

# Calculus Proficiency Test for Probability and Statistics

**Directions :** Calculus is an essential tool in probability and statistics. These questions are designed to ensure that you have a sufficient mastery of the subject for the course. If you do not answer at least 4 of the 5 questions completely correctly then you must satisfactorily complete a calculus review problem set to take the course.

**Question 1.** Suppose  $f(x) = Ae^{-4x}$  for  $x \geq 0$  (and 0 otherwise). Find the value of  $A$  so that

$$\int_{-\infty}^{\infty} f(x)dx = 1.$$

**Question 2.** Compute the following integral:

$$\int_0^{\infty} 5x e^{-x/2} dx.$$

**Question 3.** Calculate the following integral:

$$\int_0^6 x^4 (x+1) dx.$$

**Question 4.** Let  $f(x)$  be an infinitely differentiable function. Write the Taylor series expansion for  $f(x)$  about  $x = a$ .

**Question 5.** Let  $f(x) = \log(1-x)$ . Calculate the first three terms of the Taylor series expansion of  $f(x)$  about  $x = 0$  (i.e., find the degree 2 Taylor series). Calculate the first three terms of the Taylor series expansion of  $g(x) = \log(1-x^8)$  about  $x = 0$ .