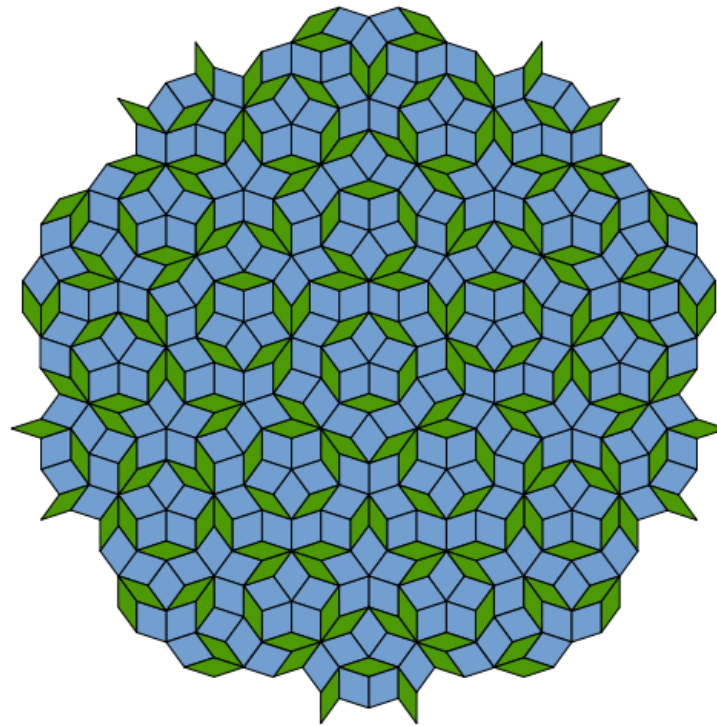# Up-down Generation

A Connection between Self-similar and Combinatorial Tilings

Holly Anderson

Vassar College

Background

- Up-down generation has been used primarily as a way to generate Penrose tilings.
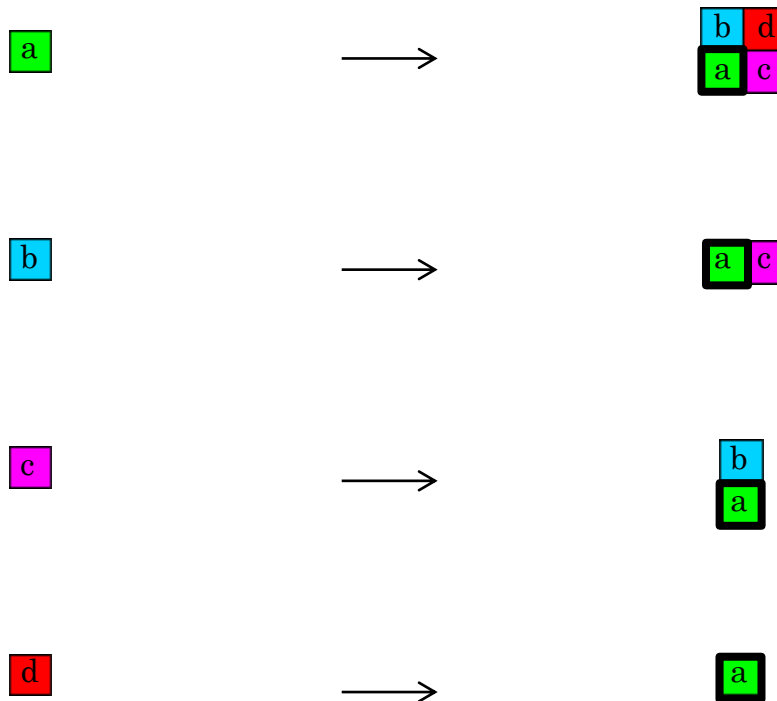

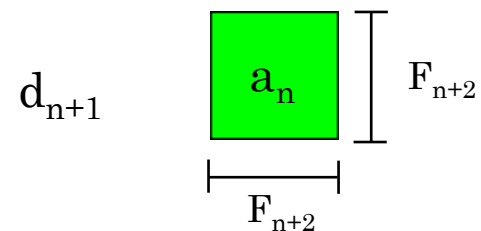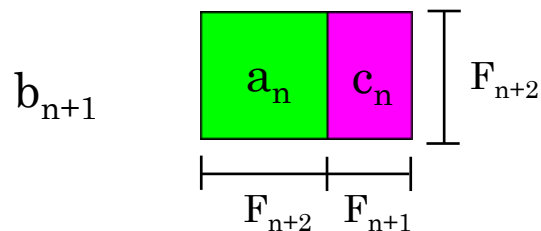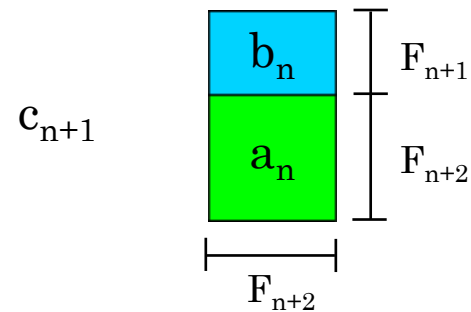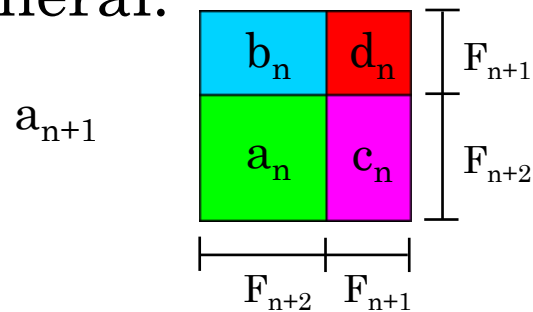- This process relies heavily on substitution rules.

FIBONACCI: COMBINATORIAL

- Basic idea: At each step, we need to choose where the tile that we have will "live" in the next level.
- Substitution rules:

a $\longrightarrow$ (tile with b, d on top; a, c on bottom)

b $\longrightarrow$ (tile with a, c)

c $\longrightarrow$ (tile with b on top, a on bottom)

d $\longrightarrow$ (tile with a)

FIBONACCI: COMBINATORIAL

- Basic idea: At each step, we need to choose where the tile that we have will "live" in the next level.
- Substitution rules:

FIBONACCI: COMBINATORIAL

- Basic idea: At each step, we need to choose where the tile that we have will "live" in the next level.
- Substitution rules:

a $\longrightarrow$
b	d
a	c

b $\longrightarrow$ a c

c $\longrightarrow$
b
a

d $\longrightarrow$ a
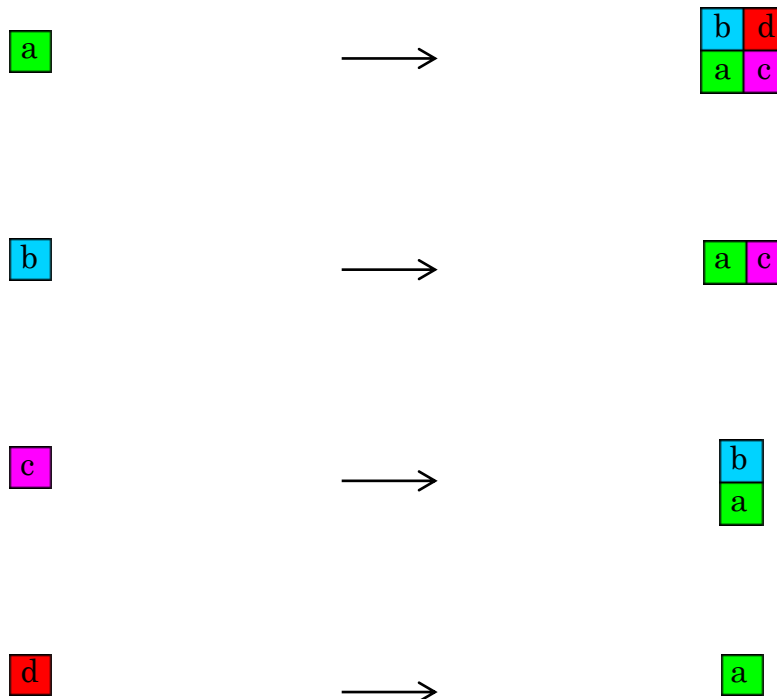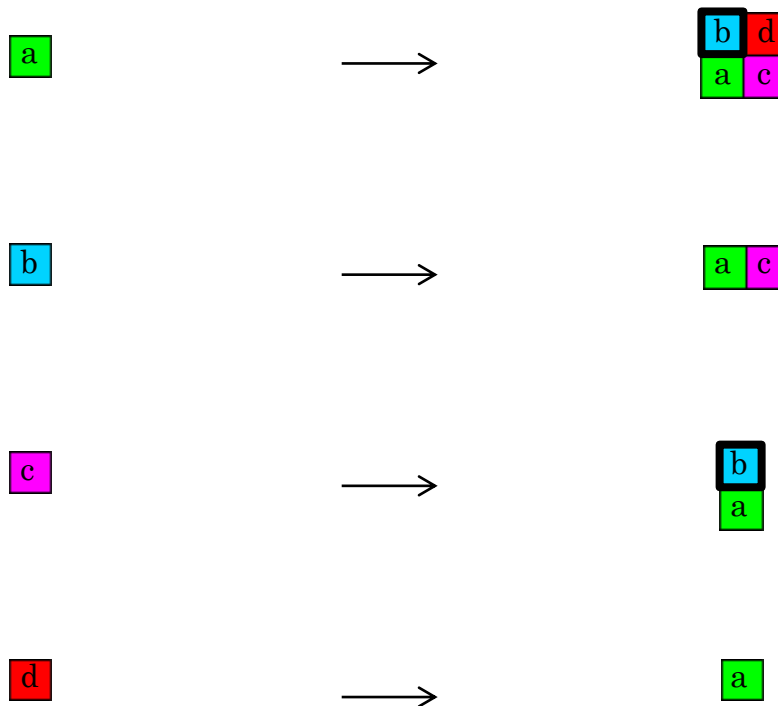
FIBONACCI: COMBINATORIAL

- Need to keep track of tile type and level.
- In general:

a_{n+1}

b_n	d_n	F_{n+1}
a_n	c_n	F_{n+2}

F_{n+2} F_{n+1}

c_{n+1}

b_n	F_{n+1}
a_n	F_{n+2}

F_{n+2}

b_{n+1}

a_n	c_n	F_{n+2}

F_{n+2} F_{n+1}

d_{n+1}

a_n	F_{n+2}

F_{n+2}

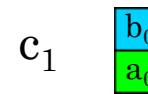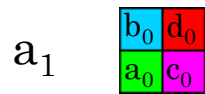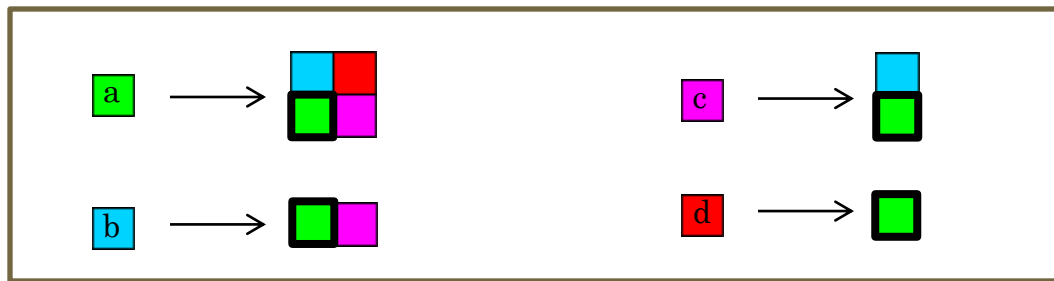| n	1	2	3	4	5	6	...
F_n	1	1	2	3	5	8	...

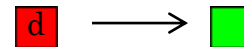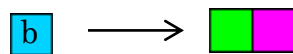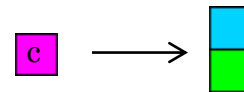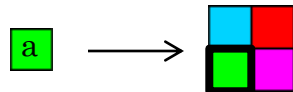# FIBONACCI: COMBINATORIAL
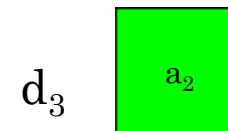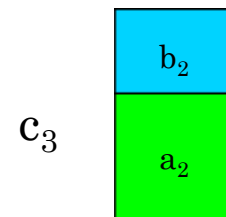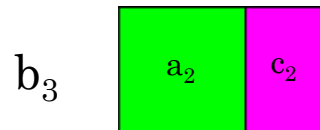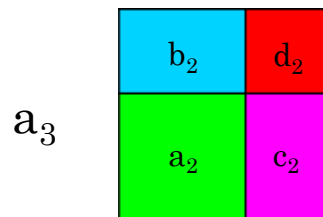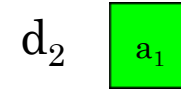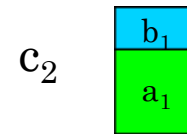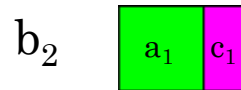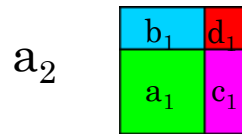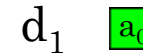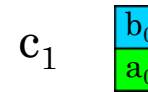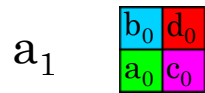
- Explicitly:

FIBONACCI: COMBINATORIAL

- Explicitly:
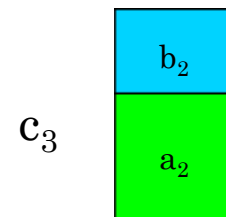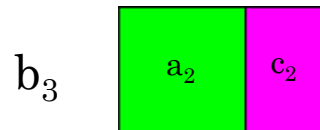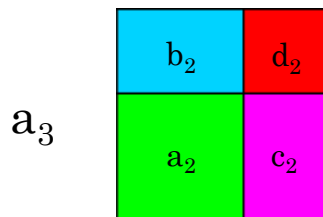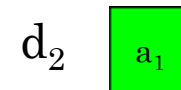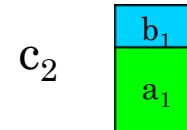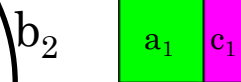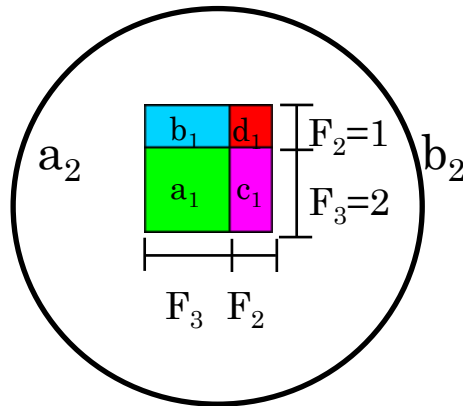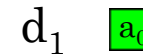
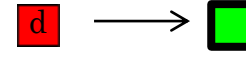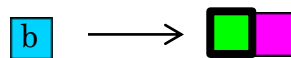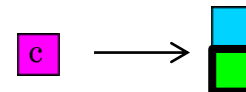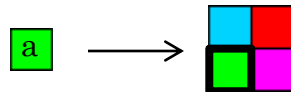# FIBONACCI: COMBINATORIAL

- Up: Level 0 Tile (a_0)

FIBONACCI: COMBINATORIAL
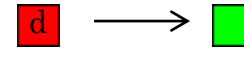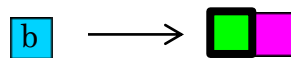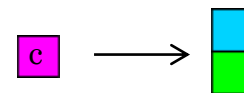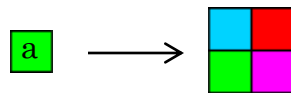
- Up: Level 1 Tile ($a_0 \longrightarrow a_1$)

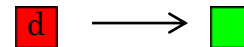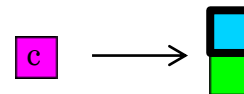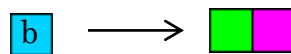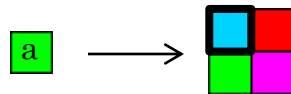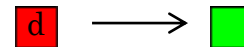# FIBONACCI: COMBINATORIAL

- Up: Level 1 Tile (a_1)

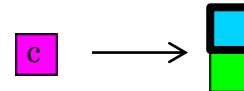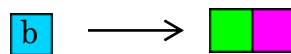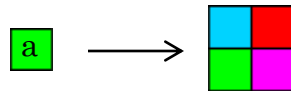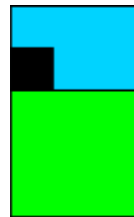# FIBONACCI: COMBINATORIAL

- Up: Level 2 Tile ($a_1 \longrightarrow b_2$)
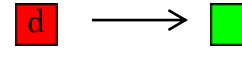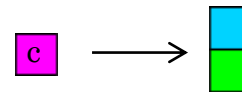
FIBONACCI: COMBINATORIAL
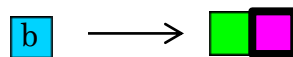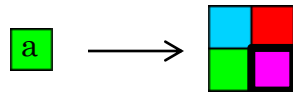
- Up: Level 2 Tile (b_2)

FIBONACCI: COMBINATORIAL

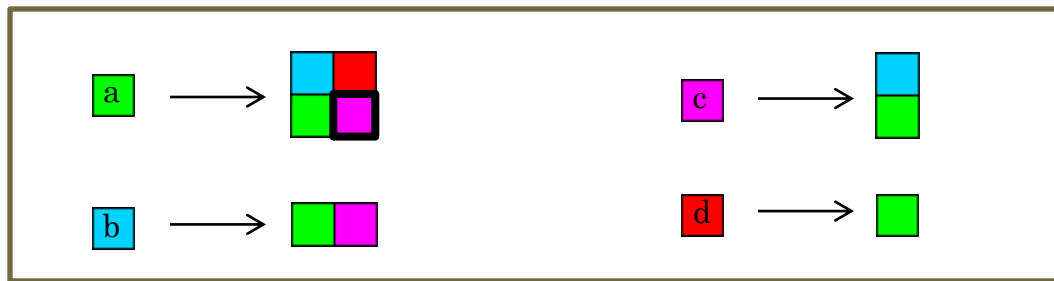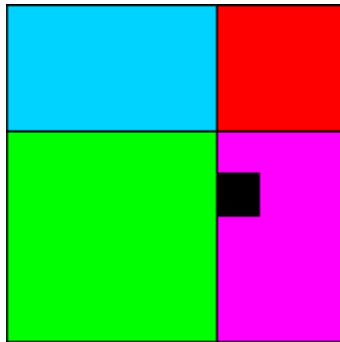- Up: Level 3 Tile ($b_2 \longrightarrow c_3$)

FIBONACCI: COMBINATORIAL
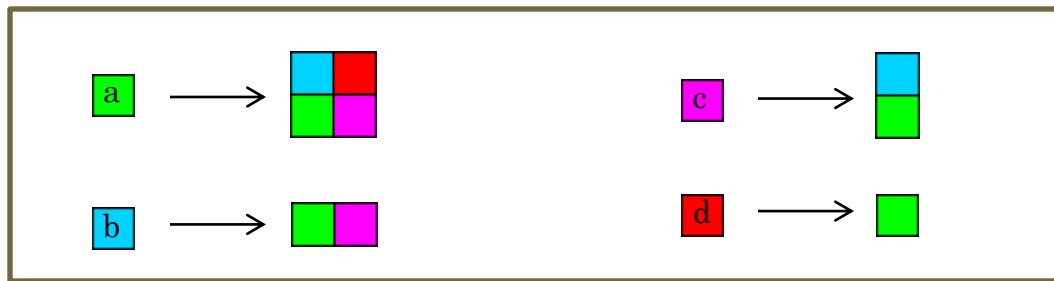
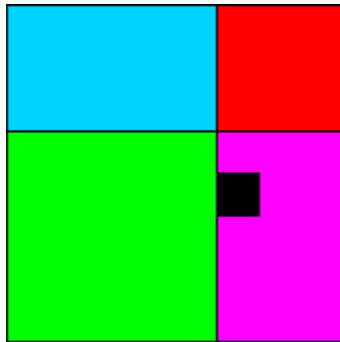- Up: Level 3 Tile (c_3)

FIBONACCI: COMBINATORIAL
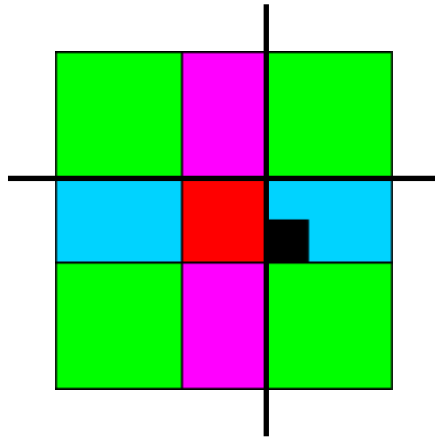
- Up: Level 4 Tile ($c_3 \longrightarrow a_4$)

FIBONACCI: COMBINATORIAL
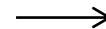
- Down: Level 3 Tiles

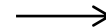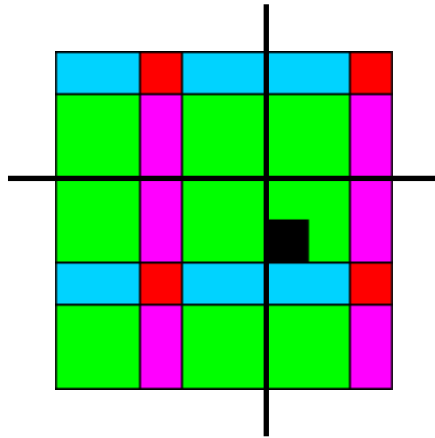# FIBONACCI: COMBINATORIAL
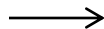
- Down: Level 2 Tiles

FIBONACCI: COMBINATORIAL

- Down: Level 1 Tiles

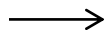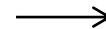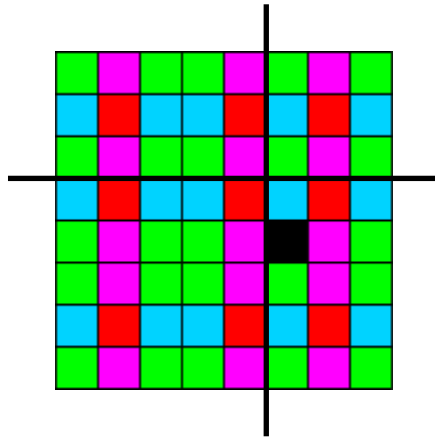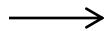# FIBONACCI: COMBINATORIAL

- Down: Level 0 Tiles
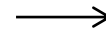
FIBONACCI: SELF-SIMILAR

- Rescaled tiles make the combinatorial tilings self-similar.
- Expansion constant: golden ratio (1.61803…)
- Substitution rules:
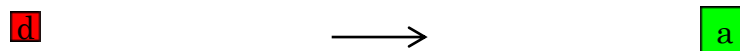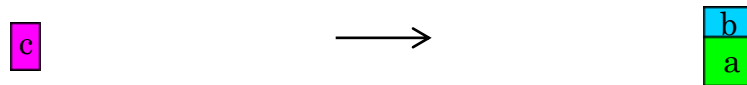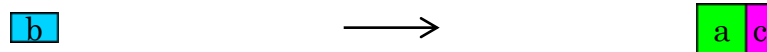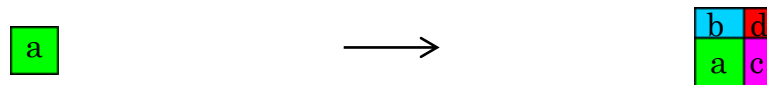
FIBONACCI: SELF-SIMILAR

- Explicitly:
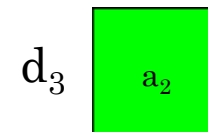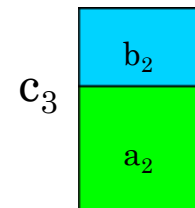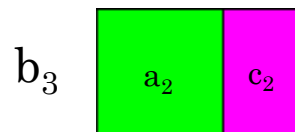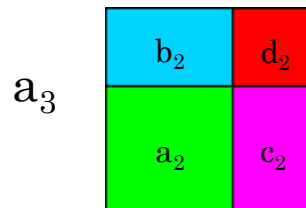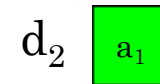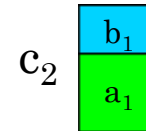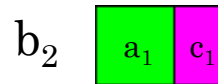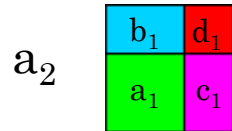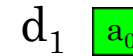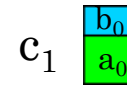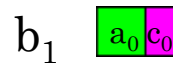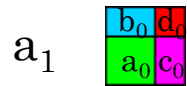
FIBONACCI: SELF-SIMILAR

- Up: Level 0 Tile (a_0)

FIBONACCI: SELF-SIMILAR

- Up: Level 1 Tile ($a_0 \longrightarrow a_1$)

FIBONACCI: SELF-SIMILAR

- Up: Level 1 Tile (a_1)

FIBONACCI: SELF-SIMILAR

- Up: Level 2 Tile ($a_1 \longrightarrow b_2$)

FIBONACCI: SELF-SIMILAR

- Up: Level 2 Tile (b_2)

FIBONACCI: SELF-SIMILAR

- Up: Level 3 Tile ($b_2 \longrightarrow c_3$)

FIBONACCI: SELF-SIMILAR

- Up: Level 3 Tile (c_3)

FIBONACCI: SELF-SIMILAR

- Up: Level 4 Tile ($c_3 \longrightarrow a_4$)

FIBONACCI: SELF-SIMILAR

- Down: Level 3 Tiles
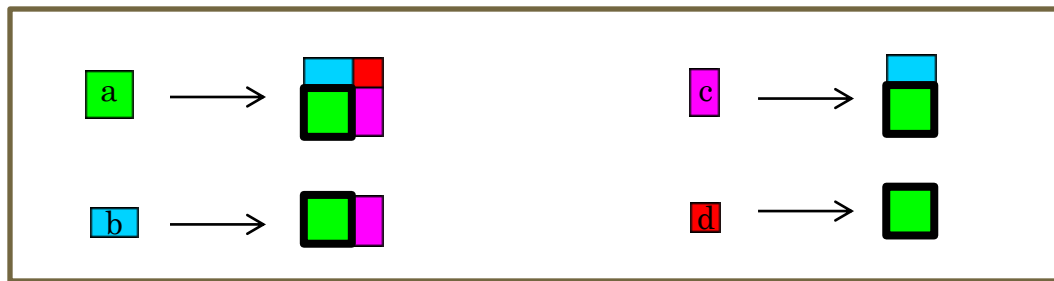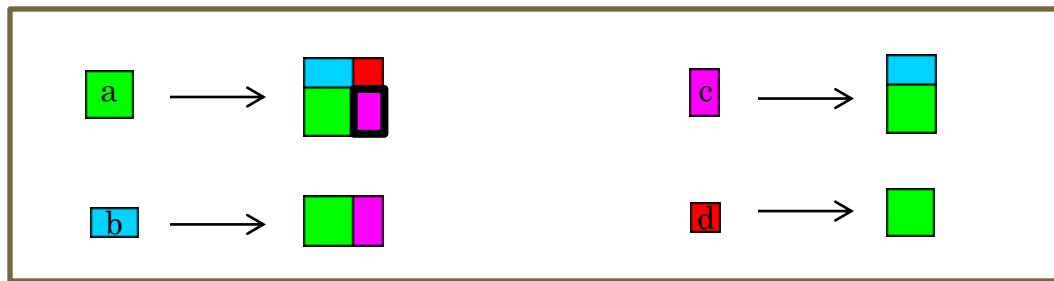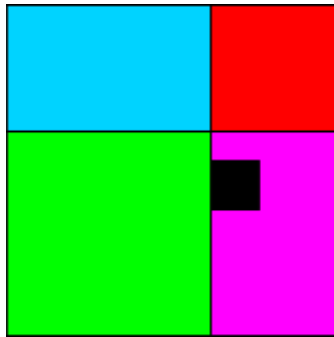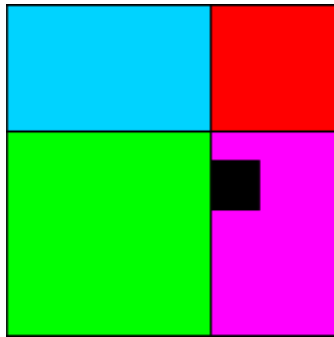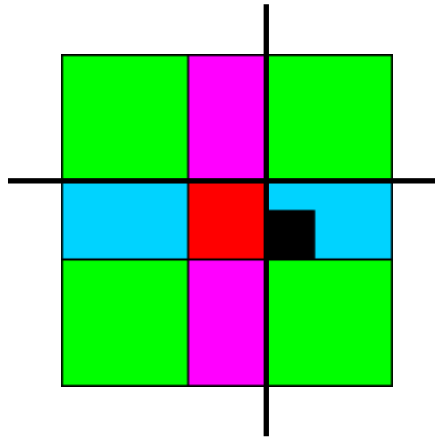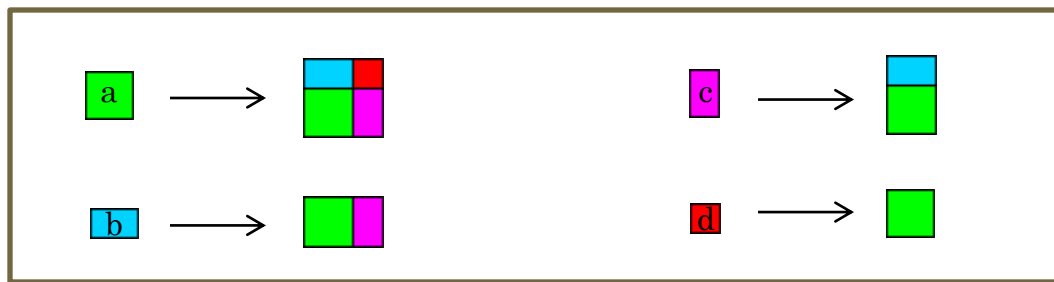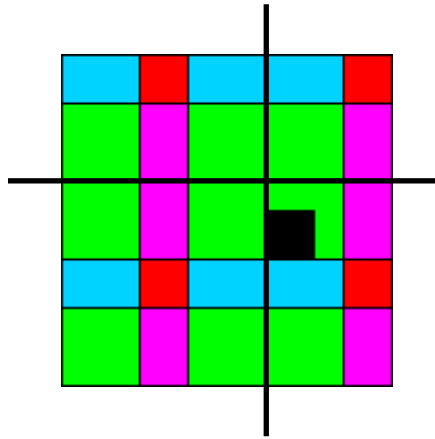
FIBONACCI: SELF-SIMILAR

- Down: Level 2 Tiles

FIBONACCI: SELF-SIMILAR
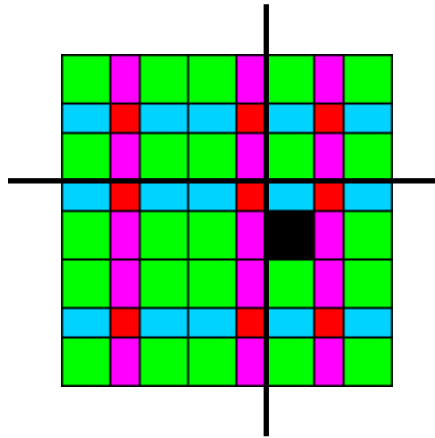
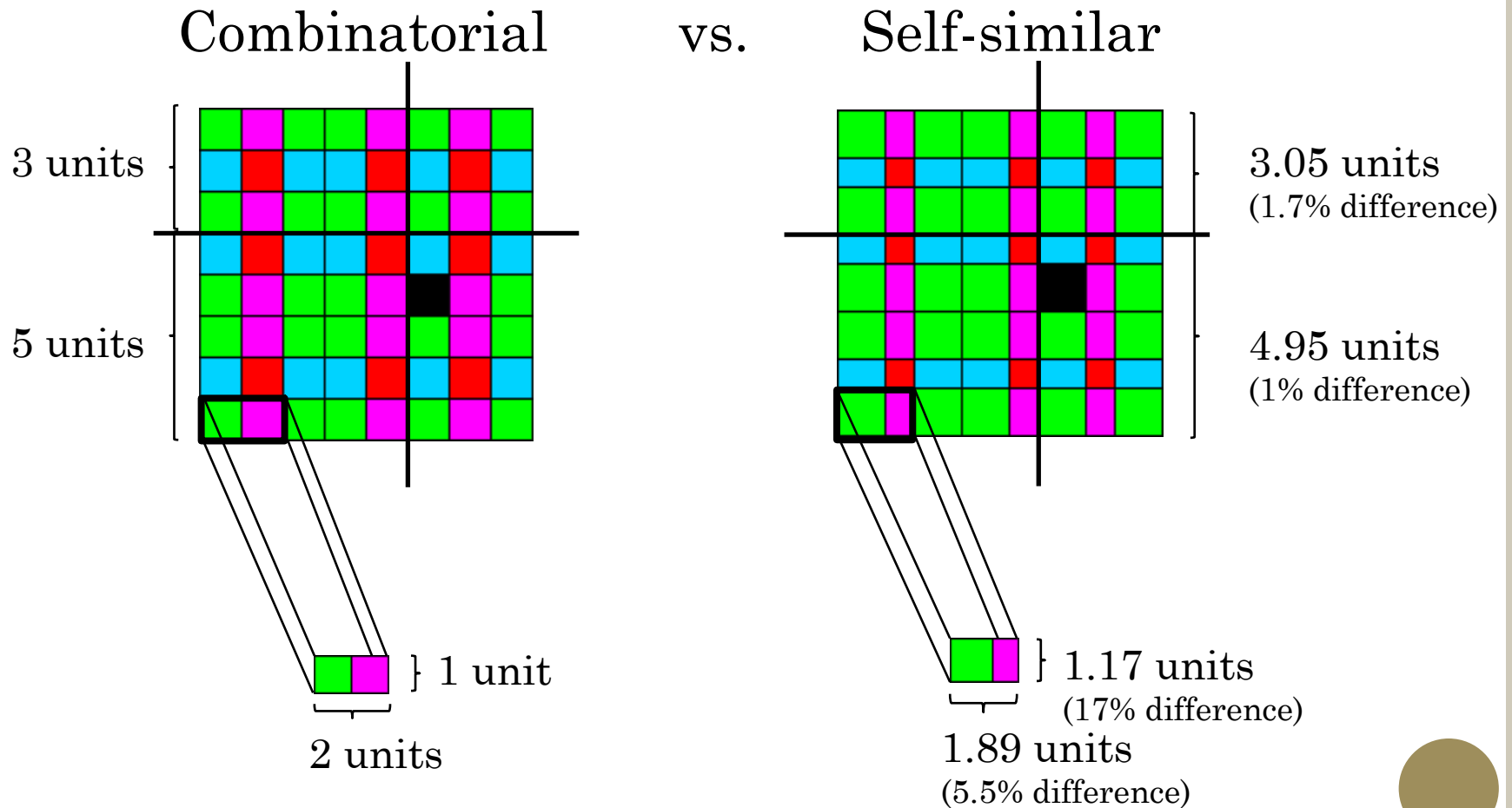- Down: Level 1 Tiles

FIBONACCI: SELF-SIMILAR

- Down: Level 0 Tiles

FIBONACCI: COMPARISON

Combinatorial   vs.   Self-similar

3 units

5 units

3.05 units
(1.7% difference)

4.95 units
(1% difference)

} 1 unit

2 units

} 1.17 units
(17% difference)

1.89 units
(5.5% difference)

Uncountably Many Tilings

- Fibonacci DP
  - a tiles can be placed in 4 locations in the next level
  - b and c tiles can be placed in 2 locations each
  - d tiles can be placed in 1 location