

2009 Green Chicken Contest

1. Let $f(n) = n - 2$ if $n \geq 5000$ and $f(n) = f(f(n+7))$ if $n < 5000$. What is $f(2009)$?
2. Evaluate the following sum:
$$\sum_{n=1}^{\infty} \sin \frac{2\pi}{3^n} \sin \frac{\pi}{3^n}.$$
3. Find the maximum value of $f(x) = x^5 - 5x$ on the set of real values for which $x^4 + 4 \leq 5x^2$.
4. Let S be a set of 100 distinct real numbers. Let A_S be the set of numbers that occur as averages of two distinct elements of S .
 - (a) What is the smallest possible number of elements of A_S ? Construct an example that attains this minimum.
 - (b) What is the largest possible number of elements of A_S ? Construct an example that attains this maximum.
5. Amy, Bob, and Colin each shop for ice cream at a shop that offers ten choices. How many ways could they select the ice cream given that collectively all ten flavors are chosen and no single flavor is selected by all three of them? (For example, Amy might choose flavors numbered 1, 2, 3, 4, 5; Bob chooses flavors 1, 3, 5, 6, 7, 8 and Colin chooses flavors 2, 6, 9, 10.)
6. Let n be a positive integer. Choose any $(n+1)$ -element subset of $S = \{1, 2, \dots, 2n\}$. Show that this subset must contain two integers, one of which divides the other.