

# Calculus Review Problems for Probability and Statistics

**Directions :** Calculus is an essential tool in probability and statistics. These questions are designed to ensure that you have a sufficient mastery of the subject for the course. You must correctly solve these problems to continue with the course.

## 1 Derivatives (one variable)

**Question 1.** Find the derivative of  $f(x) = 4x^5 + 3x^2 + x^{1/3}$ .

**Question 2.** Find the derivative of  $f(x) = (x^4 + 3x^2 + 8) \cos x$ .

**Question 3.** Find the derivative of  $f(x) = \log(1 - x^2)$ .

**Question 4.** Find the derivative of  $\log(4x) - \log(2x)$ .

**Question 5.** Find the derivative of  $e^{-x^2/2} = \exp(-x^2/2)$ .

**Question 6.** Find the second derivative of  $e^{-x^2/2} = \exp(-x^2/2)$ .

**Question 7.** Find the derivative of  $e^{x^8} \cos(3x^4) = \exp(x^8) \cos(3x^4)$ .

**Question 8.** Find the derivative of the function  $f(x) = 4x + \sqrt{2} \cos(x)$  and then use it to find the tangent line to the curve  $y = f(x)$  at  $x = \pi/4$ . Use the tangent line to approximate  $f(x)$  when  $x = \frac{\pi}{4} + .01$ .

**Question 9.** Find the second derivative of  $f(x) = \ln x + \sqrt{162}$ .

**Question 10.** Find the maximum value of  $x^4 e^{-x} = x^4 \exp(-x)$  when  $x \geq 0$ .

**Question 11.** Find the critical points of  $f(x) = 4x^3 + 3x^2$ , and decide whether each is a maximum, a minimum, or a point of inflection.

**Question 12.** Find the derivative of  $(x^2 - 1)/(x - 1)$ .

**Question 13.** Find the derivative of the function  $f(x) = \sqrt[3]{(x - 2)^2} = (x - 2)^{2/3}$ .

**Question 14.** Find the points on the graph of  $f(x) = \frac{1}{3}x^3 + x^2 - x - 1$  where the slope is (a)  $-1$ , (b)  $2$ , and (c)  $0$ .

**Question 15.** Find the second derivative of  $f(x) = (x^4 + 3x^2 + 8) \cos x$ .

## 2 Taylor Series (one variable)

**Question 16.** Find the first five terms of the Taylor Series for  $f(x) = x^8 + x^4 + 3$  at  $x = 0$ .

**Question 17.** Find the first three terms of the Taylor Series for  $f(x) = x^8 + x^4 + 3$  at  $x = 1$ .

**Question 18.** Find the first three terms of the Taylor Series for  $f(x) = \cos(5x)$  at  $x = 0$ .

**Question 19.** Find the first five terms of the Taylor Series for  $f(x) = \cos^3(5x)$  at  $x = 0$ .

**Question 20.** Find the first two terms of the Taylor Series for  $f(x) = e^x$  at  $x = 0$ .

**Question 21.** Find the first six terms of the Taylor Series for  $f(x) = e^{x^8} = \exp(x^8)$  at  $x = 0$ .

**Question 22.** Find the first four terms of the Taylor series for  $f(x) = \frac{1}{\sqrt{2\pi}}e^{-x^2/2} = \exp(-x^2/2)/\sqrt{2\pi}$  at  $x = 0$ .

**Question 23.** Find the first three terms of the Taylor series for  $f(x) = \sqrt{x}$  at  $x = \frac{1}{3}$ .

**Question 24.** Find the first three terms of the Taylor series for  $f(x) = (1+x)^{1/3}$  at  $x = \frac{1}{2}$ .

**Question 25.** Find the first three terms of the Taylor series for  $f(x) = x \log x$  at  $x = 1$ .

**Question 26.** Find the first three terms of the Taylor series for  $f(x) = \log(1+x)$  at  $x = 0$ .

**Question 27.** Find the first three terms of the Taylor series for  $f(x) = \log(1-x)$  at  $x = 1$ .

**Question 28.** Find the first two terms of the Taylor series for  $f(x) = \log((1-x) \cdot e^x) = \log((1-x) \cdot \exp(x))$  at  $x = 0$ .

**Question 29.** Find the first three terms of the Taylor Series for  $f(x) = \cos(x) \log(1+x)$  at  $x = 0$ .

**Question 30.** Find the first two terms of the Taylor series for  $f(x) = \log(1+2x)$  at  $x = 0$ .

## 3 Integrals (one variable)

**Question 31.** Find the following integral:  $\int_0^1 (x^4 + x^2 + 1)dx$ .

**Question 32.** Find the following integral:  $\int_0^1 (x^2 + 2x + 1)dx$ .

**Question 33.** Find the following integral:  $\int_0^1 (x^2 + 2x + 1)^2 dx$ .

**Question 34.** Find the following integral:  $\int_{-\pi/2}^{\pi/2} (\sin^3 x \cos x + \sin x \cos x) dx$ .

**Question 35.** Find the following integral:  $\int_{-4}^4 (x^3 + 6x^2 - 2x - 3) dx$ .

**Question 36.** Find the following integral:  $\int_0^1 \frac{x}{1+x^2} dx$ .

**Question 37.** Find the following integral:  $\int_0^3 (x^3 + 3x)^8 (x^2 + x) dx$ .

**Question 38.** Find the following integral:  $\int_0^2 x \cos(3x^2) dx$ .

**Question 39.** Find the following integral:  $\int_0^\infty x e^{-x^2/4} dx$ .

**Question 40.** Let

$$f(x) = \begin{cases} 1 & \text{if } x \in [0, 1] \\ 0 & \text{otherwise.} \end{cases}$$

Calculate  $\int_{-\infty}^\infty f(t)f(x-t)dt$ .

## 4 Derivatives (several variables)

**Question 41.** Let  $f(x) = x^2y + e^x + \sin(xy)$ . Find  $\partial f/\partial x$  and  $\partial f/\partial y$ .

**Question 42.** Let

$$f(x; \mu, \sigma) = \frac{\exp(-(x - \mu)^2/2\sigma^2)}{\sqrt{2\pi\sigma^2}}. \quad (4.1)$$

Find  $\partial f/\partial \mu$  and  $\partial f/\partial \sigma$ .

**Question 43.** Find  $\partial f/\partial x$  and  $\partial f/\partial y$  for the function  $f(x, y) = x e^{x^2+y^2} = x \exp(x^2 + y^2)$ .

**Question 44.** Find  $\partial f/\partial x$  and  $\partial f/\partial y$  for the function  $f(x, y) = e^{xy} - \log(x^2 + y^2)$ .

**Question 45.** Find  $\partial f/\partial x$  and  $\partial f/\partial y$  for the function  $f(x, y, t) = 5t^4 - 4t^5 \cos(t \sin t)$ .

## 5 Integrals (several variables)

**Question 46.** Find

$$\int_{x=0}^2 \int_{y=0}^3 5(x^2y + xy^2 + 2) dx dy.$$

**Question 47.** Find

$$\int_{x=0}^6 \int_{y=0}^5 x e^{-xy} dx dy.$$

**Question 48.** Find

$$\int_{x=0}^1 \int_{y=0}^1 x^m y^n dx dy,$$

where  $m, n > 0$ .

**Question 49.** Find

$$\int_{x=0}^1 \int_{y=0}^1 (x^2 + 2xy + y\sqrt{x}) dx dy.$$

**Question 50.** Find

$$\int_{x=0}^1 \int_{y=0}^1 (ax + by + c) dx dy.$$