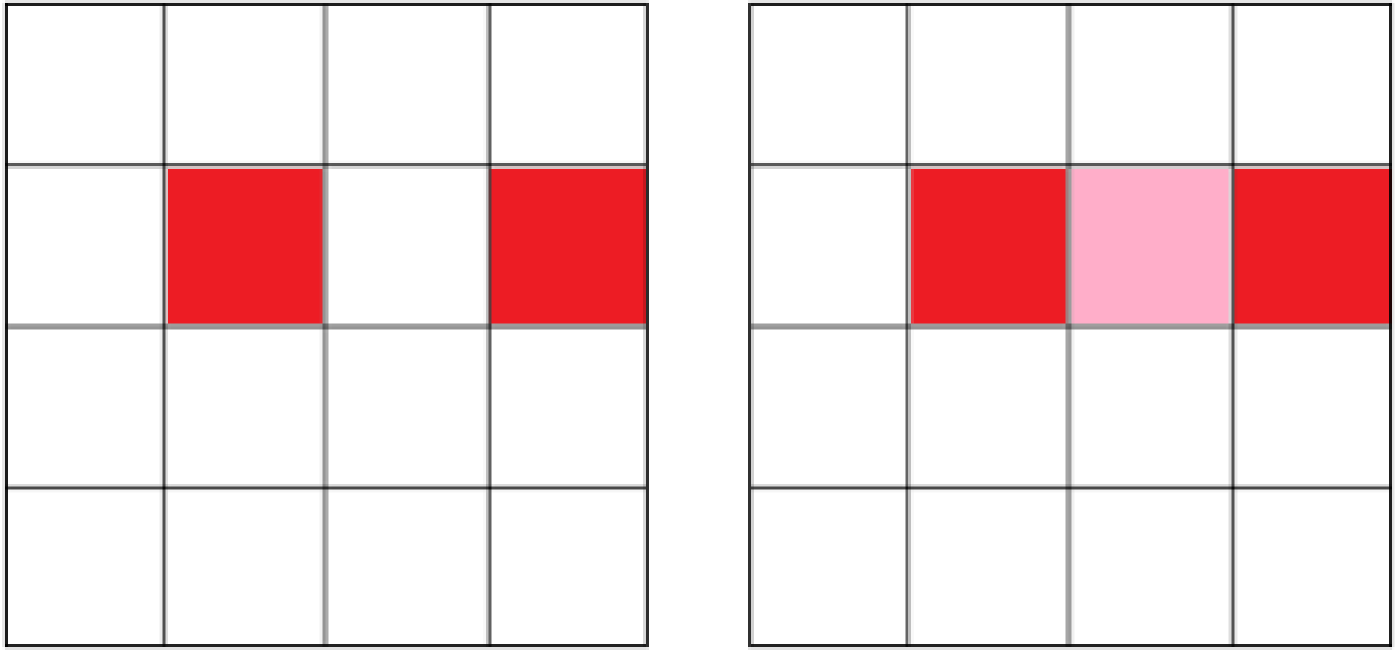

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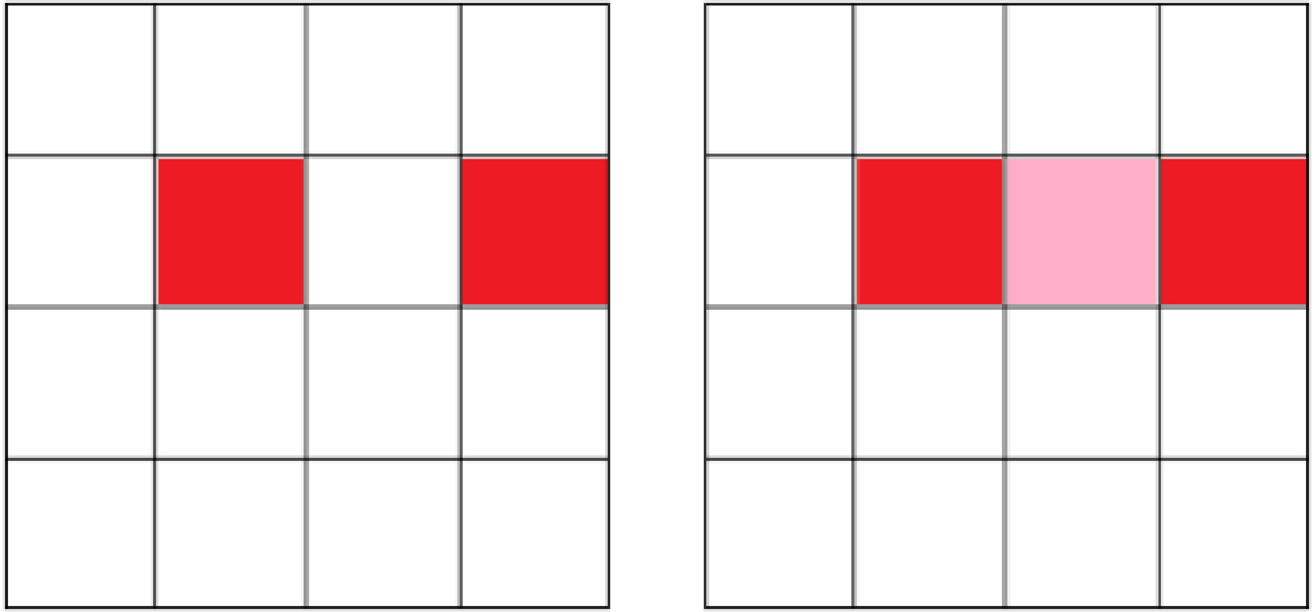
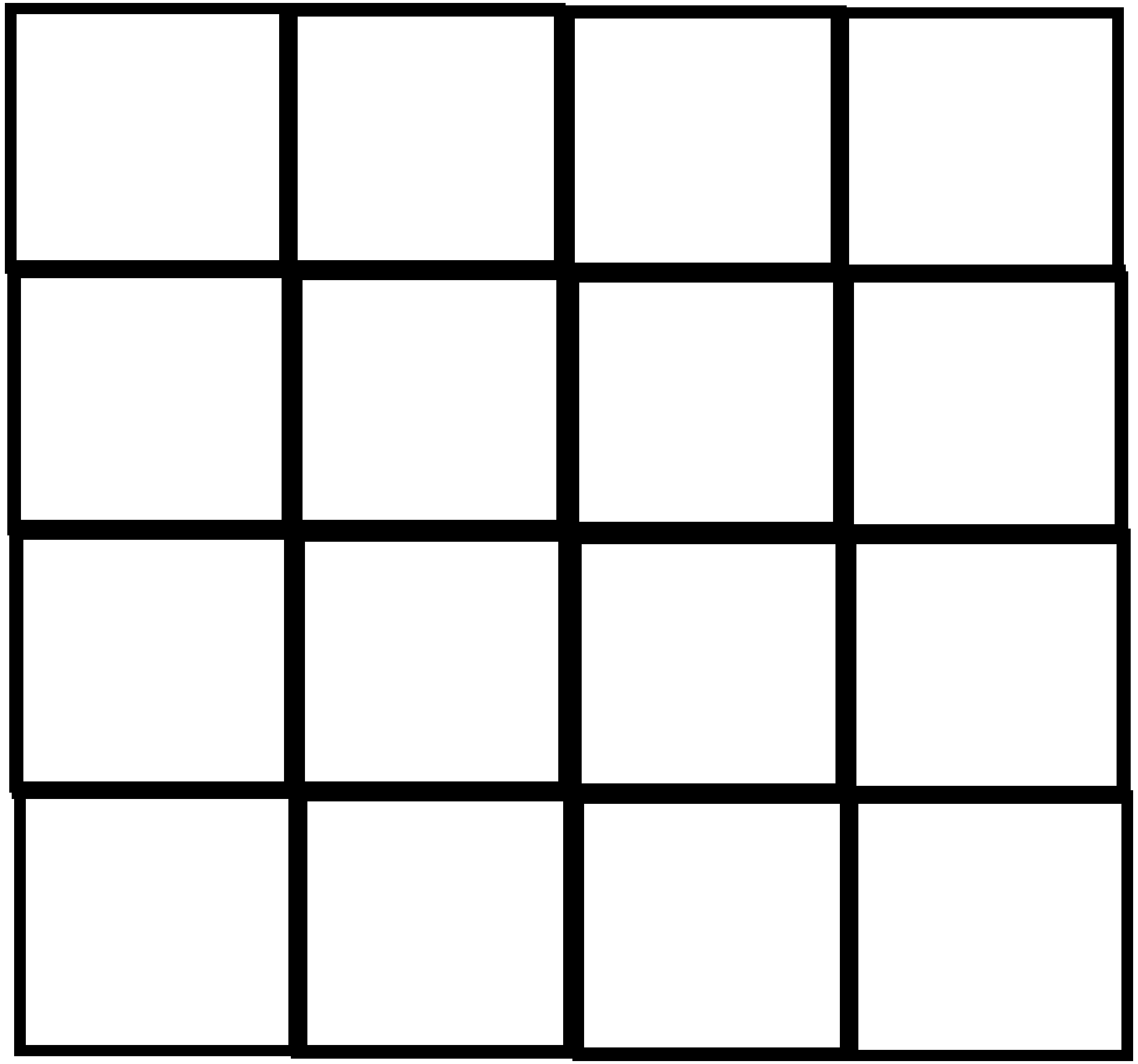
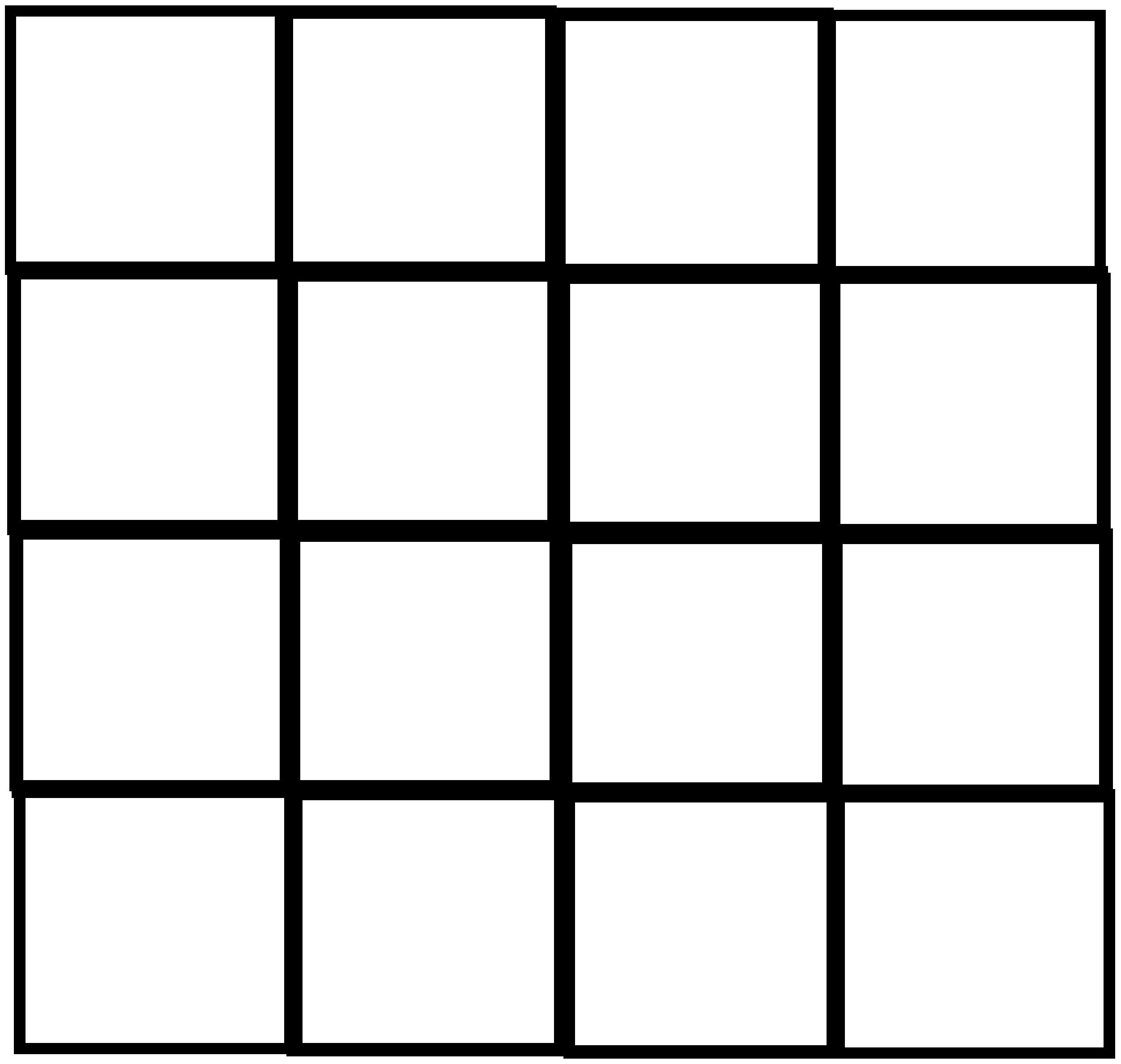
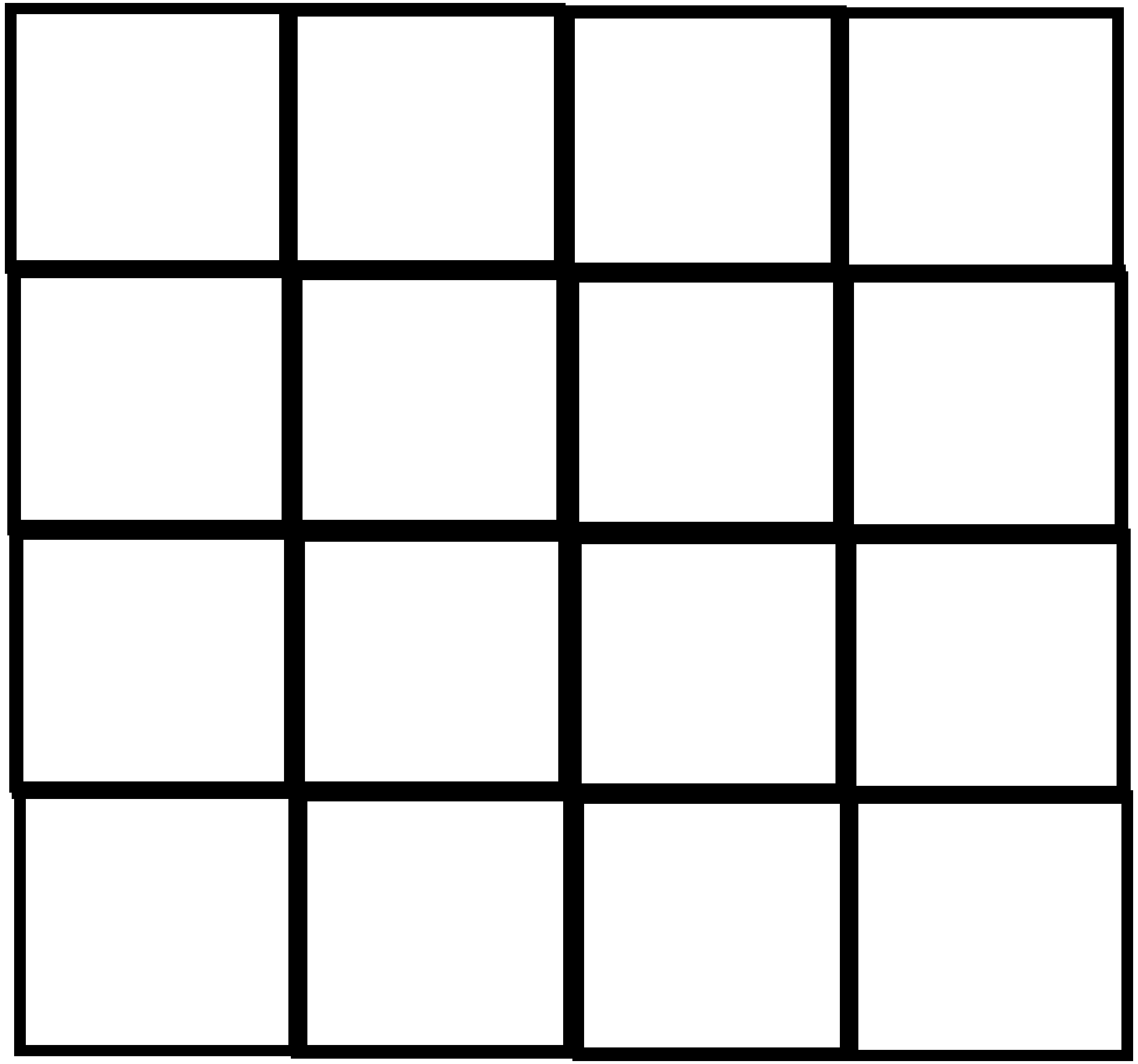


Perimeter of infection unchanged.







Monovariant: quantity that only increases or decreases as we “move”.

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Question: where else can we find monovariants?

Fibonacci numbers: $F_1 = 1$, $F_2 = 2$, $F_{n+1} = F_n + F_{n-1}$.

Zeckendorf's Theorem: Every positive integer can be written uniquely as a sum of non-adjacent Fibonacci numbers.

Example: 82 = 55 + 21 + 5 + 1 (Greedy algorithm)

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Given any decomposition as a sum of Fibonaccis, two moves:

- Split a double: $2F_n = F_n + F_{n-1} + F_{n-2} = F_{n+1} + F_{n-2}$.
- Combine adjacent: $F_n + F_{n-1} = F_{n+1}$.

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THEOREM: Among all decompositions of a number as a sum of Fibonaccis, none have fewer summands than the Zeckendorf.