GREEN CHICKEN CONTEST 2017

1. (a) Donald Trump decides to build a brick wall at his Mar-a-Lago resort. The wall is 101 feet across, 27 feet high, and 11 feet thick. Each brick is a unit cube (1 foot by 1 foot by 1 foot). A worm crawls in a straight line from the front bottom left corner to the back top right corner. How many bricks does the worm crawl through?

   (b) Due to budgetary concerns, a second brick wall is built with the same sized bricks that is just 100 feet across, 26 feet high, and 10 feet thick. How many bricks would the worm crawl through this time taking a similar path? (Disregard any bricks that the worm just meets at an edge or corner without crawling some positive distance through it.)

2. Explicitly solve as many of the following problems for n = 0, 1, 2, …, 10 where in each case p, q, r, and s are distinct prime numbers: $\det \begin{bmatrix} p & q \\ r & s \end{bmatrix} = n$.

3. Sally and her uncle both live in the same coastal town north of a river that runs directly east (orthogonally) into the ocean. Sally lives 600 feet from the river and 2000 feet from the ocean. Her uncle lives 400 feet from both the ocean and the river. Sally is asked to walk from her house with two buckets, fill one up with fresh river water, fill up the other with salt water from the ocean, and then take them to her uncle's house. What is the length of the shortest path that she could take?

4. One hundred people of equal ability sign up for a somewhat unusual single elimination tennis tournament - among them are twin sisters Jan and Jane. Each day there is just one game played determined by pulling two names out of a hat that includes all the remaining players' names. What is the probability that the sisters will be matched up to play each other at some point?

5. (a) Let $a_1 = 2$, $a_2 = 2$, and $a_{i+2} = 2a_i + a_{i+1} - 1$ for $i \geq 1$. Find a_{2017}.

   (b) Let $b_1 = 0$, $b_2 = 3$, and $b_{i+2} = 2b_i + b_{i+1}$ for $i \geq 1$. Find b_{2017}.

6. If the length of the side of a triangle is less than the average lengths of the other two sides, show that the opposite angle is less than the average of the other two angles.