

Green Chicken Contest - 2013

1. MATHew noticed that his age, that of his three children, and that of his mother MATHilda are all divisors of 2013. What is the sum of their ages? (All the children have different ages.)
2. What are the two times between noon (12:00 P.M.) and 1:00 P.M. that the hour and minute hands of a clock are perpendicular to each other?
3. Let N denote the positive integers. A function $f: N \rightarrow N$ satisfies
 - (i) $f(ab) = f(a)f(b)$ whenever $\gcd(a, b) = 1$, and
 - (ii) $f(p + q) = f(p) + f(q)$ whenever p and q are primes.Find $f(33)$.
4. Consider a 100×100 checkerboard consisting of 10,000 unit squares.
 - (a) Show that if the middle 2×2 squares are removed, then the remaining board can be tiled with a sufficient number of 3×1 sized-tiles.
 - (b) Show that if, instead, a 2×2 square is removed from the lower left corner, then the remaining board cannot be tiled in this way.
5. Define the Fibonacci sequence by $F_1 = 1, F_2 = 1$, and $F_{n+2} = F_n + F_{n+1}$ for $n \geq 1$. Show that there is a Fibonacci number that is divisible by 1000.
*1000 = 2^3 * 5^4*
6. Define a positive integer to be balanced if the number of its decimal digits equals the number of its distinct prime divisors. (For example, 12, 21, 105 are all balanced, but 25 and 210 are not.) Show that there are only finitely many balanced numbers.
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