

## MATH 162: PROBABILITY REVIEW PROBLEMS

There are many different Statistics courses at Brown, each with a slightly different emphasis. The purpose of this class is to concentrate on the theory behind the statistical tests. It is essential that you have a strong grasp of probability and are comfortable with proofs. Below are several probability review problems. If you have trouble doing these problems, you should strongly reconsider taking this class and talk to me about it; if you do stay in the course, you will need to significantly review your probability.

*This is the only homework assignment where you cannot work in groups. The purpose is to see what you remember. See how much you can do without using reference books. While you may use books / notes to solve these problems, remember the point of this assignment is to make sure you are comfortable with the probability pre-requisites for the course.*

**Question 1 :** Write the probability density function for the uniform distribution on (a) the interval  $[0, 1]$ ; (b) the interval  $[-1, 3]$ . Write the probability density function for the normal distribution (also called a Gaussian) with mean  $\mu$  and standard deviation  $\sigma$ .

**Question 2 :** Consider a group of 10 people where 3 are juniors and 7 are seniors. (a) How many ways are there to choose 5 people to serve on a committee of 5? Note that all committee members are equal. (b) How many ways are there to choose 5 people to serve on a committee of 5 given that 2 must be juniors and 3 must be seniors? (c) How many ways are there to choose 5 people where one is the president, one is the vice-president, one is the secretary, one is the treasurer, and one is the web-page administrator? Note that the positions are not equal.

**Question 3 :** A fair coin is tossed 1000 times. What is the probability that there are at least two heads?

**Question 4 :** Write down the formula for the sum of a geometric series with ratio  $r$ :  $\sum_{n=0}^{\infty} r^n = 1 + r + r^2 + r^3 + \dots$  (Optional: can you prove your formula is correct?).

**Question 5 :** What does it mean for two events to be independent? State this in words and write it out mathematically as well.

**Question 6 :** What is the relation between  $P(A)$ ,  $P(B)$ ,  $P(A \cap B)$  and  $P(A \cup B)$ , where  $P(E)$  is the probability of event  $E$ ?

**Question 7 : [VERY IMPORTANT]** Let  $X$  be a random variable with probability density function  $f_X(x)$  and cumulative distribution function  $F_X(x) = \text{CDF}_X(x)$ . Let  $Y$  denote the random variable  $X + a$  for a fixed, constant  $a$ . (a) In terms of  $f_X$ , what is the probability  $X$  takes on values in  $[\alpha, \beta]$ ? Now express your answer in terms of  $F_X$ . (b) In terms of  $f_X$ , what is the probability  $Y$  takes on values in  $[\alpha, \beta]$ ? Now express your answer in terms of  $F_X$ .

**Question 8 : [VERY IMPORTANT]** Find the first three terms of the Taylor Series expansions at  $x = 0$  for (a)  $\sin(8x)$ ; (b)  $e^{x^8}$ ; (c)  $\sin(8x) + e^{x^8}$ .