The Twenty Third Annual Green Chicken Contest

October 21, 2000(at Williams College)

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1. Suppose you start at the equator and travel continuously northwest. How long is your path? Roughly, what does your path look like? Assume that the earth is a round ball of radius 4000 miles.

2. You are told that there are two tables in a room. One of the tables is empty, the other has a large number of pennies on it, exactly 31 of which are heads. You are required to reposition the pennies using both tables so that each table has exactly the same number of heads showing. You must have all of the pennies on the tables.

You are blindfolded and cannot see the pennies, nor can you tell be feeling whether they are heads or tails. How should you proceed?

3. You have to get gas at one of the four stations on the way to work this morning. Your strategy is to observe the prices at the first k stations and then take the next station cheaper than all of them (or the final station if necessary). What value of k gives you the lowest expected (average) price? By how much?

Assume that the prices are p, p+1, p+2, and p+3 (not necessarily in that order).

4. Consider an exponential tower of 2000 5s:

$$5^{5^{5^{5^{\cdot^{\cdot^{\cdot^{5}}}}}}} \quad \text{(2000 5s)}$$

Note: $5^{5^{5}}$ means 5^{3125}, not 3125^{5}.

What is the remainder after dividing by 7?

5. a) What is the expected (average) length of the shorter piece of a meter stick with a random cut?
b) What is the expected length of the shortest of 3 pieces from 2 random cuts?

c) What is the expected length of the shortest of 4 pieces from 3 random cuts?
d) What is the expected length of the shortest of n pieces from n-1 random cuts?

6. Prove the isoperimetric theorem, that the round circle encloses the most area for a given perimeter or equivalently has the least perimeter for a given area. Or for partial credit, show that the best quadrilateral is a square, or that the best parallelogram is a square, or that the best rectangle is a square.